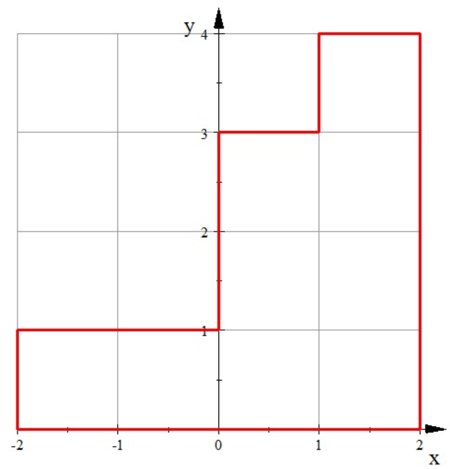
Name: Date: 4/16/2020

# M20550 Calculus III Tutorial
## Worksheet 10

1. A particle starts at the origin $(0,0)$, moves along the $x$-axis to $(2,0)$, then along the curve $y=\sqrt{4-x^2}$ to the point $(0,2)$, and then along the $y$-axis back to the origin. Find the work done on this particle by the force field $\mathbf{F}(x,y) = y^2\,\mathbf{i} + 2x(y+1)\,\mathbf{j}$.

2. Evaluate $\int_C (x^4y^5 - 2y)dx + (3x + x^5y^4)dy$ where $C$ is the curve below and $C$ is oriented in the clockwise direction.




3. Compute div $\mathbf{F}$ and curl $\mathbf{F}$ for the following vector fields.

   (a) $\mathbf{F} = x^2y\mathbf{i} - (z^3 - 3x)\mathbf{j} + 4y^2\mathbf{k}$

   (b) $\mathbf{F} = (3x + 2z^2)\mathbf{i} + \frac{x^3y^2}{z}\mathbf{j} - (z - 7x)\mathbf{k}$

4. (a) Compute div $\mathbf{F}$, where $\mathbf{F} = \langle e^y, zy, xy^2\rangle$ .

   (b) Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that curl $\mathbf{G} = \langle xyz, -y^2z, yz^2\rangle$? Why?

5. Show that any vector field of the form

$$\mathbf{F}(x,y,z) = f(y,z)\mathbf{i} + g(x,z)\mathbf{j} + h(x,y)\mathbf{k}$$

   is incompressible (i.e. has div $\mathbf{F}$= 0 everywhere).

6. Fill in the sentences below by circling one option:

   We can take the gradient of a (vector field/scalar function), and the output is a (vector field/scalar function).

   We can take the divergence of a (vector field/scalar function) and the output is a (vector field/scalar function).

   We can take the curl of a (vector field/scalar function), and the output is a (vector field/scalar function).

Name: Date: 4/16/2020

7. Which of the following combinations of grad, div, and curl make sense?

$$\begin{array}{ccc} \nabla(\nabla(f)) & div(\nabla(f)) & curl(\nabla(f)) \\ \nabla(div(\mathbf{F})) & div(div(\mathbf{F})) & curl(div(\mathbf{F})) \\ \nabla(curl(\mathbf{F})) & div(curl(\mathbf{F})) & curl(curl(\mathbf{F})) \end{array}$$