Symplectic Manifolds, Seiberg-Witten Gauge Theory, and Taubes Minimal Conjecture
Mathematics 658, Professor Connolly
MWF 10:40am-11:30am Room 326, CCMB
First meeting: Wednesday, January 15
DESCRIPTION OF THE COURSE

(This course will presume no previous knowlege of Gauge Theory. However a knowlege of vector bundles, transversality for smooth manifolds, algebraic topology, Riemannian metrics and differential forms will be assumed).

An oriented smooth compact 4-manifold X^4, is *irreducible* if it admits no connected sum decomposition, $X = Y\#Z$, except when Y or Z is homeomorphic to S^4. X is *minimal* if X admits no decomposition, $X = Y\#\overline{CP^2}$. Finally X is symplectic if X comes with a closed real 2-form ω such that at every point x ω_x is nonsingular.

In 1995 , Taubes proved that a simply connected minimal symplectic manifold must be irreducible in the above sense. Then Taubes, Mrowka, and others conjectured that, conversely, a simply connected irreducible 4-manifold must be minimal symplectic. This is the Minimal Conjecture. But three months ago, Z.Szabo proved that the Minimal Conjecture was false. He was aided in this by the *Product Formula* of Morgan-Szabo-Taubes.

All of the above work in smooth-4-manifold-topology was proved using ideas pioneered by physicists. Beginning with Yang-Mills theory, and culminating, in 1994, with the work of Seiberg and Witten, Gauge Theory has become the unrivalled method for investigating smooth 4-manifolds.

This course will introduce the student to Seiberg-Witten Gauge Theory. The heart of this theory is a Z-valued invariant on the set of $Spin^c$-structures on an oriented 4-manifold X. This is the Seiberg-Witten Invariant, $SW : Spin^c(X) \to Z$. It vanishes on manifolds expressible as "non-trivial" connected sums. After some applications of this, the course will focus on the Product Formula mentioned above, which gives a means of computing SW in certain cases. This is the heart of the course.

The first major consequence of this formula is the result that a nonsingular holomorphic curve in a Kaehler surface, say $C \subset X$, with $C\dot{C} \geq 0$, must have minimal genus among all smooth embedded 2-manifolds in its homology class. The second major consequence (in combination with other work) is Szabo's counterexample. We will outline (at least) the counterexample, show how the minimal-genus theorem follows from the Product Formula, and then concentrate on the proof of the product formula.

BIBLIOGRAPHY

E. Witten: *Monopoles and Four Manifolds,* Math. Research Letters, 1994.

P. Kronheimer and T. Mrowka: *The Genus of Embedded Surfaces in the Projective Plane,* Math. Research Letters, 1995.

Morgan, J. : *Seiberg Witten Invariants and Kahler Manifolds,* Princeton University Press, 1996.

Salamon, D. : *Spin Geometry and Seiberg-Witten Invariants,* Oxford University Press, to appear.

Taubes, C.: *Gromov Invariants and Seiberg-Witten Invariants,* Jour. AMS, 1996.