Testing for and Correcting for Missing Data in Survey Responses:
A Survey of Downtown Businesses in Minneapolis

Shelley S. Baxter and **Lawrence C. Marsh**
Baxter Econometrics University of Notre Dame

Introduction

Business and government agencies often use surveys to obtain information about consumers or other groups. The individuals who fill out these surveys are not always willing or able to answer all questions. Often such missing data are assumed to be missing-at-random. If the data are not missing-at-random, statistical analyses may be distorted and generate reports that are misleading.

We provide a test to determine if survey data are missing-at-random. For data that are not missing-at-random, we provide a method of correcting the probability distribution of the response variables for the systematic pattern in the missing data. Previous studies used time-series, panel data or the ordinal nature of responses to provide the structure for handling systematically missing data (see Fitzmaurice et al.(1996)). Davis(1993) presented useful methods for the case when all of the independent variables are categorical. Our method uses cross-sectional data with nominal responses and both nominal and continuous independent variables. This methodology simply uses a joint probability distribution framework. It can be extended to incorporate both detailed and aggregated responses so that all observations can be used without sacrificing any of the detail. We require the response outcomes to be mutually exclusive and exhaustive in the form of a composite variable.[1] Specifically, this paper provides a methodology using SAS®[2] CATMOD, REG, NLIN and IML to produce distributions of the missing variable probabilities and GPLOT to produce graphs of the difference between the probability distributions with and without adjusting for the missing data, thereby providing a visual check of the nature of the missing responses.

We examine a survey of different types of businesses in downtown Minneapolis measuring their satisfaction with downtown conditions, services, etc. This survey was in conjunction with the first author's dissertation (see Baxter(1994)). One question on the survey required self-reporting of the type of business as a check on the Dun's database categorization used to acquire addresses. Coders later transformed the responses to standard industrial classification (SIC) codes. Fourteen percent of these responses were left blank. The authors faced the all-too-common research dilemma of either throwing away the observations when the SIC code was missing or ignoring this check all together and trusting that the current designation according to Dun's database was correct. Preliminary investigation had indicated a fairly large discrepancy between the two.

The Problem of Missing Survey Data

Missing data can pose a significant problem when the goal is to understand the behavior of all subjects rather than just the ones with complete data. Often researchers have simply ignored missing data with the hope that they are missing-at-random. Unfortunately, data are frequently systematically missing in a non-ignorable manner that can lead to substantial distortions when only complete data are used. Some researchers have estimated models with the complete data and then used the estimated model to fill in the missing data. Using predicted values for the missing data simply covers up rather than explains the systematic nature of the missing data. A more constructive approach in this context has been to hypothesize a model for the complete data, $Y_{i1} = f(X_{i1},\beta_1) + \varepsilon_{i1}$, a model for the missing data, $Y_{i0} = f(X_{i0},\beta_0) + \varepsilon_{i0}$, and a latent variable model explaining the difference between missing and nonmissing status, $I_{i1} = f(Z_i,\delta) + \nu_i$.

It is important to note that even in the special case where the X's and the β's are the same, the distribution of the errors may be quite different. Therefore, econometricians have

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[1] For a more extensive discussion of using a composite variable in statistical analysis in place of a set of characteristic dummy variables, see Marsh and Wells (1995). We have benefited from the helpful comments of Yuichi Kitamura on a related paper (see Marsh and Wells(1996)).

[2] SAS® is a registered trademark of SAS Institute, Inc., Cary, NC.

focused on determining the distribution of the triple $\{\varepsilon_{i0}, \varepsilon_{i1}, \nu_i\}$. On the other hand, statisticians have viewed this problem as one involving a mixture of the error distributions.

