Math 621: Topics in Algebraic Geometry

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1 Course Content

I plan to build this course around the Castelnuovo-Mumford regularity conjecture of Gruson, Lazarsfeld and Peskine [1], which states that if $X \subset \mathbb{P}^r$ is a smooth non-degenerate variety of dimension n and degree d, then X is k-regular for all $k \geq d - (r - n) + 1$. I expect to spend at least the first four weeks covering basic elements of algebraic geometry (relevant parts of chapters 2,3 and 4 in [2] with examples) and in particular to get a good handle on the statement of the conjecture. Once we have developed some tools, we will go through the proof of the conjecture for the case $n = 1$ (curves) [1] and $n = 2$ (surfaces) [5]. If time permits, we will also look at the recent work of Sijong Kwak [3, 4] for threefolds.

References

[1] L. Gruson, R. Lazarsfeld and C. Peskine, On a theorem of Castelnuovo and the equations defining projective varieties, Inv. Math. 72 (1983) 491-506.

[2] R. Hartshorne, Algebraic Geometry GTM 52, Springer-Verlag, Berlin, Heidelberg and New York, 1977.

[3] S. Kwak, Castelnuovo regularity for smooth subvarieties of dimensions 3 and 4, J. Alg. Geom. 7 (1998) 195-206.

[4] S. Kwak, Castelnuovo-Mumford regularity bound for smooth threefolds in $\mathbb{P}^5$ and extremal examples, J. reine angew. Math. 509 (1999) 21-34.