LIE ALGEBRAS AND THEIR APPLICATIONS
652 - Professor Billig - Spring 1995
Department of Mathematics

This will be a rather informal course on Lie algebras and their applications. It is oriented to graduate students whose area of specialization is outside of Lie algebras but who are interested in their applications and want to get an insight on the general methods of Lie theory.

The first part of the course is an introduction to Lie algebras and their representations. We shall consider here as many examples of Lie algebras as possible to provide a good feeling for these objects. We shall discuss the structure of nilpotent, solvable and free Lie algebras and prove the Poincaré- Birkhoff- Witt theorem.

The second part of the course is devoted to the applications of Lie theory to differential equations. Many differential equations possess continuous groups (Lie groups) of symmetries. The knowledge of these symmetries allows one to produce new solutions from the known ones. Also, for ordinary differential equations it makes it possible to reduce the order of the equation. It turns out that instead of working with the symmetry groups it is much easier to work with their Lie algebras. We shall discuss an effective procedure of computing such a Lie algebra of "infinitesimal" symmetries for a given differential equation. The general correspondence between Lie groups and Lie algebras will be studied.

In the last part of the course we shall review the theory of finite-dimensional simple complex Lie algebras (root systems, Weyl group, Killing form etc.) and their representations which are extremely important for quantum theory. We shall also give an introduction to the theory of Kac-Moody algebras.

[1] J.-P. Serre, Lie algebras and Lie groups

[2] N. Jacobson, Lie algebras

[3] N. Bourbaki, Groupes et algébres de Lie, Chapitre 1

[4] P. Olver, Applications of Lie groups to differential equations

[5] V. Kac, Infinite dimensional Lie algebras