In this paper, the problem is a special case where outcomes are discrete random variables and multinomial probabilities form the basis of the data generating process. Consequently, we are able to bypass the latent variable approach and focus directly on the joint probability structure. In this case, the joint probabilities are broken down into two sets. One set of joint probabilities is observed and the second set is unobserved (missing). If we were just interested in behavior of those with observed observations and if group membership was determined strictly by exogenous variables, then we would be content with knowing the joint probability structure for the observed observations alone. This would also be satisfactory if group differences were ignorable and the missing data were truly missing-at-random. Since we know which observations have missing data, we have a censored rather than a truncated distribution. Since we test for data to be missing-at-random (MAR) and not missing-completely-at random (MCAR), we have to control for whatever independent variables are needed to ensure that the model errors have mean zero. For further discussion of this issue see Rubin(1976).

An Analysis of the Joint Multinomial Density

The probability of an observation being missing may be viewed as the sum of the probabilities of the unobserved outcomes. In other words, the probability of being missing is simply the marginal probability corresponding to the outcome designated "missing" for a discrete random variable representing "missing" versus "observed" status.

Figure 1. Multinomial Probabilities for Observed Outcomes Only

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Missing	
p_{1oi}	p_{2oi}	p_{3oi}	p_{4oi}	p_{5oi}	p_{6oi}	p_{7oi}	p_{mi}	1.0

Figure 2. Joint Multinomial Probabilities for Observed/Unobserved

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	
observed	p_{1oi}	p_{2oi}	p_{3oi}	p_{4oi}	p_{5oi}	p_{6oi}	p_{7oi}	p_{oi}
missing	p_{1mi}	p_{2mi}	p_{3mi}	p_{4mi}	p_{5mi}	p_{6mi}	p_{7mi}	p_{mi}
	p_{1i}	p_{2i}	p_{3i}	p_{4i}	p_{5i}	p_{6i}	p_{7i}	**1.0**

The objects of interest here are the marginal probabilities on the bottom line of Figure 2 which are obtained by summing over the corresponding observed and missing joint probabilities. If the missing data are missing-at-random, then for each observation the missing joint probabilities will just be some constant multiple, Ci of the observed joint probabilities (i.e. constant over the

alternatives). However, if the missing data are not missing-at-random, then separate Cji's must be estimated for each of the j = 1, . . . ,7 alternatives as shown in Figure 3.

Figure 3. Joint Multinomial Distribution of Observed Probabilities[3]

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	
observed	p_{1oi}	p_{2oi}	p_{3oi}	p_{4oi}	p_{5oi}	p_{6oi}	p_{7oi}	p_{oi}
missing	$c_{1i}p_{1oi}$	$c_{2i}p_{2oi}$	$c_{3i}p_{3oi}$	$c_{4i}p_{4oi}$	$c_{5i}p_{5oi}$	$c_{6i}p_{6oi}$	$c_{7i}p_{7oi}$	p_{mi}
	p_{1i}	p_{2i}	p_{3i}	p_{4i}	p_{5i}	p_{6i}	p_{7i}	**1.0**

In the context of this framework the problem becomes one of appropriately estimating the Cji terms. For example, a simple structure for this problem would be $C_{ji} = C_j \eta_i$. The Cj provides weights appropriate for the j = 1, ..., J outcomes while the η_i provides the necessary observation specific adjustment to ensure the condition:

$$C_1 \eta_i P_{1oi} + C_2 \eta_i P_{2oi} + C_3 \eta_i P_{3oi} + C_4 \eta_i P_{4oi} + C_5 \eta_i P_{5oi} + C_6 \eta_i P_{6oi} + C_7 \eta_i P_{7oi} = P_{mi} \quad (1)$$

Our procedure in this case is straightforward. Since for each observation the original data generates a value of one for the outcome that occurs and a zero for each of the other outcomes, it is easy to estimate the corresponding multinomial logistic probabilities corresponding to those in Figure 1. Applying a least squares regression of P_{mi} onto all of the other probabilities in equation (1) with no intercept term produces estimates of the Cj's. The fitted equation for the predicted values for P_{mi} are then multiplied by the appropriate η_i value necessary to ensure the equality in equation (1) and, thus, to fully restore the original P_{mi} value for each of the i=1,...,n observations. Thus, the η_i value serves as the sample realization of the population error, or, in other words, a residual term. The null hypothesis, Ho: $C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7$ can be used to test the missing-at-random assumption. A rejection of this hypothesis supports the claim that the data are not randomly missing and, therefore, in that case, imposing the missing-at-random assumption could produce misleading estimates of the corresponding marginal probabilities.

