

4 - Exercises

Monday, September 21, 2015 6:58 AM

(1) Show that if E is a connective ring spectrum, and

$$X, Y \in \text{Sp}$$

$$E_*X \text{ flat}/E_*$$

that the Künneth map:

$$E_*X \otimes_{E_*} E_*Y \rightarrow E_*(X \wedge Y)$$

is an isomorphism.

[Hint: induction on a skeletal filtration of X]

(2) Show that localization w.r.t. $M[\mathcal{P}^{-1}]$ (\mathcal{P} = set of primes)

is given by

$$X \rightarrow X_{M[\mathcal{P}^{-1}]} \cong X \wedge M[\mathcal{P}^{-1}]$$

(3) Show that localization w.r.t. $M(\mathfrak{p}) = M\mathbb{Z}/\mathfrak{p}$ is given by

$$X \rightarrow X_{M(\mathfrak{p})} \cong \varprojlim_i (X \wedge M\mathbb{Z}/\mathfrak{p}^i)$$

If $\pi_n X$ is f.g. in each degree,

$$\text{then } \pi_n(X_{M(\mathfrak{p})}) \cong (\pi_n X)_{\mathfrak{p}}^{\wedge}$$