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Interpreting Permanent Shocks to Inflation When Money Growth is Endogenous

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Abstract: Permanent inflation shocks in VAR models have been used to try to test for superneutrality, the hypothesis that a permanent change in the growth rate of money has no permanent effect on certain real variables. A permanent increase in inflation was found to be associated with a significant permanent increase in the level of output for a number of low inflation countries. These results have been interpreted as evidence of Mundell-Tobin effects. However, if central bank behavior has made money growth endogenous, that interpretation of the findings is questionable. This paper shows that endogenous policy implies these are downward biased estimates of the intended effect. An important implication is that this evidence can not be explained by reverse causation, but it can still be interpreted as support for Mundell-Tobin effects . If policy is endogenous, the actual Mundell-Tobin effects are larger than the estimated effects. Another interesting result is that endogenous monetary policy does not provide a good explanation for the finding that countries with higher inflation rates tend to have smaller long-run output responses to permanent inflation shocks. Variation in the effect on output of exogenous changes in money growth does, however, serve as a reasonable explanation for that empirical finding. In addition, the cross country variation in impulse response of output to permanent inflation shocks suggest that exogenous changes in money growth have smaller short-run effects on output when the inflation rate is higher and/or monetary policy is more endogenous in countries with higher inflation. The paper also shows how to extend this analysis of a bivariate system to the general multi-variate case, and then applies this result to a particular trivariate structural VAR model that was used to assess superneutrality and Fisher's theory of interest rate determination.

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Introduction

Macroeconomic theory often generates long-run neutrality propositions. Long-run neutrality of exogenous changes in the money stock is a feature of almost all modern monetary theories. Consequently, much of the early work with structural VARs used neutrality assumptions to identify a model. Shapiro and Watson (1988), Blanchard and Quah (1989) and Gali (1992), for example, assumed aggregate demand shocks have no long-run effects on output. Also, theory is sometimes able to show that a permanent changes in the growth rate of the money supply will have no long-run effect on certain real variables, When that is the case, money is said to be superneutral.

King, Plosser, Stock and Watson (1991) identified a system with permanent shocks to balanced growth, inflation, and the real rate of interest. They allowed inflation shocks to affect the real rate of interest, and as a result these shocks could influence the percentage of output devoted to consumption and to investment. (These effects were found to be small and statistically insignificant.) An interesting feature of their model is that it allows inflation to be affected in the long-run by balanced growth shocks which they attribute to permanent changes in productivity. This type of response can be motivated by the assumption that the central bank allows money growth to permanently change in response to a supply shock. However, King et.al. restrict inflation shocks to have no long-run output effect. Thus because the assumption of superneutrality with respect to output is used to identify the model, they were unable to test that hypothesis about output.

Subsequently, a number of papers (REF) have attempted to estimate the effect on output from a permanent change in the growth rates of nominal variables. Superneutrality is an interesting hypothesis to test because it does not hold under all structural assumptions. The results are quite varied with a permanently higher money growth rate yielding a positive effect , a negative effect or no effect on output in the long run depending on particular features of the theory.

A number of papers use inflation rather than money growth in empirical models designed to address questions about superneutrality. One reason for this substitution is that there have been serious disagreements about which measure is best to use. The well-received principle that inflation in the long run is a monetary phenomenon allows economists to substitute inflation for money growth when formulating tests of superneutrality propositions. This notion receives a fair amount of empirical support from cross-country studies that compare long-run averages of inflation and money growth as well as time series studies that have found money growth and inflation are cointegrated. This evidence supports the hypothesis that permanent shocks to money growth and inflation are really one and the same, or if I may borrow from perhaps Milton Friedman's most famous quote: Permanent shocks to inflation are everywhere and always caused by permanent shocks to the growth rate of money. An important corollary of this idea is that permanent shocks to inflation are exogenous if and only if permanent shocks to money growth are exogenous.

As is well documented by Fisher and Seater (1993) and King and Watson (1997) (following important work by Sargent (1971) and Lucas (19??)) that time series tests of long-run propositions require permanent shocks of some kind in the sample period. Much of the recent empirical literature uses linear models and specifies permanent changes by assuming the data have a unit root. Bullard and Keating (1995), for example, studied whether inflation and real output each had a unit root for a large number of countries. When both series have a unit root, they show how to estimate the long-run effect on output from a permanent change in the inflation rate. They use these estimates to infer whether changes in the money growth rate are non-neutral with respect to output in the long run. Their key identification assumption is that permanent changes in inflation are exogenous. Fisher and Seater (1993) identify permanent changes in US money growth as exogenous, and economic theory tells us that these two exogeneity assumptions are really the same assumption.