Analyzing the Minneapolis Survey Data

Because we are dealing with a polychotomous dependent variable and both continuous and nominal independent variables, we utilize the multinomial logit response function in PROC CATMOD. We chose a large number of control variables (thirty in all) in an effort to satisfy the zero mean assumption in the error term. These variables, indicated in Figure 4, included number

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[3] The corresponding likelihood function for this situation is given as follows:

$$L = \prod_{i=1}^{n} P_{1oi}^{y_{i1}} P_{2oi}^{y_{i2}} P_{3oi}^{y_{i3}} P_{4oi}^{y_{i4}} P_{5oi}^{y_{i5}} P_{6oi}^{y_{i6}} P_{7oi}^{y_{i7}} (C_{1i}P_{1oi} + C_{2i}P_{2oi} + C_{3i}P_{3oi} + C_{4i}P_{4oi} + C_{5i}P_{5oi} + C_{6i}P_{6oi} + C_{7i}P_{7oi})^{y_{i8}}$$

Notice that each probability occurs twice: once when it makes a direct contribution and a second time when its contribution is indirect as a component of the missing value. The y_{ij}'s take on the value one when the i^{th} observation has the j^{th} outcome and zero otherwise.

of employees as a continuous variable, location of primary market as four independent dummy variables, and the importance of various downtown characteristics such as cost of space and access to parking ramps as 26 independent variables each with four response levels/magnitudes which we treated as continuous for this study. The SIC's were grouped into seven industry sectors which with the missing response category provided eight possible responses to serve as the dependent variable for our multinomial logistic model. This model was then estimated using PROC CATMOD which produced the following analysis of variance table:

Figure 4. Analysis of Variance Table from PROC CATMOD

MAXIMUM-LIKELIHOOD ANALYSIS-OF-VARIANCE TABLE

Source	Source	DF	Chi-Square	Prob
CONSTANT	intercept	7	10.42	0.1658
Q018A1	primary mkt in Minneapolis	7	34.91	0.0000
Q018A2	in twin cites outside Mpls	7	10.83	0.1463
Q018A3	in MN but outside of TC's	7	12.36	0.0892
Q018A4	outside of Minnesota (MN)	7	20.32	0.0049
Q004A	number of employees	7	8.24	0.3120
Q025A	parking ramps	7	17.60	0.0139
Q025B	freeway access	7	1.04	0.9941
Q025C	public transportation	7	49.95	0.0000
Q025D	communications facilities	7	32.23	0.0000
Q025E	face to face communication	7	5.45	0.6050
Q025F	private research & develop	7	8.86	0.2630
Q025G	proximity to suppliers	7	33.99	0.0000
Q025H	proximity to customers	7	6.94	0.4356
Q025I	cultural activities	7	19.65	0.0064
Q025J	restaurants and shops	7	21.51	0.0031
Q025K	employee residences	7	8.27	0.3097
Q025L	owner residence	7	8.71	0.2743
Q025M	employee preference	7	9.30	0.2319
Q025N	management preference	7	10.24	0.1752
Q025O	costs of space	7	23.02	0.0017
Q025P	quality of office space	7	22.71	0.0019
Q025Q	central location metro	7	4.98	0.6630
Q025R	labor market access	7	18.74	0.0091
Q025S	business regulation climate	7	12.85	0.0759
Q025T	property taxes	7	14.82	0.0383
Q025U	city help with financing	7	19.40	0.0070
Q025V	employee education opportns	7	24.22	0.0010
Q025W	near to university research	7	22.50	0.0021
Q025X	near to city libraries	7	16.84	0.0184
Q025Y	prestige	7	24.84	0.0008
Q025Z	easy location recognition	7	11.77	0.1083
	LIKELIHOOD RATIO:	5796	chi-square:	2737.03

Since the focus of this paper is on the probability distributions and not on the regression coefficients, we will not present the details of each of the seven logistic equations that were produced here. Rather, Figure 5 presents the results of regressing the probability of falling into the missing data cell onto the other seven probabilities: OP1, OP2, OP3, OP4, OP5, OP6 and OP7.

