2 - exercises

Wednesday, September 9, 2015 6:40 AM

1) Prove: if X is a CW spectrum

(a) $\omega X_i := \varinjlim_n \Omega^n X_{i+n}$

forms an Ω-spectrum

(b) the map

$$X \longrightarrow \omega X$$

is a stable equivalence.

2) Prove that every spectrum is levelwise equivalent to a CW spectrum.

3) Prove that if

$F : \mathcal{C} \to \mathcal{D}$ is a functor

$W \subseteq \mathrm{Mor}\, \mathcal{C}$ class of weak equivalences

and if $X \longrightarrow p(X)$

is a right F-approximation

(in the sense of the notes)

then $RF(X) = F(pX)$ is

a right derived functor of F.

4.) Let G be a discrete group.
Consider the functor

$$(-)_G : Top_G \longrightarrow Top$$

(Top_G = cat. of G-spaces)

$$X_G = G\text{-orbits} = X/\sim \qquad x \sim gx \quad \forall\, x \in X,\ g \in G$$

The left derived functor is

Homotopy fixed points

$$(-)_{hG} : Ho(Top_G) \longrightarrow Ho(Top)$$

{ $/_G$ weak equivalence in $\top$ is an equivariant

a weak eq.

Top_G is an equivariant map which is an underlying weak equiv.

Let EG = free G-CW cx such that $EG \to *$ is a w.e.

a free G-CW cx Y is a colimit $Y = \varinjlim_n Y^{[n]}$ in Top_G so that the squares are pushouts

$$\begin{array}{ccc} \coprod G \times S^{n-1} & \longrightarrow & Y^{[n-1]} \\ \downarrow & \ulcorner & \downarrow \\ \coprod G \times D^n & \longrightarrow & Y^{[n]} \end{array}$$

Thm $X_{hG} \simeq (EG \times X)_G$

give this the diagonal G-action.