Bullard and Keating found that the long-run effect of inflation on output is inversely related to the in-sample average level of inflation across countries. For low inflation countries a permanently higher rate of inflation tends to have a significant positive effect on the level of output. However, as the average inflation rate increases, this estimate becomes zero or negative, with the only significantly negative long-run estimate obtained for the country with the highest average rate of inflation. Bullard and Keating interpreted their results as being mostly consistent with superneutrality, except primarily for some low inflation countries where the inflation shock permanently and significantly raised the level of output. Others have used different variables to investigate superneutrality. Crosby and Otto (2000) find evidence of Mundell-Tobin effects for certain low inflation countries by using the capital stock instead of output. Rapach (2003) uses a model with three variables and three shocks and obtains a similar finding for output. His model also permits him to test the hypothesis that inflation and nominal interest rates move one for one in the long run.

The assumption that permanent shocks to inflation are exogenous is not, however, impervious to criticism. There are various ways that central banks may have reacted to real disturbances. For example, the oil price shocks in the 1970s are often cited as the source of persistently high inflation during that period. Economic theory shows that if money growth is held constant, a permanent supply shock can have a permanent effect on the price level, but only a transitory effect on the inflation rate. (assuming supply shocks affect the level of output and not the long-run growth rate of output). Hence, money growth must permanently change in response to oil price shocks for these shocks to have permanent effects on inflation. Others have claimed the Fed pursued a policy of opportunistic disinflation in the 1990s, waiting for beneficial supply shocks that would allow the central bank to disinflate with less chance of instigating a recession. There is also a long literature on how and why governments or central banks may choose to purchase debt rather than issue it to the public. An adverse supply shock will cause tax revenues to decline. If a fraction of the consequent budget deficit is purchased by the monetary authority money may grow at a faster rate growth and that could lead to more inflation eventually. Any of these policies could permit supply shocks to cause permanent changes in the rate of inflation. If monetary policy is endogenous and policy reacts to the economy in such a way that money growth is permanently affected by real shocks, then empirical models associating permanent changes in inflation with exogenous changes in money growth will be of questionable relevance. In other words, it is possible that evidence against superneutrality from these models might be explained by reverse causation whereby permanent changes in real output cause permanent changes in inflation, and not the other way. The fact that many nations had their highest postwar inflation rates following the 1970s oil price shocks seems consistent with a reverse causation interpretation. This paper investigates what can be learned about economic structure from permanent shocks to inflation when money growth is endogenous.

I develop a structural time series model that is driven by two shocks: Exogenous technology disturbances and exogenous money growth shocks. For nearly all of the analysis this paper assumes that non-superneutrality is possible. Permanent exogenous shocks to aggregate supply arise from shocks to the production function, with changes in the relative price of energy a particular source of such disturbances. I assume the central bank may allow money growth to rise in response to the decline in real output caused by adverse supply shocks.

I construct the statistical model of permanent and transitory shocks to inflation under this set of structural assumptions. I show that if superneutrality holds and long-run money growth is endogenous to supply shocks, a permanent positive shock to inflation is associated with a permanent decline in the level of output. This result is inconsistent with the positive effects, also known as Mundell-Tobin effects, that have been found in the empirical literature. Therefore, superneutrality combined with endogenous monetary policy can not explain the finding that permanent shocks to inflation are associated with a permanent increase in output for a number of low inflation countries.

Then I extend the previous analysis by augmenting the model of endogenous monetary policy with non-superneutral effects from changes in the money growth rate. Under these assumptions, I show that the estimated Mundell-Tobin effects are downward-biased estimates of the long-run output effect of a permanent exogenous increase in the money growth rate. In other words, if monetary policy has an important endogenous component, then the long-run effect on output of an exogenous permanent increase in the growth rate of money exceeds the Mundell-Tobin effect estimates in Bullard and Keating. Exogenous money growth shocks may actually have a stronger long-run effect on output than is implied by their estimates.

