The Zermelo-Fraenkel Axioms for Set Theory
Math 609

The language for set theory is the language $\mathcal{L} = (\in)$ with a single binary function symbol. By ZF (Zermelo-Fraenkel) Set Theory, we mean the set of axioms (or axiom schemata) 1 - 9 below. By Z (Zermelo) Set Theory is meant the set of axioms 1 - 7 and the axiom schema 8. By ZFC (ZF with Choice) Set Theory, we mean the following list of axioms.

1. Extensionality. $\forall x, y\, (x \doteq y \leftrightarrow \forall z\, (z \in x \leftrightarrow z \in y))$.

2. Null set. $\exists x\, \forall y (\neg y \in x)$.

3. Pairs. $\forall x, y\, \exists z\, \forall u (u \in z \leftrightarrow u \doteq x \vee u \doteq y)$.

4. Unions. $\forall x\, \exists y\, \forall z (z \in y \leftrightarrow \exists w\, (z \in w \wedge w \in y))$.

5. Power set. $\forall x\, \exists y\, \forall z (z \in y \leftrightarrow \forall w\, (w \in z \rightarrow w \in x))$.

6. Infinity. $\exists x\, [\exists y\, ((y \in x) \wedge \exists z\, (y \in z \wedge z \in x))]$.

7. Regularity. $\forall x\, [\exists y\, (y \in x \wedge \neg \exists z\, (z \in y \wedge z \in x))]$.

8_φ. Definable subsets. $\forall x\, \exists y\, \forall z (z \in y \leftrightarrow z \in x \wedge \varphi(z, x_1, \ldots, x_n))$, where φ is a formula in which y does not occur.

9_φ. Collection. $\forall x\, [x \in u \rightarrow \exists z\, \varphi(x, z, u, x_1, \ldots, x_n)] \rightarrow \exists y\, \forall x\, [x \in u \rightarrow \exists z\, (z \in y \wedge \varphi(x, z, u, x_1, \ldots, x_n))]$, where φ is a formula in which y does not occur.

10. Choice. $\forall x\, \exists y\, [(y \text{ is a function with domain } x) \wedge \forall z\, (z \in x \wedge \exists u\, u \in z \rightarrow y(z) \in z)]$.