5 - Hopf Algebroids of cooperations

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The E_2-term

(✱) Suppose E_*E is flat $/E_*$

$\Rightarrow$ (E_*E, E_*) is a Hopf algebroid. (c.f. Ravenel appendix A)

$$\begin{tikzcd} E_* \arrow[r, shift left=2, "\eta_L"] \arrow[r, shift right=2, "\eta_R"'] & E_*E \arrow[l, "\varepsilon" description] \arrow[loop below, "c"] \arrow[r, "\Delta"'] & E_*E \otimes_{E_*} E_*E \arrow[l, bend right, "\mu"'] \end{tikzcd}$$

$$\begin{tikzcd} E \arrow[r, shift left=2, "1\wedge\eta"] \arrow[r, shift right=2, "\eta\wedge 1"'] & E\wedge E \arrow[l, "\mu" description] \arrow[loop below, "\tau"] \arrow[r, "1\wedge\eta\wedge 1"'] & E\wedge E\wedge E \arrow[l, bend right, "1\wedge\mu"'] \end{tikzcd}$$

(use "Künneth thm")

e.g. $E = H\mathbb{F}_p$ $\quad$ $p=2$: $(H\mathbb{F}_2)_* H\mathbb{F}_2 = \mathbb{F}_2[\xi_1, \xi_2, \ldots]$ $\quad$ $|\xi_i| = 2^i - 1$ $\quad$ $\eta_L = \eta_R$

$$\psi(\xi_i) = \sum_{i_1+i_2} \xi_{i_1}^{2^{i_2}} \otimes \xi_{i_2} \qquad [\xi_0 := 1]$$

p odd: $(H\mathbb{F}_p)_* H\mathbb{F}_p = \mathbb{F}_p[\xi_1, \xi_2, \ldots] \otimes \Lambda_{\mathbb{F}_p}(\tau_0, \tau_1, \ldots)$

$$|\xi_i| = 2(p^i - 1)$$
$$|\tau_i| = 2p^i - 1$$

$$\psi(\xi_i) = \sum_{i_1+i_2=i} \xi_{i_1}^{p^{i_2}} \otimes \xi_{i_2}$$

$$\psi(\tau_i) = \sum_{i_1+i_2=i} \xi_{i_1}^{p^{i_2}} \otimes \tau_{i_2} + \tau_i \otimes 1$$

$\forall X$ E_*X is an E_*E-comodule

$$E_*X \longrightarrow E_*E \otimes_{E} E_*X$$

$$E \wedge X \xrightarrow[1 \wedge \eta \wedge 1]{} E \wedge E \wedge X$$

e.g. $E = H\mathbb{F}_p$

$(H\mathbb{F}_p)^*X$ is an $\mathcal{A}$-module

$\Longrightarrow$ $(H\mathbb{F}_p)_*X$ is an $\mathcal{A}_*$-comodule