

1 - Exercises

Monday, August 31, 2015 1:46 PM

1.) Prove that a W -equivalence between W -local objects is an isomorphism

Hint: I found it useful to consider
 $W \subset \bar{W} \subset \text{Mor}(\mathcal{C})$
 $(f: X \rightarrow Y \in \bar{W}) \Leftrightarrow (\mathcal{C}(Y, Z) \xrightarrow[\mathcal{C}(X, Z)]{f^*} \mathcal{C}(X, Z) \cong \mathcal{C}(X, Z) \quad \forall \begin{matrix} W\text{-local} \\ Z \end{matrix})$

2.) Prove that if a W -localization functor exists, then

$$\mathcal{C}[W^{-1}] \cong \mathcal{C}^{W\text{-loc}} \quad (\text{the full subcat of } \mathcal{C} \text{ of } W\text{-loc objects})$$

3.) Prove that if E^* is a coh thry, then there is a long exact sequence (LES of a pair)

$$\dots \rightarrow E^k(X, A) \rightarrow E^k(X) \rightarrow E^k(A) \rightarrow E^{k+1}(X, A) \rightarrow \dots$$

4.) Prove that if $\{E_i, \sigma_i\}$ is an Ω -spectrum,

$$\tilde{E}^i(-) := [-, E_i]_* \quad \text{is a cohomology theory.}$$

5.) Prove that if $f: E \rightarrow F$ is a stable equivalence of Ω -spectra

then $f^*: E^i(-) \rightarrow F^i(-)$

is an isomorphism of coh theories.

6.) Show CW- Ω -spectra are stably colocal.

7.) Construct a stable colocalization functor for Sp^{Ω} .