Then **...**

A Model of Permanent and Transitory Shocks to Inflation

The bivariate decomposition of inflation into permanent and transitory shocks can be constructed essentially by replacing output with inflation in Blanchard and Quah's (1989) decomposition. The choice of second variable depends on the question under investigation. Bullard and Keating (1995) use output growth as the second variable because they want to determine if a permanent change in the inflation rate affects the level of output, particularly in the long run. While this paper emphasizes their model, the methods developed here are suitable for interpreting empirical results from models that use another variable in place of output. Crosby and Otto (2000), for example, use capital stock as the second variable in a permanent-transitory shock decomposition for inflation. Furthermore, the appendix shows how the basic method can be applied to a VAR model with an arbitrary number of variables, and Section ? applies the method to the trivariate structural VAR used by Rapach (2003) to investigate the effect of permanent shocks to inflation on output and nominal interest rates.

Bullard and Keating's (1995) bivariate decomposition of inflation into permanent and transitory shocks can be written as:

$$\begin{bmatrix} \Delta\pi_t \\ \Delta y_t \end{bmatrix} = \begin{bmatrix} R^{\pi P}(L) & R^{\pi T}(L) \\ R^{yP}(L) & R^{yT}(L) \end{bmatrix} \begin{bmatrix} u_t^P \\ u_t^T \end{bmatrix} \tag{1}$$

where Δ = 1-L is the first difference operator, $R^{ki}(L) = \sum_{j=0}^{\infty} R_j^{ki} L^j$ for k=π,y and i=P,T , permanent shocks are defined as u^P and temporary shocks by u^T. Constants and deterministic functions of time have been omitted without loss of generality. This statistical model is identified by two assumptions:

(i) permanent and transitory shocks are uncorrelated:

$$Eu_t^P u_t^T = 0\,;$$

(ii) a temporary shock has no long-run effect on inflation:

$$\sum_{j=0}^{\infty} R_j^{\pi T} = 0$$

The statistical model can be written as:

$$\Delta X_t = r(L) u_t \tag{2}$$

where $X_t = (\pi_t, y_t)'$, $u_t = (u_t^P, u_t^T)'$, $r(L) = \begin{bmatrix} R^{\pi P}(L) & R^{\pi T}(L) \\ R^{yP}(L) & R^{yT}(L) \end{bmatrix}$

and given that the shocks are uncorrelated, it is convenient to normalize the shock variances to one and thus make the shock covariance matrix an identity: $Eu_t u_t' = \Sigma_u = I$. Recursive substitution allows us to rewrite the statistical model as:

$$X_t = X_0 + \sum_{k=0}^{t-1} Q_k u_{t-k} \tag{3}$$

$$\text{where } Q_k = \sum_{j=0}^{k} r_j \ . \tag{4}$$

This last equation implies that:

$$\frac{\partial X_t}{\partial u_{t-k}} = \sum_{j=0}^{k} r_j \quad \text{for } k = 0, 1, 2 \ldots, t\text{-}1 \ . \tag{5}$$

By letting k go to infinity, the equation yields:

$$\lim_{k\to\infty}\left(\frac{\partial X_t}{\partial u_{t-k}}\right) = \sum_{j=0}^{\infty} r_j \equiv r(1) \ \ , \tag{6}$$

where r(1) is the sum of parameters in r(L). This matrix provides the long run effect of each shock on each variable. From this last equation, we also see that the second identifying assumption implies:

$$\lim_{k\to\infty}\left(\frac{\partial \pi_t}{\partial u^T_{t-k}}\right) = \sum_{j=0}^{\infty} R_j^{\pi T} = 0 \ \ , \tag{7}$$

which is of course what it means for a shock to not have a permanent effect on inflation. Hence, this second identifying assumption makes r(1) a lower triangular matrix:

$$r(1) = \begin{bmatrix} R_{\pi P} & 0 \\ R_{yP} & R_{yT} \end{bmatrix} \ . \tag{8}$$

Identifying the Parameters in the Statistical Model

To obtain the statistical model's coefficients, the identifying assumptions are applied to a reduced-form vector autoregression (VAR):

$$\beta(L)\Delta X_t = e_t \tag{9}$$

where e_t is the vector of residuals to the inflation and output equations and $\beta(L) = I - \beta_1 L - \beta_2 L^2 - \ldots - \beta_\ell L^\ell$ represents VAR coefficients in which the identity matrix, I, and each β_j for j=1,2, ... ,ℓ are 2×2 matrices and ℓ is the number of lags in the VAR. In general, a VAR representation exists and is unique.

