13 - chromatic picture

Wednesday, November 11, 2015 8:54 AM

$\mathcal{M}_{FG}$ = moduli stack of FGL's

$$\left[\mathcal{M}_{FG}(R) = \left(\text{FGL's}/R,\ \text{strict isos}\right)\right]$$

$\mathcal{M}_{FG^{p\text{-}typ}}$ = moduli stack of p-typ. FGL's

$$\begin{array}{ccc} \mathrm{Ext}_{MU_*MU}(MU_*, MU_*)_{(p)} & \cong & \mathrm{Ext}_{BP_*BP}(BP_*, BP_*) \\ \| \wr & & \| \wr \\ H^*(\mathcal{M}_{FG})_{(p)} & & H^*(\mathcal{M}_{FG^{p\text{-}typ}}) \end{array}$$

[Since groupoids $\mathcal{M}_{FG}(R)$, $\mathcal{M}_{FG^{p\text{-}typ}}(R)$ are equivalent for R a $\mathbb{Z}_{(p)}$-alg]

$R = \mathbb{F}_p$-alg

Def $F \in FGL(R)$ has ht n if $[p]_F(x) = v_n x^{p^n} + \cdots \quad v_n \in R^\times$

$\mathcal{M}_{FG}^{\leq n}(R) = $ FG's ht $\leq n$

$$(\mathcal{M}_{FG})_{\mathbb{Q}} \hookrightarrow \mathcal{M}_{FG}^{\leq 1} \hookrightarrow \mathcal{M}_{FG}^{\leq 2} \hookrightarrow \cdots\cdots \to \mathcal{M}_{FG}$$

$$\begin{array}{ccccccc} H^*((\mathcal{M}_{FG})_{\mathbb{Q}}) & \leftarrow & H^*(\mathcal{M}_{FG}^{\leq 1}) & \leftarrow & H^*(\mathcal{M}_{FG}^{\leq 2}) & \leftarrow & \cdots \\ \| & & \uparrow & & \uparrow & & \\ H^*(\mathcal{M}_{\mathbb{Q}}) & & H^*(\mathcal{M}^{\leq 1}, \mathcal{M}_{\mathbb{Q}}) & & H^*(\mathcal{M}^{\leq 2}, \mathcal{M}^{\leq 1}) & & \end{array}$$

<u>Get SS</u> (chromatic SS)

Ht SS (chromatic SS)

$$E_1^{*,*,n} = H^*(\mathcal{M}^{\le n}, \mathcal{M}^{\le n-1}) \Rightarrow H^*(\mathcal{M}_{FG})$$

Thm k = separably closed fld, $F_i \in FGL(k)$

$$\mathrm{ht}\, F_1 = \mathrm{ht}\, F_2 \iff F_1 \cong F_2$$

Consequence $\mathcal{M}^{\le n}_{FG} - \mathcal{M}^{\le n-1}_{FG}$ looks like (single object F_n, $\mathrm{Aut}\, F_n$ = morph)

$$\begin{tikzcd} F_n \arrow[loop below] \end{tikzcd}$$

$\mathrm{Aut}(F_n) = \mathbb{G}_n$

$\mathbb{G}_n$ = Morava stabilizer gp (profinite gp)

Morava Change of Rings

$$H^*(\mathcal{M}^{\le n}, \mathcal{M}^{\le n-1}) \cong H^*(\mathbb{G}_n, -)$$