Figure 5. PROC REG of Probability of Missing onto the Other Seven Probabilities

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	7	23.08677	3.29811	116.156	0.0001
Error	853	24.21998	0.02839		
U Total	860	47.30675			

Root MSE	0.16850	R-square	0.4880
Dep Mean	0.18140	Adj R-sq	0.4838
		C.V.	92.89369

Group	Variable	Parameter	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
Group1:	OP1	C1	1	0.290663	0.06714980	4.329	0.0001
Group2:	OP2	C2	1	0.205629	0.04876047	4.217	0.0001
Group3:	OP3	C3	1	0.269572	0.06617726	4.073	0.0001
Group4:	OP4	C4	1	0.520265	0.04241483	12.266	0.0001
Group5:	OP5	C5	1	0.042962	0.03278431	1.310	0.1904
Group6:	OP6	C6	1	-0.016225	0.04004514	-0.405	0.6855
Group7:	OP7	C7	1	0.091259	0.02875196	3.174	0.0016

Notice that no intercept was used in this regression since it would be inappropriate given the statistical theory discussed in the previous section. The estimated regression coefficients are all positive and statistically significantly greater than zero as expected except for the fifth one which is positive but not significantly greater than zero and the sixth one which has a negative sign but is not statistically significant.

In order to determine whether the marginal probabilities implied by the above regression results indicate that the missing data are missing-at-random or not, we ran a regression that restricted the coefficients in the above model to all be equal (still with no intercept) where PCOMBINE = (OP1 + OP2 + OP3 + OP4 + OP5 + OP6 + OP7). This enabled us to run the restricted regression where the fitted value of P_{mi} is equal to an estimate of the common coefficient C times PCOMBINE: fitted $P_{mi} = C\ (P_{1oi}+P_{2oi}+P_{3oi}+P_{4oi}+P_{5oi}+P_{6oi}+P_{7oi})$.

Figure 6. Restricted PROC REG of Probability of Missing onto the Combined Probability Variable

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	19.84560	19.84560	620.782	0.0001
Error	859	27.46115	0.03197		
U Total	860	47.30675			

Root MSE	0.17880	R-square	0.4195
Dep Mean	0.18140	Adj R-sq	0.4188
		C.V.	98.56815

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
PCOMBINE	C 1	0.182584	0.00732812	24.915	0.0001

Figures 5 and 6 provide the information necessary to carry out a conditional test of the null hypothesis that the missing data are missing-at-random. A conditional F-statistic can be computed treating the nonmissing probability variables as exogenous (independent) variables in the regressions reported in Figures 5 and 6. This ignores the first stage which was the logistic (PROC CATMOD) stage of the analysis so it does not qualify as a legitimate unconditional test for randomness. This conditional or pseudo F-statistic can be calculated using the standard F value for testing a linear restriction on a "linear" model using error sums of squares (SSE) from the restricted , R, and unrestricted, U, regressions and corresponding degrees of freedom (df) as follows:

$$\text{Conditional } F = \frac{(SSE_R - SSE_U)/(df_R - df_U)}{SSE_U / df_U} = \frac{(27.46115 - 24.21998)/(859 - 853)}{24.21998/853} = 19.025$$

With 6 numerator degrees of freedom and 853 denominator degrees of freedom, at a 0.01 level of significance the Table-F value is approximately 2.80, and, therefore, the missing-at-random hypothesis is rejected. This confirms our suspicion that missing-at-random was not a good assumption here. However, one disturbing aspect of the results in Figure 5 is the negative sign for C6. Even though in this case this negative value is not even close to being statistically significant at any of the usual levels of significance, clearly negative values for these parameters are inappropriate. Instead of dealing with these negative values on an ad hoc basis, it would be better to impose the restriction that the Cj's must be positive. A simple way of imposing this restriction is to define a new parameter $\alpha j = \ln(Cj)$ for j = 1, . . . , J.