The statistical model and the VAR are related by two equations:

$$\beta(L) = r_0 r(L)^{-1}, \tag{10}$$

and

$$e_t = r_0 u_t \tag{11}$$

which are obtained by mapping the statistical model into the VAR. Defining β(1) as the sum of VAR coefficients matrix, the first of these two equations implies:

$$\beta(1) = r_0 r(1)^{-1}. \tag{12}$$

Then defining Σ_e as the covariance matrix for residuals, the second of these two equations along with the assumption of an identity covariance matrix for shocks to the statistical model implies:

$$\Sigma_e = r_0 r_0'. \tag{13}$$

Combining the last two equations we obtain the following relationship for long-run parameters in the statistical model:

$$r(1)r(1)' = \beta(1)^{-1}\Sigma_e \beta(1)'^{-1} \quad . \tag{14}$$

Given that r(1) is a triangular matrix, its parameters are uniquely determined as is shown by Hamilton *1994,p.?). The appropriate Cholesky decomposition of the right hand side of this equation would typically be used to estimate r(1). Given these long-run parameters, the VAR is used to calculate the dynamic response of each variable to temporary and permanent shocks (from 3 equations on the previous page):

$$r(L) = \beta(L)^{-1}\beta(1)r(1) \quad . \tag{15}$$

But rather than use this relationship to generate the dynamic response of each variable to each type of shock, economists generally take an easier approach and just simulate the time series model.

Empirical Evidence on Permanent Shocks to Inflation

The primary focus of Bullard and Keating (1995) is the long run relationship between output and inflation. They estimate values of $\frac{R_{yP}}{R_{\pi P}}$ from VAR models for countries that satisfy the necessary condition that a unit root in both inflation and output can not be rejected. This ratio of parameters can be interpreted as an estimate of the long run effect on output from a 1 percentage point increase in the rate of inflation.

Most of the empirical results are common across countries. Transitory shocks to inflation cause the price level to permanently rise, and the level of output to permanently fall. By construction this shocks has no long-run effect on inflation. These effects are consistent with a permanent aggregate supply shock interpretation. And permanent shocks to inflation cause inflation to be higher, in general.

The responses of output to shocks that permanently increase inflation yield four interesting findings. First, for most of the countries there is no statistically significant long-run relationship between permanent inflation shocks and the level of output. This finding comes from the countries for which their VAR method is appropriate and also for a number of countries where inflation has a unit root but output is trend stationary. Fisher and Seater’s (1993) long-run derivative is used to interpret these test results as evidence of superneutrality. Intuitively, if an economy has permanent shocks to inflation but no permanent shocks to output, then the permanent inflation shocks must be having no long-run effect on output. Since permanent inflation movements are the result of permanent changes in money growth, the absence of a unit root in output would imply that superneutrality is a feature of an economy. Second, the primary exceptions to the first finding are 4 countries with low average rates of inflation for which a permanent increase in inflation has a permanent positive effect on the level of output. There is also one country that had a significantly negative long-run output response to a permanent increase in inflation, and that country had the highest average inflation rate in the sample period. Third, the output effect of a permanent inflation shock is related to the average inflation rate. For 9 of the 10 countries with inflation averaging less than 11 percent, a permanent increase in inflation has a long-run positive effect on output whereas that effect is zero or negative for all 6 countries with inflation averaging 15 percent or more. The US is the only low inflation country (averaging roughly 5 percent inflation over the sample period) for which the point estimate is negative, although it is not statistically significant. And fourth, the dynamic response of output to a permanent inflation shock is inversely related to the average rate of inflation. As the inflation rate rises the entire impulse response tends to be lower. In fact, for the highest inflation countries (with average inflation rates of ? and ?), the output response to a permanent increase in inflation is non-positive at all points.

Bullard and Keating interpret the first result as some support for superneutrality. The second result is interpreted to mean that superneutrality fails to hold in some cases, and in particular that a Mundell-Tobin effect is operational in a number of low inflation countries. The third result can be interpreted as evidence consistent with theoretical work of Azariadis and Smith (1996) in which a Mundell-Tobin effect exists when inflation is low but weakens, possibly disappearing, when the inflation rate rises beyond a certain level. Their result is derived from a theory of credit market imperfections. Lagos and Rocheteau (2003) obtain a similar result using search theory. Bullard and Keating do not discuss the fourth result.

