Introduction to Algebraic Geometry

Math 621

Hurley 168, MWF 11:45 - 12:35 (subject to change)

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Classical algebraic geometry is the study of sets (called varieties) of solutions of systems of polynomial equations. The nature of algebraic geometric inquiry is to relate the geometric features of a variety to the algebraic properties of the polynomials describing it. In modern algebraic geometry, the notion of variety is generalized to that of a scheme, whereby an arbitrary commutative ring may be associated with a "geometric" object.

In addition to its intrinsic appeal as a subject of study, algebraic geometry is closely related to commutative algebra, complex analysis, and number theory, and has applications to PDE, mathematical physics, topology, geometric optimization, and engineering.

We shall introduce basic notions of classical algebraic geometry. Our main objects of consideration will be varieties in affine or projective space.

- Basic objects: a variety and its coordinate ring
- Examples: rational normal curves, Veronese and Segre varieties, Grassmannians, flag varieties
- attributes of varieties, such as dimension, degree, smoothness, irreducibility
- classical constructions and techniques
- birational maps, blow-ups, resolution of singularities
- Bertini theorems
- divisors, linear systems, intersection theory, Riemann-Roch theorems

Prerequisite:

A graduate course in algebra and/or the candidacy syllabus for algebra is strongly recommended as background.

Specifically, it is essential to have some familiarity with the following notions: rings, homomorphisms, ideals; factorization, PID's, UFD's; prime ideals, local rings, localization; polynomial rings, formal power series rings, Hilbert's basis theorem; modules, exact sequences, tensor products; fields, algebraic extensions, transcendence bases; Noetherian rings and modules.

Also, it is useful to have acquaintance with integral ring extensions, Noether's normalization lemma, and Hilbert's Nullstellensatz.

A reference for these topics is: Hungerford, *Algebra*, Chapters III, IV, V, VI, and VIII.

Main reference:

Shafarevich, *Basic algebraic geometry 1*

Additional references:

Eisenbud, *Commutative algebra with a view toward algebraic geometry*

Harris, *Algebraic geometry*

Hartshorne, *Algebraic geometry*