This simply means replacing the Cj's in the least squares regression above with exp(αj) and estimating the resulting regression using nonlinear least squares. The starting values for the αj's are the natural logs of the corresponding Cj's. In the cases where negative values were initially obtained for any of the Cj's, the OLS regression was first rerun with the negative Cj values set to zero (dropped out those variables) to obtain new starting values more consistent with the nonnegative restriction before running the nonlinear least squares regression for that group. The nonlinear least squares results are presented in Figure 7.

Figure 7. The Results of the PROC NLIN

```
Non-Linear Least Squares Iterative Phase
Dependent Variable OPZ999       Method: Marquardt

NOTE: Convergence criterion met.

Non-Linear Least Squares Summary Statistics
Dependent Variable  PMISS

Source                DF Sum of Squares     Mean Square

Regression             6   23.082111446     3.847018574
Residual             854   24.224640659     0.028366090
Uncorrected Total    860   47.306752106
(Corrected Total)    859   19.009077722

NOTE: The Jacobian is singular.


Parameter Estimate Asymptotic       Asymptotic 95 %    corresponding
                   Std. Error     Confidence Interval      C-value
                                  Lower         Upper

A1    -1.2466842 0.231858449   -1.70177097   -0.79159746  0.287456
A2    -1.5816872 0.237013610   -2.04689244   -1.11648204  0.205628
A3    -1.3081609 0.244599426   -1.78825537   -0.82806642  0.270317
A4    -0.6622891 0.079217914   -0.81777627   -0.50680184  0.515670
A5    -3.2798392 0.797587280   -4.84532630   -1.71435210  0.037634
A6  -372.2241151 0.000000000 -372.22411509 -372.22411509  0.000000
A7    -2.4047374 0.317181541   -3.02729452   -1.78218035  0.090289
```

```
                Asymptotic Correlation Matrix

Corr   A1       A2       A3        A4       A5     A6     A7
 ------------------------------------------------------------------
A1        1  -0.0929  -0.5683   -0.3208  -0.1073    .   0.0935
A2  -0.0929        1  -0.0796   -0.0946  -0.1420    .  -0.1380
A3  -0.5683  -0.0796        1    0.0635  -0.1581    .  -0.1366
A4  -0.3208  -0.0946  0.06351         1  -0.2568    .  -0.2838
A5  -0.1073  -0.1420 -0.15815   -0.2568        1    .  -0.0126
A6        .        .        .         .        .    .         .
A7  0.09355  -0.1380  -0.1366   -0.2838  -0.0126    .         1
```

The Jacobian was singular for the asymptotic variance-covariance matrix so a conditional Wald test statistic could not be calculated to test the hypothesis that all seven coefficients are equal because of the A6 coefficient. However, it is possible to generate a conditional Wald statistic to test the hypothesis that the other six groups all have the same population coefficient value.

