Name: ______________________________

Instructor: __________________________

**Math 20550, Old Exam 1**
**February 19, 2019**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.
- Each multiple choice question is 6 points. Each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

| | | | | | |
|---|---|---|---|---|---|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

**Please do NOT write in this box.**

| | |
|---|---|
| Multiple Choice | ________ |
| 11. | ________ |
| 12. | ________ |
| 13. | ________ |
| Extra Points. | 4 |
| Total: | ________ |

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Multiple Choice

**1.**(6 pts) Which of the following vectors has the same direction as $\boldsymbol{v} = \langle -1, 2, 2 \rangle$ but has length 6?

(a) $\langle -2, 4, 4 \rangle$ (b) $\langle 2, 4, 4 \rangle$ (c) $\langle 4, 2, 4 \rangle$

(d) $\left\langle -\sqrt{2}, 2\sqrt{2}, 2\sqrt{2} \right\rangle$ (e) $\langle 0, 6, 0 \rangle$

**2.**(6 pts) Compute the vector projection of the vector $\langle 1, 0, 0 \rangle$ on the vector $\langle 2, -1, 1 \rangle$.

(a) $\frac{1}{3}\langle 1, 2, 3 \rangle$ (b) $\frac{1}{3}\langle 2, -1, 1 \rangle$ (c) $\langle 1, 1, -1 \rangle$

(d) $\langle 2, -1, 1 \rangle$ (e) $\langle 3, 1, 4 \rangle$

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**3.**(6 pts) Determine which of the following expressions gives the length of the curve defined by $\boldsymbol{r}(t) = 2t\boldsymbol{i} + \cos(t)\boldsymbol{j} + 2\sin(t)\boldsymbol{k}$ between the points $(0, 1, 0)$ and $(2\pi, -1, 0)$.

(a) $\displaystyle\int_0^{\pi} \sqrt{4t^2 + \cos^2(t) + 4\sin^2(t)}dt$

(b) $\displaystyle\int_0^{2\pi} \sqrt{4 + \sin^2(t) + 4\cos^2(t)}dt$

(c) $\displaystyle\int_0^{2\pi} \sqrt{4t^2 + \cos^2(t) + 4\sin^2(t)}dt$

(d) $\displaystyle\int_0^{\pi} (2t\boldsymbol{i} + \cos(t)\boldsymbol{j} + 2\sin(t)\boldsymbol{k})dt$

(e) $\displaystyle\int_0^{\pi} \sqrt{4 + \sin^2(t) + 4\cos^2(t)}dt$

**4.**(6 pts) Find the volume of the parallelopiped determined by the vectors $\langle 3, 1, 4\rangle$, $\langle 2, 0, 2\rangle$ and $\langle -3, -1, 0\rangle$.

(a) $8$ (b) $-4$ (c) $20$ (d) $\langle 2, -6, -2\rangle$ (e) $4$

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**5.**(6 pts) What is the center of the sphere given by the equation

$$x^2 - 2x + y^2 - 4y + z^2 - 6z + 25 = 10$$

(a) $(3, 2, 1)$

(b) $(1, 2, 3)$

(c) This is not the equation of a sphere

(d) $(0, 0, 0)$

(e) $(2, 1, 3)$

**6.**(6 pts) The two curves

$$\boldsymbol{r}(t) = \left\langle t^2 - 1, \ln(t^2), t^4 - t^3 - t^2 + t \right\rangle$$
$$\boldsymbol{s}(t) = \left\langle 2\sqrt{t+3} - 4, \cos(\pi t) + 1, \sqrt{t} - 1 \right\rangle$$

intersect at the origin when $t = 1$. What is the cos of the angle of intersection?

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\frac{1}{2}$ (e) $1$

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**7.**(6 pts) The plane $S$ contains the points $(0, 1, 3)$, $(2, 2, 2)$, and $(3, 2, 1)$. Which of the following is an equation for $S$?

(a) $2x - 4y + 3z = 5$ (b) $x + y + z = 6$ (c) $x + 3y + z = 6$

(d) $y + z = 4$ (e) $x - y + z = 2$

**8.**(6 pts) For which of the following function does the contour plot consist of concentric circles?

(a) $f(x, y) = 8 - x - y$ (b) $f(x, y) = e^{-(x^2+y^2)}$ (c) $f(x, y) = e^{4x^2-y}$

(d) $f(x, y) = \sin(x + y)$ (e) $f(x, y) = x^2 - y^2$

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**9.**(6 pts) Find symmetric equations for the tangent line to the helix $\boldsymbol{r}(t) = \langle 2\cos(t), \sin(t), t\rangle$ at the point $(0, 1, \pi/2)$.

(a) $\dfrac{x}{-2} = \dfrac{z - \frac{\pi}{2}}{1}$ and $y = 0$

(b) $\dfrac{x}{-2} = \dfrac{z - \frac{\pi}{2}}{1}$ and $y = 1$

(c) $\dfrac{x}{2} = \dfrac{y-1}{2} = \dfrac{z - \frac{\pi}{2}}{1}$

(d) $\dfrac{x}{-2} = \dfrac{y-1}{1} = \dfrac{z - \frac{\pi}{2}}{1}$

(e) $\dfrac{x}{2} = \dfrac{z - \frac{\pi}{2}}{1}$ and $y = 1$

**10.**(6 pts) The position vector of a flying cardinal at second $t$ is given by $\boldsymbol{r}(t) = \langle 4t, \cos(3t), \sin(-3t)\rangle$. What is the normal component of its acceleration, $a_N$, at time $t$?

(a) $9t$

(b) $\cos(3t)$

(c) $1$

(d) $9$

(e) $-9$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

**11.**(12 pts.) Find the area of the triangle with vertices $P(1,4,6)$, $Q(-2,5,-1)$ and $R(1,-1,1)$.

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**12.**(12 pts.) Let $z = z(x, y)$ be the function of $x, y$ given implicitly by the equation

$$x^2 + y^3 + z^4 + 2xyz = 1$$

Find $\dfrac{\partial x}{\partial y}$ and $\dfrac{\partial y}{\partial z}$.

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**13.**(12 pts.) The acceleration of a particle at time $t$ is

$$\boldsymbol{a} = \left\langle 36t^2, e^t, \cos(t) \right\rangle$$

Determine its location at time $t = 1$ if it is known that at time $t = 0$ the particle was passing through the origin with velocity $\langle 1, 1, 1 \rangle$.