

$$F = \text{formal gp}/R \quad R = \overset{\text{torsion-free}}{\mathbb{Z}(p)\text{-alg}}$$

$$F \text{ is } \underline{p\text{-typical}} \text{ if } \log_F(x) = \sum_i k_i x^{p^i}$$

Thm

$$\begin{array}{ccc} F & \xrightarrow{f} & F_{p\text{-typ}} \\ & \searrow \sum m_i x^{p^i} & \swarrow \sum k_i x^{p^i} \\ & & F_{\text{add}} \end{array}$$

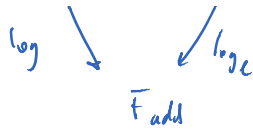
$$\log_{F_{p\text{-typ}}}(f(x)) = \log_F(x)$$

$$\begin{aligned} \text{or } f(x) &= \exp_{F_{p\text{-typ}}}(\log_F(x)) \\ &= \sum_i^F g_i \left( \sum_j m_j x^{p^j} \right)^{p^i} \end{aligned}$$

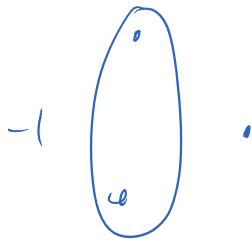
Consider:  $f_\ell: F \rightarrow F_\ell \quad \ell \neq p$

$$f_\ell^{-1}(x) = x - \frac{1}{\ell} \left[ \frac{1}{\ell} \right]_F \left( \sum_{i=1}^{\ell-1} g_i x \right)$$

$$F \xrightarrow{f} F_\ell$$



$$\begin{aligned} \log_e(x) &= \log_F(F_e^{-1}(x)) = \log_F(x) - \frac{1}{\ell} \left( \sum_{i=1}^{\ell} \log_F(\sigma^i x) \right) \\ &= \sum_{i,j} m_{i,j} \frac{1}{\ell} (1 - \sigma^{(j+1)i}) x^{j+1} \end{aligned}$$



$$= \sum_j \left\{ \begin{array}{l} 0, \ell | j+1 \\ m_j, \ell \nmid j+1 \end{array} \right\} x^{j+1}$$

Do this for all  $\ell \neq p$

Get  $f_{p\text{-typ}} : F \rightarrow F_{p\text{-typ}}$

$$\log_{F_{p\text{-typ}}}(x) = \sum_i \lambda_i x^{p^i}$$

So every formal sp law can be uniquely  
 "p-synthesized"  $(F_{p\text{-typ}})_{p\text{-typ}} = F_{p\text{-typ}}$

Universal case:

$$MU_{(p)} \xrightarrow{\varepsilon} MU_{(p)}$$

$$\varepsilon^2 = \varepsilon$$

$$BP \begin{array}{c} \hookrightarrow \\ \leftarrow \end{array} MU_{(p)}$$

lemmng:

$$f \in R[[x]]$$

$$f: F_1 \rightarrow F_2$$

$F_1$   $p$ -typical

$F_2$  is  $p$ -typical

(iff if  $f$  is an iso)

$$\Rightarrow f(x) = \sum_{i \in \mathbb{F}_2} \varepsilon_i x^{p^i}$$

Cor:  $[p]_F(x) = \sum_{i \in \mathbb{F}_2} v_i x^{p^i}$

$$p \log(x) = \sum \lambda_i v_j^{p^i} x^{p^{ij}}$$

e.g.  $m_1 p^p + v_1 = p^{m_1}$   
 $\Rightarrow m_1 = \frac{v_1}{p-p^p}$

$$p \sum_k \lambda_k x^{p^k}$$

$$\Rightarrow p \lambda_k = \sum_{i+j=k} \lambda_i v_j^{p^i}$$

get

classification of  $p$ -tp.

