Final Exam

Math 610

Prove the statements below. Give complete proofs and write legibly. You may use any non-human reference, but collaboration with other humans is not permitted. If you have questions, you may reach me at 233-4111 (home) or 631-8849 (office). Return the test to me or to the secretary Patti by 5PM, Friday, May 9.

1. $\mathrm{ZFC} \vdash \mathrm{GCH} \leftrightarrow \forall \kappa \geq \omega\, (2^{<\kappa} = \kappa^+)$.

2. Let $\kappa = \aleph_\omega$. Note that κ is a singular cardinal. There is a function $f : \kappa \to \kappa$ such that $\forall \gamma > 0\, (f(\gamma) < \gamma)$, but for all $\alpha < \kappa$, $f^{-1}(\{\alpha\})$ is not stationary.

3. Let $\mathbf{M} \subseteq \mathbf{V} \models \mathrm{ZF}$ be a transitive class which is a model of the Comprehension Axiom Schema. Suppose furthermore that
$$\mathbf{V} \models \forall x \subseteq \mathbf{M}\, \exists y \in \mathbf{M}\, (x \subseteq y).$$
Then $\mathbf{M}$ is a model of ZF.

4. Let $\kappa > \omega$ be a regular cardinal and $f : \kappa \to \kappa$ a strictly increasing continuous function. The set $\{\alpha : f(\alpha) = \alpha\}$ of fixed points of f is a club in κ.