Assignment 2

Let N be an infinite countable set. Let K be the abstract simplicial complex consisting of all finite subsets of N. The collection of all *finite* subsets of a countable set is countable so K has only countably many simplicies. Since a finite complex is a finite union of simplicies, K also has only countably many finite subcomplexes.

On the other hand, any finite complex L is simplicially isomorphic to a finite subcomplex of K: just embed the vertices of L in N and observe that this embedding provides a simplicial isomorphism between L and its image as a subcomplex of K. This proves that the collection of finite simplicial complexes is at most countable. The 0–dimensional simplicial complexes, L_n, where L_n has n vertices, is a countably infinite collection of distinct complexes, so the collection of finite simplicial complexes is exactly countably infinite.