Dynamical Systems

Math 675 - Fall 98

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The goal of this course is to give an *introductory* exposition to the theory of dynamical systems. This course might be of interest to graduate students and researchers in differential geometry, topology and complex dynamics.

The theory of dynamical systems is a major mathematical discipline that is intertwined with all the major areas of mathematics: analysis, number theory, differential geometry, topology, probability. Ideas coming from dynamical systems stimulate research in many sciences, and give rise to the vast new area called nonlinear dynamics, or chaos theory.

The course will start with a discussion of the following elementary examples: rotations of the circle, translations of the torus, linear flows on the torus, expanding maps, hyperbolic toral automorphisms, geodesic and horocycle flows on surfaces of constant negative curvature, and symbolic dynamical systems.

These examples are used to formulate a general program aimed toward the study of the asymptotic properties (minimality, topological transitivity, growth of orbits, ergodicity, mixing) of dynamical systems, as well as to introduce the principal invariants (differential and topological equivalence, moduli, asymptotic orbit growth, entropy, cocycles and cohomological equations), and several important methods (fixed point methods, coding, KAM-type Newton method, local normal forms).

The last part of the course will be dedicated to the study of the hyperbolic dynamical systems: existence of the stable and unstable manifolds, closing lemma, shadowing of pseudo-orbits, stability of hyperbolic sets, Markov partitions, local product structure, ergodicity of geodesic flows on compact Riemannian manifolds with negative curvature, Livsic theorem, SRB measures).

Main reference: Introduction to the modern theory of dynamical systems, by A. Katok and B. Hasselblatt.