Compendium of Absolute Notions

Math 609

Corollary 3.6. If $\mathbf{M}$ is transitive, then every Δ_0 formula is absolute for $\mathbf{M}$.

Theorem 3.9. If $\mathbf{M} \models \mathrm{ZF}^- - \mathrm{P} - \mathrm{Inf}$ is transitive, then the following notions are absolute for $\mathbf{M}$.

$x \in y$	$< x, y >$	$S(x)$	z is an ordered pair	R is a function
$x = y$	0	x is transitive	$A \times B$	$R(x)$
$x \subseteq y$	$x \cup y$	$\bigcup x$	R is a relation	R is an injection
$\{x, y\}$	$x \cap y$	$\bigcap x$	$\mathrm{dom}(R)$	R is a surjection
$\{x\}$	$x \setminus y$		$\mathrm{ran}(R)$	R is a bijection.

Theorems 5.1, 5.3, 5.4, 5.5 and 5.7. Let $\mathbf{M} \models \mathrm{ZF}^- - \mathrm{P}$ be transitive.
The following notions are absolute for $\mathbf{M}$.

x is an ordinal	0
x is a limit ordinal	1
x is a successor ordinal	2
x is a finite ordinal	3
ω	4.

x is finite	R well-orders A	$\alpha + 1$	α^β
A^n	$\mathrm{type}(A, R)$	$\alpha - 1$	$\mathrm{rank}(x)$
$A^{<\omega}$		$\alpha + \beta$	$\mathrm{tr\ cl}(x)$
		$\alpha \cdot \beta$	