Midterm Exam
Math 610
(Recursion Theory)

You have one hour to complete this exam. Answer the following questions as clearly and completely as possible. You may use your notes and Soare as references.

1. Let A, $B \subseteq \omega$. Prove that $A \leq_1 B$ implies $A \leq_m B$ and that $A \leq_m B$ implies $A \leq_T B$. Show that if $A \leq_m B$ and B is recursive (respectively, recursively enumerable) then so is A.
2. Let $\mathrm{Tot} = \{e : \varphi_e \text{ is total}\}$ be the index set of the recursive functions. Prove using a diagonalization argument that Tot is not recursive. Give a Π_2 definition of Tot.
3. Show that an infinite recursively enumerable subset $A \subseteq \omega$ is recursive if and only if it is the range of an increasing recursive function.
4. Let $W_e = \operatorname{rng} \varphi_e$ be an infinite recursively enumerable set. Prove there exists a recursive subset $A_e \subseteq \omega$ such that φ_e restricted to A_e is 1:1 and onto W_e.
5. Let W be an infinite recursively enumerable set. By the two previous exercises, $W = \operatorname{rng} f$ where f is a 1:1 recursive function. Let $B = \{s : (\exists t > s)[f(t) < f(s)]\}$ be the *deficiency* set of W. Show that if W is not recursive, then $\bar{W}$ is infinite.
6. Denote by $\mathcal{E}$ the lattice of the recursively enumerable sets under inclusion. Verify that the set $\mathcal{F} := \{A \subseteq \omega : A \text{ is simple or cofinite}\}$ forms a filter in $\mathcal{E}$.