MATH 658, Spring 1999 - Seiberg-Witten theory

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This course is the sequel to Math 657 currently running. What is Seiberg-Witten theory?

Before giving you a general idea I want to say I find this subject very exciting because it has something to offer to anyone, and it does that in an unconventional way.

This very young theory is about some nonlinear elliptic equations and their applications to low dimensional differential topology. These equations are formulated in geometrically sensitive terms (such as spinors and connections) and because of this, the geometry has a considerable impact on their analytic features. The topological relevance of these equations comes into play when we start counting in a suitable manner their solutions. The resulting integer turns out to be a very fine invariant of a 4-manifold.

The fall semester covers the general theory: what are these equations, what are their properties how to count their solutions and why the above count is a topological invariant.

In spring I want to put all these facts to work. I want to show how to *effectively* count solutions and then discuss the topological and geometric implications of these computations. This count is easier if the background geometry is richer. That is why I will focus on algebraic surfaces and, towards the end, on symplectic 4-manifolds. With a relatively modest technical luggage one can prove results inaccessible before November 1994, the official birthdate of this theory. I plan to describe Kronheimer and Mrowka's proof of the Thom conjecture, to give a proof of Donaldson's theorem on negative definite *smooth* 4-manifolds and to discuss Taubes' symplectic nonvanishing theorem.

Math 657 is a pre-requisite for the Spring sequel, but if you have a basic knowledge of elliptic equations and Dirac operators you will have little trouble catching-up. Even if you do not plan to become an expert in this subject, the ideas and techniques we will discuss in class have a wide applicability, and may prove to be relevant in your research as well.

Helpful reference

J.W. Morgan: *The Seiberg-Witten equations and applications to the topology of smooth 4-manifolds*, Princeton University Press, 1996.