Figure 8. PROC IML Code for Conditional Wald Test for Missing-at-Random

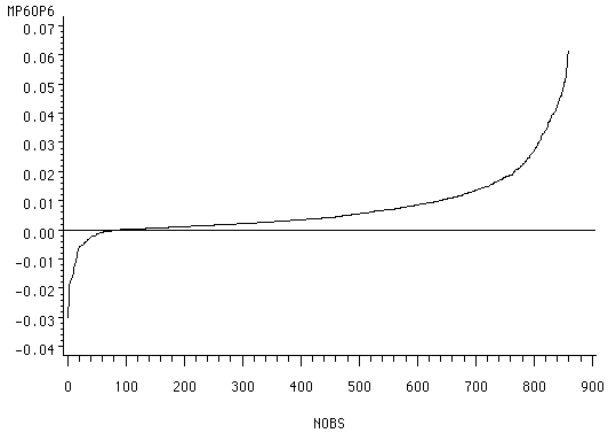
```
354  proc iml;
IML Ready
355  a1= -1.2466842;
356  a2= -1.5816872;
357  a3= -1.3081609;
358  a4= -0.6622891;
359  a5= -3.2798392;
360  a7= -2.4047374;
361  first= a1||a2||a3||a4||a5;
362  second= a2||a3||a4||a5||a7;
363  pre=first - second;
364  v={0.231858449  0.237013610  0.244599426  0.079217914
                                    0.797587280  0.317181541};
365  D=diag(v);
366  CORR={ 1  -0.0929  -0.5683  -0.3208  -0.1073   0.0935,
367    -0.0929       1  -0.0796  -0.0946  -0.1420  -0.1380,
368    -0.5683  -0.0796       1   0.0635  -0.1581  -0.1366,
369    -0.3208  -0.0946  0.0635        1  -0.2568  -0.2838,
370    -0.1073  -0.1420 -0.1582  -0.2568        1  -0.0126,
371     0.0936  -0.1380 -0.1366  -0.2838  -0.0126       1};
372   cov=D*CORR*D;
373  AU={1 -1 0 0 0 0, 0 1 -1 0 0 0, 0 0 1 -1 0 0,
                                      0 0 0 1 -1 0, 0 0 0 0 1 -1};
374  w=pre*inv(au*cov*au`)*pre`;
374                                 prob=1-probchi(w,5);
375  print first second pre v d corr cov au; print w; print prob;
```

The Wald Test Statistic value is 51.777303 for the null hypothesis Ho: A1 = A2 = A3 = A4 = A5 = A7. With five degrees of freedom the table value for the chi-square is 11.07 at the 0.05 level of significance and 15.086 at the 0.01 level. Again, as in the case of the linear model, the null hypothesis that the data were missing-at-random is rejected.

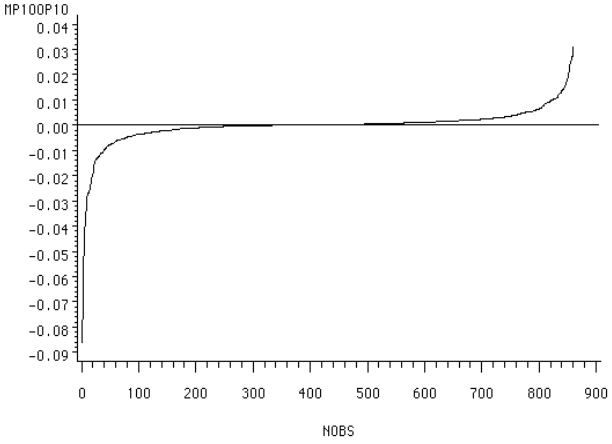
The calculations above have enabled us to distribute the missing data probability to probabilities associated with the other outcome categories. Having done this we were then able to combine the observed and missing probabilities to get rid of the observed and missing data categories altogether. By this method, we recover the marginal distribution with probabilities that represent the full population and not just its observed members.

Figure 9. Adjustments to the Original Probabilities (plot of diff = final minus original)

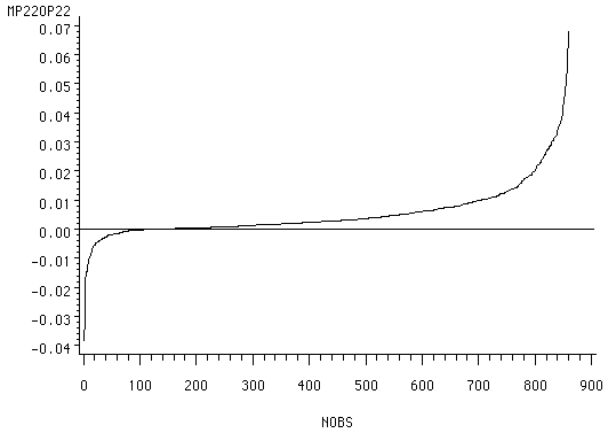
Group 1: Transportation-Utilities-Wholesale



