Syllabus for Math. 609
Fall 2002
Professor Julia Knight

Math. 609-610 is divided into three parts, model theory, computability theory, and set theory. For Fall, 2002, model theory came first, followed by the first part of computability. In model theory, the usual material includes some propositional logic, and then focuses on predicate logic. There are the basic Completeness, Compactness, and Löwenheim-Skolem-Tarski Theorems. There are examples of theories with different numbers of countable models, including the Ehrenfeucht examples and something involving quantifier elimination. We do the Omitting Types Theorem, and the Ryll-Nardjewski Theorem. There is a discussion of prime, atomic, and saturated models. The final goal is the theorem of Vaught saying that there is no countable complete theory with exactly two countable models, up to isomorphism.

The first part of computability includes Turing machines, primitive recursive functions, partial computable functions, Kleene normal form, and a sketch of the proof of equivalence of Turing computable and partial computable functions. We discuss computable and computably enumerable sets. There are some interesting examples such as Ackermann's function, the busy-beaver function, and the halting set. There is a discussion of many-one reducibility and one-one reducibility, the Myhill Isomorphism Theorem, and productive and creative sets. Adding the existence of a simple set, we get the fact that among c.e. sets, there is one which is neither computable nor 1-complete. We describe oracle machines and relative computability, Turing reducibility, Turing degrees, the jump function on sets, and on degrees, and the Kleene-Post Theorem.

NOTE: *This course was tailored for one student.*

For Fall, 2002, there was only one student in Math. 609. Because this student was already familiar with some of the material, it was possible to cover the basic material quickly, and add supplementary material. Thus, the course included Morley's Categoricity Theorem, plus some notes of Marker on model theory of fields. Instead of a mid-term, the student (by his own choice) gave several lectures on Marker's notes. There was regular homework. The final will be of the usual kind—written, covering just the basic material—but with problems that are more challenging than usual.