MATH 655, Topics in Complex Analysis
Fall 2001, Mei-Chi Shaw

In this course we will discuss several main topics in several complex variables and complex differential geometry using partial differential equations. It is intended as an introductory course for students who are interested in partial differential equations arising from problems in several complex variables and complex geometry. We will start from the beginning with no assumption of previous knowledge other than familiarity with the basic real analysis and one complex variable. Our first goal is to study the Cauchy-Riemann equations and tangential Cauchy Riemann equations in the simplest setting, i.e., $\mathbb{C}^n$. Several different methods will be introduced to solve these equations, including the L^2 technique, the $\bar{\partial}$-Neumann problem and the integral representation method. Applications will also be given, including function theory in pseudoconvex domains, regularity of the Bergman projection up to the boundary. Recent progress on Lipschitz domains and the irregularity of the Bergman projection on the so-called worm domains will also be discussed.

The second half of the semester will be concentrated on complex geometry. Here the curvature of the manifolds will come into play. The topics include

1. Harmonic functions on Riemannian manifolds.

2. Stein and Kähler manifolds, curvature of Kähler manifolds, Chern classes.

3. Bergman metrics, Kähler-Einstein metrics and the Calabi conjecture.

4. Cauchy-Riemann equations in complex manifolds.

References: 1. "Partial Differential Equations in Several Complex Variables"

So-Chin Chen and Mei-Chi Shaw, AMS/IP Studies in Advanced Mathematics, volume 19, American Mathematical Society, Providence, RI; International Press, Boston, MA, 2001. 2. "Canonical Metrics in Kähler Geometry"
Gang Tian, Lectures in Mathematics, Birkhäuser Verlag, Basel, Boston, Berlin. 2000. 3. "Complex Differential Geometry"
Fangyang Zheng, AMS/IP Studies in Advanced Mathematics, volume 18. American Mathematical Society, Providence, RI; International Press, Boston, MA, 2000. **Prerequisite**: Basic real analysis and one complex variable.