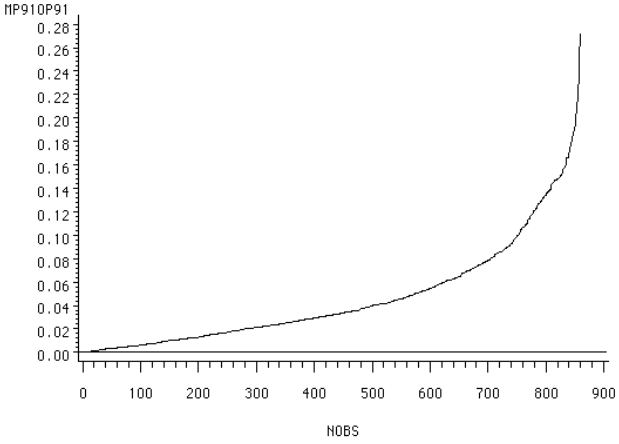
Group 2: Non-profit Organizations



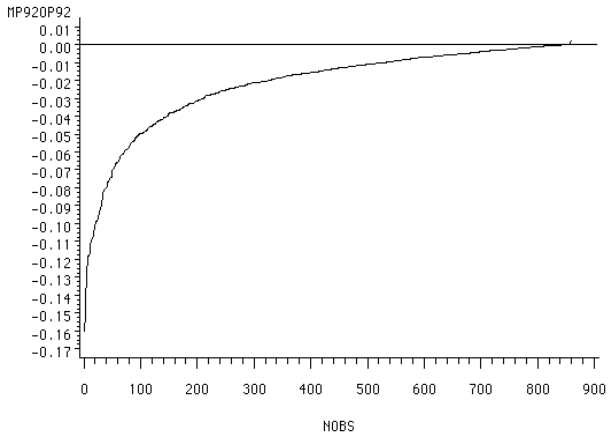
Group 3: Mining and Manufacturing



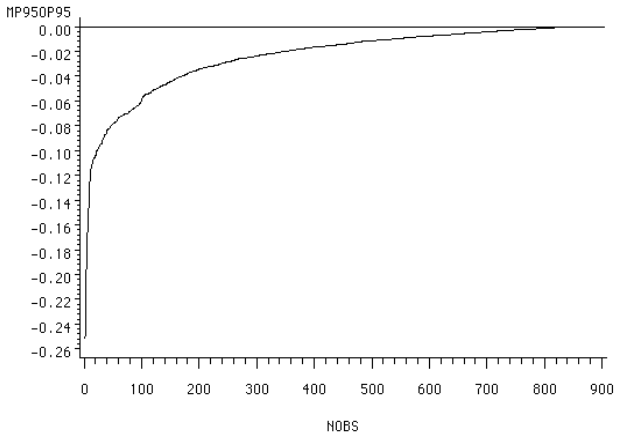
Group 4: Finance, Insurance, Real Estate



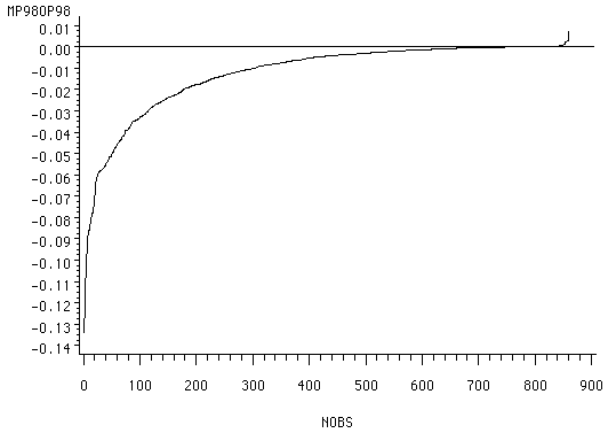
Group 5: Business Services



Group 6: Engineering



Group 7: Education and Social Services



Interpreting PROC GPLOT Graphs of Probability Differences (Final minus Original)

