Math 622: Numerical Algebraic Geometry

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Time: MWF 10:40-11:30

Algebraic geometry is an ancient subject, with intimate connections to science, engineering, and the rest of mathematics. One original motivation for the subject is the explicit solution of systems of polynomials - often motivated by engineering. In the last few years powerful numerical analysis techniques, based on ideas from algebraic geometry, have been developed to numerically solve polynomial systems and manipulate the possibly positive dimensional solution sets of such systems.

This course will cover those parts of algebraic geometry that have been useful in the recent numerical work, plus the new algorithms for numerically solving systems of polynomials. Polynomial systems from engineering are often sparse, and possess special structure, that efficient algorithms take advantage of.

Though, previous knowledge of algebraic geometry will be useful, this introductory course does not assume it. There will be an emphasis on the concrete description of algebraic geometric objects. For the first part of the course, we will follow D. Mumford's "Algebraic geometry I : complex projective varieties," Grundlehren Math. Wiss. 221, Springer-Verlag, New York, (1976). Ideas from numerical analysis and several complex variables will be developed as needed.

Specifically the will course cover the following topics and material around the topics.

1. Basic correspondence, including the Nullstellenzatz, between affine and projective algebraic sets and ideals;
2. the irreducible/primary decomposition;
3. local theory of algebraic sets;
4. sheaf cohomology and coherent algebraic sets; projection formula;
5. basic examples such as ruled surfaces and their very ample line bundles;
6. numerical solution of polynomial systems; and
7. numerical manipulation of irreducible algebraic sets.