Name: Date: 02/27/2020

M20550 Calculus III Tutorial
Worksheet 5

1. Find $\dfrac{dz}{dt}$ when $t = 2$, where $z = x^2 + y^2 - 2xy$, $x = \ln(t-1)$ and $y = e^{-t}$.

2. Let $r = r(x, y)$, $x = x(s, t)$, and $y = y(t)$. Given that

$$\begin{array}{lll} x(1,0) = 2, & x_s(1,0) = -1, & x_t(1,0) = 7, \\ y(0) = 3, & y(1) = 0 & y'(0) = 4, \\ r(2,3) = -1, & r_x(2,3) = 3, & r_y(2,3) = 5, \\ r_x(1,0) = 6, & r_y(1,0) = -2, & \end{array}$$

   calculate $\dfrac{\partial r}{\partial t}$ at $s = 1, t = 0$.

3. (a) Let $f(x, y, z) = x^2 - yz$. If $\mathbf{v} = \langle 1, 1, 0\rangle$, find the directional derivative of f in the direction of $\mathbf{v}$ at the point $(1, 2, 3)$.

   (b) Interpret your result in part (a) by filling in the blanks and circling the correct word of the statement below:

   At the point ____________, the value of the function f is *increasing / decreasing* at the rate of ____________ as we move in the direction given by the vector ____________.

4. Let $f(x, y) = \ln(xy)$. Find the maximum rate of change of f at $(1, 2)$ and the direction in which it occurs.

5. Identify the absolute maximum and absolute minimum values attained by $g(x, y) = x^2y - 2x^2$ within the triangle T bounded by the points $P(0, 0)$, $Q(2, 0)$, and $R(0, 4)$.

6. Identify the absolute maximum and absolute minimum values attained by $z = 4x^2 - y^2 + 1$ on the region $R = \{(x, y) \mid 4x^2 + y^2 \leq 16\}$.

7. Find all points on the surface $z = x^2 - y^3$ where the tangent plane is parallel to the plane $x + 3y + z = 0$.

   More Practice Problems:

8. *(This usually is a challenging problem to students)* Find **all** points at which the direction of fastest change of the function
   $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

9. If $h = x^2 + y^2 + z^2$ and f is a differentiable function of two variables that satisfies the equation

$$y \cos f(x, y) + f(x, y) \cos x = 0$$

at every point (x, y) in its domain, find

$$\frac{\partial(h(x, y, f(x, y)))}{\partial x}$$

.

10. A cylinder containing an incompressible fluid is being squeezed from both ends. If the length of the cylinder is *decreasing* at a rate of 3m/s, calculate the rate at which the radius is changing when the radius is 2m and the length is 1m. (Note: An incompressible fluid is a fluid whose volume does not change.)