Figure 9 on the next page shows the adjustments that were made to the original probabilities. The horizontal line at 0.00 represents the base line before adjustment. The monotonically increasing line represents the change in the probability of a firm being in a particular industrial classification that occurs when taking into account the effects of the missing data.[4] Note that these are plots of the conditional means and not plots of the error distribution. The amount of the adjustment varies from observation to observation so the data were ordered by the amount of the needed adjustment before the data were plotted. Since the measurement is final probability minus original probability, values above the 0.00 line represent upward adjustments while those below the 0.00 line are downward adjustments.

The first three industry groups which include Transportation, Utilities, Wholesale, Mining, Manufacturing and Non-profit Organizations (Groups 1, 2 and 3) show rather minor adjustments with most in the middle range showing that virtually no adjustment was needed but with a few at either end (especially the upper end) showing some, but still relatively small, adjustments were needed.

However, the industrial group (Group 4) consisting of Finance, Insurance and Real Estate required consistently positive adjustments which were large in some cases. This suggests that this group was underrepresented in the observed data by virtue of being overrepresented in the missing data. In fact, the statistics reported in Figure 10 confirm that all of the adjustments for this group were positive with at least one adjustment as high as 0.27.

One the other hand, the last three graphs in Figure 9 representing Business Services, Engineering, Education and Social Services (Groups 5, 6 and 7) show that predominantly negative adjustments were needed (as much as -0.25 in Engineering) with a few barely visable exceptions as confirmed by the last column in Figure 10. This suggests that the proportion of these types of businesses was overstated by the originally observed (nonmissing) data.

Figure 10. Adjustments to the Original Probabilities (PROC MEANS)

Variable	N	Mean	Std Dev	Minimum	Maximum
GROUP1	860	0.0076355	0.0113956	-0.0300980	0.0612586
GROUP2	860	-0.0001481	0.0077354	-0.0860430	0.0311354
GROUP3	860	0.0055269	0.0096257	-0.0383066	0.0678353
GROUP4	860	0.0470900	0.0458737	0.0001259	0.2719392
GROUP5	860	-0.0224777	0.0250306	-0.1600833	0.0016888
GROUP6	860	-0.0251142	0.0297314	-0.2514728	-5.903096E-6
GROUP7	860	-0.0125124	0.0187359	-0.1338352	0.0070185

[4] For an explanation of this method of plotting and interpreting nonlinear models, see Marsh et al.(1994) and Chilko (1983).

The danger inherent in summary statistics such as those in Figure 10 (PROC MEANS) is in missing the intrinsically nonlinear nature of the data distribution which can be seen so clearly in the ordered (PROC SORT) data graphs (PROC GPLOT). Researchers are tempted to try to interpret a nonlinear model by looking at the average effect of a variable when all other variables are held at their means (or medians). The problem with this approach is that the group implied by this averaging process may not even approximately exist. Such an approach runs the danger of developing policy recommendations that turn out to benefit no one and ignore individual differences that reflect the real behavioral patterns inherent in the data.

Summary and Conclusions

In this paper we have demonstrated a method of testing and correcting for distortions in the probability distribution of categorical response data that have some missing values when the data include both continuous and categorical independent variables. In particular, we have applied these methods to survey data of businesses in downtown Minneapolis to correct for distortions in the observed (nonmissing) frequency data for seven types of business categories that were based on SIC codes. We have determined through the use of conditional F and Wald tests that the missing data are not missing-at-random. We then corrected the probability distribution for the distortions caused by the nonrandom nature of the missing data. PROCs CATMOD, NLIN and IML serve as useful tools for applying this method of analysis.

Moreover, we has also demonstrated a graphical way of displaying the differences in conditional group membership probabilities that more fully takes advantage of the nonlinear information inherent in this analysis. While PROC MEANS provides a useful but limited summary of the data, PROC SORT and PROC GPLOT give us additional important information when interpreting the results of nonlinear statistical analysis.

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