Peano's Axioms for Arithmetic
Math 609

The language for arithmetic is $\mathcal{L} = (+, \cdot, S, 0)$ where $+$ and $\cdot$ are binary function symbols, S is a unary function symbol and 0 is a constant symbol. By PA (Peano Arithmetic), we mean the following set of axioms.

1. $\neg(S(x) \doteq 0)$.

2. $S(x) \doteq S(y) \rightarrow x \doteq y$.

3. $x + 0 \doteq x$.

4. $x + S(y) \doteq S(x + y)$.

5. $x \cdot 0 \doteq 0$.

6. $x \cdot S(y) \doteq (x \cdot y) + x$.

7_φ. For each formula $\varphi(x_0, x_1, \ldots, x_n)$ where x_0 does not occur bound in φ, the axiom

$$[\varphi(0, x_1, \ldots, x_n) \wedge \forall x_0\, (\varphi(x_0, x_1, \ldots, x_n) \rightarrow \varphi(S(x_0), x_1, \ldots, x_n))] \rightarrow \forall x_0\; \varphi(x_0, x_1, \ldots, x_n).$$