Syllabus for Mathematics 517
Foundations of Computational Mathematics
CALL Number 5681

Instructor: Andrew Sommese

MWF at 10:40-11:30

The course is a solid theoretical introduction to numerical analysis. Topics covered will include:

1. Polynomial interpolation (including generalized Hermite interpolation); two dimensional interpolation; splines; trigonometric interpolation
2. least squares and the basic theory of orthogonal functions
3. numerical integration in one variable, including adaptive methods; Romberg integration; Gauss quadrature and the relations to orthogonal functions
4. numerical linear algebra
   (a) direct methods and the analysis of error based on the condition number;
   (b) basic numerical factorizations of matrices and the singular value decomposition;
   (c) iterative methods, e.g.,the Jacobi method and the method of successive over-relaxation; and
   (d) methods to find eigenvalues and eigenvectors such as the QR Method, power method, and inverse power method.
5. methods to solve systems of nonlinear equations, e.g., Newton like methods and constrained Newton's methods such as homotopy continuation;
6. numerical solution of ordinary differential equations by marching methods, multistep methods, finite differences, and the finite element method (including such variants as the Galerkin and Rayleigh-Ritz method);
7. solution of some simple partial differential equations by difference methods; and by the finite element method.

We will devote at least a week to an extended discussion of what is known about the numerical solution of systems of polynomials in several variables.

References

[1] P.J. Davis, *Interpolation and approximation*, Dover, 1975.

[2] C. De Boor, *A practical guide to splines*, Springer, 1978. .

[3] E. Isaacson and H.B. Keller, *Analysis of numerical methods*, Dover Publications, 1994.

[4] A. Iserles, *A first course in the numerical analysis of differential equations*, Cambridge University Press, 1996.

[5] C. Johnson, *Numerical solution of partial differential equations by the finite element method*, Cambridge University Press, 1987.

[6] A.P. Morgan, *Solving polynomial systems using continuation for engineering and scientific problems*, (1987) Prentice-Hall, Englewood Cliffs, New Jersey.

[7] J. Stoer and R. Bulirsch, *Introduction to numerical analysis*, 2nd Edition, Texts in Applied Mathematics 12, Springer-Verlag, 1993, New York.

[8] E.E. Tyrtyshnikov, *A brief introduction to numerical analysis*, Birkhäuser, 1997.