14 - chromatic ss

Wednesday, November 11, 2015 9:09 AM

$$\mathrm{Ext}_{BP_*BP}(M) := \mathrm{Ext}_{BP_*BP}(BP_*, M)$$

$$\begin{tikzcd}
 & & BP_*/v_1^\infty \arrow[dr] & & & & \cdot \arrow[dr] & \\
BP_* \arrow[r] \arrow[dr, equal] & BP_*[v_1^{-1}] \arrow[ur] \arrow[r] & BP_*/v_1^\infty[v_2^{-1}] \arrow[rrr] \arrow[dr] & & & BP_*/(v_1^\infty, v_2^\infty)[v_3^{-1}] \arrow[ur] \arrow[r] & \cdot & \cdots \\
 & BP_* \arrow[ur] & & BP_*/(v_1^\infty, v_2^\infty) \arrow[urr] & & & &
\end{tikzcd}$$

get <u>Chromatic SS</u>

$$E_1^{s,t,n} = \mathrm{Ext}^{s,t}_{BP_*BP}\left(BP_*/(p^\infty, v_1^\infty, \ldots, v_{n-1}^\infty)[v_n^{-1}]\right) \Rightarrow \mathrm{Ext}^{s+n,t}_{BP_*BP}(BP_*)$$

BP_* is a comodule via η_R:

$$\begin{tikzcd}
BP_* \arrow[r, "\psi"] \arrow[d, "\eta_R"'] & BP_*BP \otimes_{BP_*} BP_* \arrow[dl, equal] \\
BP_*BP &
\end{tikzcd}$$

$\eta_R(v_n) \equiv v_n \mod (p, v_1, \ldots, v_{n-1})$

$\Rightarrow$ <u>Prop</u> ("Alg periodicity thm")

Suppose $I = (p^{i_0}, v_1^{i_1}, \ldots, v_{n-1}^{i_{n-1}})$ is an invariant ideal

(i.e. BP_*/I is a BP_*BP comodule)

Thm $\exists\ i_n \gg 0$ s.t.

$$\cdot\, v_n^{i_n} : BP_*/I \longrightarrow BP_*/I \quad \text{is a map of comodules}$$

(and consequently $BP_*/(I, v_n^{i_n})$ is a BP_*BP-comodule)

Cor: $BP_*/(p^\infty, v_1^\infty, \ldots, v_{n-1}^\infty)$, $BP_*/(p^\infty, v_1^\infty, \ldots, v_{n-1}^\infty)[v_n^{-1}]$ are BP_*BP-comodules

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Computations:

(1) $$\mathrm{Ext}^{**}_{BP_*BP}(BP_{\mathbb{Q}}) \simeq \begin{cases} \mathbb{Q}, & s=t=0 \\ 0, & \text{o/w.} \end{cases}$$

(2)
$$\begin{tikzcd}
\mathrm{Ext}^0_{BP_*BP}(BP_*/p^\infty) \arrow[d, Rightarrow] & \mathbb{Z}/p^k\left\{\dfrac{v_1^{sp^{k-1}}}{p^k}\right\} \arrow[l, hook'] \arrow[d, Rightarrow] \\
\mathrm{Ext}_{BP_*BP}(BP_*) & \mathbb{Z}/p^k\left\{\alpha_{sp^k/k}\right\} \arrow[l, hook']
\end{tikzcd}
\qquad
\begin{tikzcd}
v_1^i/p^j \arrow[d, Rightarrow] \\
\alpha_{i/j}
\end{tikzcd}$$