

# Math648: Topics in Differential Geometry

## Spring 2002

**TIME:** MWF: 12:50 - 1:40.

**INSTRUCTOR:** Xiaobo Liu.

**COURSE DESCRIPTION:**

*Calibrated geometry* studies distinguished submanifolds of a Riemannian manifold whose tangent spaces realizes maximum values of some closed differential forms. Such submanifolds necessarily have the smallest volume among all submanifolds in the same homology classes (In particular, they are always minimal submanifolds in the sense that mean curvature is constantly equal to zero). Well known examples include all complex submanifolds of Kähler manifolds, special Lagrangian submanifolds of Calabi-Yau manifolds, associative cycles and Cayley cycles in manifolds with  $G_2$  or  $Spin(7)$  holonomy. Recently, calibrated submanifolds have attracted lots of attention of physicists because they provide best candidates for supersymmetric cycles in string theory. Special Lagrangian submanifolds also get lots of popularity in mirror symmetry due to a well known conjecture by Strominger-Yau-Zaslow which predicts that the mirror of a Calabi-Yau manifold should be obtained by dualizing some special Lagrangian submanifolds which are diffeomorphic to tori. In this course we will first study basic theories of calibrated geometry and related materials. If time permits, we will also study some recent papers in this field, especially about special Lagrangian geometry.

**REFERENCE:**

1. Reese Harvey, *Spinors and Calibrations*, Academic Press, Inc., 1990.
2. Reese Harvey, and B. Lawson, *Calibrated Geometries*, Acta Math. 148 (1982) 47-157.