Math 651, Topics in Algebra,
Fall 2001
Coxeter Groups
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Coxeter groups are an interesting and important family of discrete reflection groups.

The finite groups generated by reflections in a real Euclidean space are exactly the finite Coxeter groups; they include thedihedral groups and more generally the symmetry groups of regular polytopes such as the Platonic solids. Important subclasses of Coxeter groups are the finite Weyl groups (the finite Coxeter groups which stabilize a ``root lattice'' in their reflection representation) and affine Weyl groups(the semidirect product of a finite Weyl group with its root lattice); these arise naturally in many areas of Lie theory, such as in the study of algebraic groups, Lie groups and Lie algebras, quantum groups and finite groups of Lie type. More general ``crystallographic'' Coxeter groups appear in the study of Kac-Moody Lie algebras and groups.In many of these situations, the Iwahori-Hecke algebra of the Coxeter group(an associative algebras arising as aspecial deformation of the group algebra) also plays a fundamental role; for example, answers to important questions such as composition factor multiplicities of Verma modules and dimensions of local intersection cohomology spaces of Schubert varieties are expressed in terms of the coefficients of certain polynomials(Kazhdan-Lusztig polynomials) which arise as entries of the change of basis matrix between two natural bases of the Iwahori-Hecke algebra.The study of Kazhdan-Lusztig polynomials and their applications is the focus of much current research activityand there are many interesting and important open problems in this area(for example, it is only in special cases where geometric or representation-theoretic interpretations such as the above are known that their coefficients have been proved to be non-negative, as is conjectured in general).

In this course, we shall develop the basic theory of Coxeter groups and then treat some deeper algebraic and combinatorial topics related to Kazhdan-Lusztig polynomials.We hope to cover

1) Coxeter groups, reflection subgroups, root systems

2) classification offinite Coxeter groups and finite and affine Weyl groups

3) BN-pairs

4) invariant theory of Coxeter groups

5) Chevalley-Bruhat order

6) Iwahori-Hecke algebra, Kazhdan-Lusztig polynomials, cells

7) Highest weight representation categories associated to Chevalley-Bruhat intervals and applications to the Iwahori-Hecke algebra\noindent

These topics will be treated in various levels of detail (some possibly just surveyed) as time permits given our intention to reach the material in 7).There is no prerequisite for this course other than a thorough knowledge of basicalgebra and rudimentary point-set topology; basic material fromLie theory, commutative andhomological algebra, representation theory, combinatorial topology, category theory etc will be explained as needed. However, for motivation and to provide indications of some of the applications of the subject matter, more advanced ideas will sometimes be mentioned.

Text: Humphreys; Reflection groups and Coxeter groups (treats 1--2, 4--

Reference: Bourbaki: Group\`{e}s et algebres de Lie Ch 4--6 (classic reference on 1--4).|