$$\mathbb{Z}_p[v_1, v_2, \dots] \longrightarrow \bigvee \longrightarrow \bar{\bigvee}$$

$$\begin{array}{ccccc}
 \mathbb{Z}_{(p)}[v_1, v_2, \dots] & \longrightarrow & V & \hookrightarrow & \bar{V} \\
 & & \uparrow & & \uparrow \\
 & & L & \longrightarrow & \bar{L} \\
 & & & & \mathbb{Z}_{(p)}''[\lambda_1, \lambda_2, \dots]
 \end{array}$$

$$(1) \quad H\mathbb{F}_p \ast BP = \mathbb{F}_p \langle \lambda_1, \lambda_2, \dots \rangle$$

$$H\mathbb{F}_p \ast MU = \mathbb{F}_p \langle b_1, b_2, \dots \rangle = E_p \langle m_1, m_2, \dots \rangle$$

$$x_{mu} = \sum_i b_i x_E^{i+1}$$

$$x_E = \sum_i m_i x^{i+1}$$

$$(2) \quad \pi_* BP = \mathbb{Z}_{(p)}[\bar{v}_1, \bar{v}_2, \dots]$$

$$(3) \quad \mathbb{Z}_{(p)}[\bar{v}_1, \dots] \xrightarrow{\cong} \pi_* BP \longrightarrow \bar{V}$$

"  
V

natural iso

$\Rightarrow x_i$

hits independently =  $x_{ij}$

$$[P]_{F_R}(\omega) = \sum_{F_R} \pi_R(v_i) x^{p^i}$$

$$\begin{array}{ccc}
 F_L & \xleftarrow{f^{-1}} & F_R \\
 & \searrow \log_{F_L} & \downarrow \log_{F_R} \\
 & & F_{\text{add}}
 \end{array}$$

$$\log_{F_R}(\alpha) = \sum_i n_R(m_i) x^{p^i}$$

$$\log_{F_L}(f^{-1}(\alpha)) = \log_{F_R}(\alpha) = \sum_i n_R(m_i) x^{p^i}$$

$$\parallel$$

$$\log_{F_L}(\sum_i t_i x^{p^i})$$

$$\parallel$$

$$\sum_i m_i t_i x^{p^{i+1}}$$

$$\text{So } n_R(m_i) = \sum_{i_1+i_2=i} m_{i_1} t_{i_2}^{p^{i_1}}$$

$$\text{e.g. } n_R(m_1) = t_1 + m_1$$

$$\Rightarrow n_R(v_1) = n_R((p-p^p)m_1) = (p-p^p)t_1 + v_1$$

$$\equiv v_1 + p t_1 \pmod{p^2}$$

$$\mathbb{Z}_p \text{ BP}_\bullet [t_0, t_1, \dots] \xrightarrow{\mathcal{F}} \mathbb{Z}_p \text{ BP}_\bullet [s_1, s_2, \dots]$$

$$F_0 \xrightarrow{f_1} F_1 \xrightarrow{f_2} F_2$$

$$f_2^{-1} = \sum_i^{F_1} s_i x^{p^i}$$

$$f_1^{-1} = \sum_i^{F_0} t_i x^{p^i}$$

$$f_1^{-1}(f_2^{-1}(\alpha)) = \sum_i^{F_0} \mathcal{N}(t_i) x^{p^i}$$

$$\parallel$$

$$f_1^{-1}(\sum_i^{F_1} s_i x^{p^i})$$

$$\begin{aligned} & \int_1^{\mathbb{1}} \left( \sum_i^{F_1} s_i x^{p^i} \right) \\ & \quad \parallel \\ & \sum_i^{F_0} f_i^{-1}(s_i x^{p^i}) \\ & \quad \parallel \\ & \sum_i^{F_0} t_i s_i^p x^{p^{i+j}} \end{aligned}$$


---

Apply  $\log_{F_0}$

$$\sum_i m_i t_i^p s_i^p x^{p^{i+j+k}} = \sum_j m_j \Psi(t_j)^{p^i} x^{p^{i+j}}$$

is

$$m_1 + t_1 + s_1 = m_1 + \Psi(t_1)$$

$$\Rightarrow \Psi(t_1) = t_1 \otimes 1 + 1 \otimes t_1$$

---


$$\Psi(t_2) = t_2 \otimes 1 + 1 \otimes t_2 + t_1 \otimes t_1^p$$

$$+ \frac{v_1}{(p-1)} \sum_{i=1}^{p-1} \frac{1}{p} \binom{p}{i} t_1^i \otimes t_1^{p-i}$$


---