Last time \[ SW = \text{Ho}(\text{Top})(\Sigma^{-}) \quad \text{Spanner = Unordered Category} \]

objects = \((X, n) \quad X \in \text{CW} \quad \sigma X^{-n} \)

morphisms = \(\left[ (X, n), (Y, m) \right]^{SW} = \coprod_{i} \left( \Sigma^{-i} X, \Sigma^{-i} Y \right) \)

\(\text{Coh. Thys.} \)

objects: \(\{ E^k : \text{Top} \to \mathbb{A} \}_{k \geq 0} \quad \) bonding morphisms

\(\sigma : E^k (-) \to E^{k+1} (-) \)

We claimed \( SW \to \text{Coh. Thys.} \)

Definition was flawed \((X, n) \to E^k (\eta, n) \)

\[ E^k_{(\eta, n)}(Z) = \left[ (Z, 0), (X, n-k) \right]^{SW} \]

Problem: This does NOT satisfy wedge axiom.

Problem: \(\mathbb{Z}_{\omega} \), \((V \mathbb{Z}_{\omega}, 0)\) is not:

in general a coproduct in \( SW \).

\(\xi : S^{n+1} \to S^{n} \quad n = \text{inherit}, \quad n \to \omega \)

\(\xi : S^{n+1} \to S^{n} \quad n = \text{inherit}, \quad n \to \omega \)

\(\text{then } \xi : (S^{n+1}, 0) \to (S^{n}, 0) \)

thus is \(\xi : (V S^{n}, 0) \to (S^{n}, 0) \)

Therefore \( (V X_{n}, n) \to (X_{n}, n) \)

\[ \left[ (V X_{n}, n), Z \right]^{SW} \to \prod_{i} \left( X_{n}, n \right) \]

\(\text{not an isomorphism!} \)
Don't try to make sense of the candidates for a balanced
\((C_{\omega_i}, n_i) \in \text{SW} \quad \forall (C_{\omega_i}, n_i) \in \text{SW}\)

Indeed, define \(\tilde{E}_{(x, y)}(z)\)

\[
\tilde{E}_{(x, y)}(z) := \left[ Z, \lim_{\to} \Omega^{\sum_{z=0}^{\infty} X} \right]
\]

Note: this agrees with the old definition if \(Z\) is finite \(\text{SW}\).

Q: What are our ions?

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Big problem: \(\text{SW}\) does not allow for infinite unions/intersections
\(\forall (C_{\omega_i}, n_i) \in \text{SW}\)

\[
(C_{\omega_i}, n_i) \to (C_{\omega_i} \cup C_{\omega_i}, n_i) \to \cdots
\]

\[
\sum_{x=0}^{\infty} C_{\omega_i} \to \sum_{x=0}^{\infty} (C_{\omega_i} \cup C_{\omega_i})
\]

\[
\sum_{x=0}^{\infty} \left( \sum_{y=0}^{\infty} (C_{\omega_i} \cup C_{\omega_i}) \right) \to \sum_{x=0}^{\infty} C_{\omega_i} \cup \sum_{y=0}^{\infty} C_{\omega_i}
\]

etc...

Good answer: from

\[
\lim \left( K_0 \hookrightarrow \sum K_i \hookrightarrow \sum \sum K_i \hookrightarrow \cdots \right)
\]

Define: \(\sum K_i \hookrightarrow K_{\infty}\)

Inclusion of simplices

Solution: make these the objects of a category

Define: Category of spectra \(\text{Sp}\)

\[
\text{Obj} \ X \cdot \left\{ X_{\omega_i} : \Omega_{\omega_i} X_{\omega_i} \to \text{Top} \right\}
\]

\[
\sigma_i : \sum X_{\omega_i} \to K_{\infty}
\]

\[
(\text{equiv} \sigma_i : X_{\omega_i} \to \Omega_{\omega_i} K_{\infty})
\]
Net: \[ \{ f_i : X_i \to Y_i \} \]

Note: Spheres complete if complete.

\[ \Sigma X_i \to \Sigma Y_i \]

\[ \begin{array}{c}
\downarrow \phi \\
\Sigma X_i \to \Sigma Y_i
\end{array} \]

A CW complex \( X_i = \text{pointed} \) CW complex

\[ \Sigma X_i \to X_{i+1} \quad \text{inclusion of subcomplexes.} \]

\[ (x, n) \quad \to \quad \cdots \quad \to \quad \Sigma^\infty X \to \Sigma^\infty Y \quad \to \]

\[ (x, n) = \sum_{i=0}^{\infty} x_i, \quad \in \quad \Sigma^\infty X = \Sigma^\infty Y \]

Let \( [x, y] \) denote "spheres homotopy classes of maps between spaces".

What does this mean?

Say \( K \) is finite CW complex, \( X \) is CW complex

\[ [k, n), X \to [k, n), \lim \phi (X_i, i)] \to [k, n), (X_i, i)] \to \]

\[ \lim \phi \to [k, n), (X_i, i)] \to [k, n), (X_i, i)] \to \]

\[ \lim \phi \to [\Sigma^\infty K, X] \]

\[ \lim \phi \to [\Sigma^\infty K, X] \]

Define \( \pi_0 K \), with equivalence in \( SHC \)

\[ \lim \phi \to \pi_0 K \]

\[ \text{will be } \pi_0 \text{ isos.} \]

\[ \lim \phi \to \text{stable cones.} \]

\[ \Sigma W \to SHC \quad \to \quad \text{full subcat} \]

\[ \Sigma W \to \text{full subcat} \]

\[ \text{"Objects in SHC are } \]

\[ \text{infinite unions of objects of } \]

\[ \Sigma W \]

\[ \text{full subcat spanned by } \]

\[ (k, n) \quad \text{if } K \text{ finite} \]
Exercise  A map $f: (X, x) \to (Y, y)$ is a SW$^s$ isomorphism if and only if

$$f_*: \pi^s_*(X, x) \to \pi^s_*(Y, y)$$

is an iso.

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Čech theory

$$\{\tilde{E}, \sigma_i\} \to \{E_i, \sigma_i\}$$

Brown representability

$$E^i(Z) \cong [Z, E]_i$$

$$\sigma_i$$

$$E^m(\Sigma Z) = [\Sigma Z, E_m] \cong [Z, \Sigma E_m]$$

Get an $\Omega$-spectrum.

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Exercise

(i) Show $E^k(S^n) \cong \pi^s_{n-k} E$

(ii) Suppose $f: X \to Y$ is a map of $\Omega$-spectra w/ induced map on cols. Then:

$$f_*: \tilde{X}^*(-) \to \tilde{Y}^*(-)$$

Show $f_*$ is an iso.

$$f$$ is a stable equivalence.

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We will define $\text{SHC} = \text{Sp}[\text{stably}]$ (Need to show localization).