Other evidence .... ????

A Structure with Endogenous Monetary Policy

The key identifying assumption for the statistical model is that permanent inflation shocks are associated with exogenous changes in money growth. However, it is easy to make the case that changes in money growth have sometimes been endogenous. The narrative history of monetary policy (Romer and Romer?) shows how central banks have reacted to supply shocks and possibly made inflation exhibit permanent, or at least highly persistent changes as a result. The high inflation of the 1970s and early 1980s, for example, is often attributed to adverse supply shocks. Economic theory predicts that if a central bank holds the growth rate of money constant, a permanent change in oil prices will have at most a temporary effect on inflation, although the price level would be permanently higher. Central banks apparently increased the growth rate of money in an attempt to counter balance the negative output effects resulting from these adverse supply shocks (REF). Some have argued (REF) that in the 1990s a policy of opportunistic disinflation was followed whereby a central bank would lower inflation only when doing so would not be too painful for the real economy. For example, when the economy experiences beneficial supply shocks, a skillful central banker might be able to gradually bring the rate of inflation down without causing a period of negative or even weak output growth. Money growth may also be endogenous when a central bank monetizes government debt. Aggregate supply shocks cause output to fall, reducing tax revenues and forcing the deficit to rise. If some portion of the debt is financed by an increase in the money supply and if the deficits persist then a policy of monetizing debt can lead to a persistently higher money growth rate. This mechanism for endogenous money growth may be more relevant for less developed economies which have tended to experience the most severe fiscal problems and also have tended to have relatively underdeveloped financial markets. In such cases, a large and persistent deficit may yield more debt than the market will bare at a reasonable cost of financing, possibly forcing a government or its central bank to purchase all of the new debt.

According to this analysis, the central bank's reaction to supply shocks may cause the growth rate of money to move in opposite direction to the supply shock's effect on real output. Should these responses lead to permanent changes in the money growth rate, then every permanent movement in inflation would clearly not be the result of exogenous changes in money growth. This plausible criticism of empirical models that assume permanent shocks to inflation are exogenous leads to fundamental questions. Is there any value in decomposing inflation into permanent and transitory components? Do these decompositions provide any information about the structure of the economy? It turns out that even when we take account of endogenous money growth the answer to each question is yes.

Assume that the economy is described by a structure with exogenous shocks to money growth, μ, and exogenous shocks to technology, λ:

$$\begin{bmatrix} \Delta\pi_t \\ \Delta y_t \end{bmatrix} = \begin{bmatrix} \alpha^{\pi\mu}(L) & \alpha^{\pi\lambda}(L) \\ \alpha^{y\mu}(L) & \alpha^{y\lambda}(L) \end{bmatrix} \begin{bmatrix} \mu_t \\ \varepsilon_t \end{bmatrix} \tag{16}$$

where $\alpha^{ki}(L) = \sum_{j=0}^{\infty} \alpha_j^{ki} L^j$ for $k=\pi,y$ and $i=\mu,\lambda$ specifies the dynamic responses of variables to the structural shocks. The structure can be written as:

$$\Delta X_t = a(L)\varepsilon_t \tag{17}$$

where $X_t = (\pi_t, y_t)'$, $\varepsilon_t = (\mu_t, \lambda_t)'$ and $a(L) = \begin{bmatrix} \alpha^{\pi\mu}(L) & \alpha^{\pi\lambda}(L) \\ \alpha^{y\mu}(L) & \alpha^{y\lambda}(L) \end{bmatrix}$.

And using the common assumption in the structural VAR literature that these shocks are uncorrelated, we can normalize the structural shock variances to be unity and write the covariance matrix of these shocks as: $E\varepsilon_t\varepsilon_t' = \Sigma_\varepsilon = I$. Using recursive substitution on the structure we obtain:

$$X_t = X_0 + \sum_{k=0}^{t-1} \varphi_k \varepsilon_{t-k} \tag{18}$$

$$\text{where } \varphi_k = \sum_{j=0}^{k} a_j \,. \tag{19}$$

Later it will be convenient to write this matrix:

$$\varphi_k = \begin{bmatrix} \Phi_k^{\pi\mu} & \Phi_k^{\pi\lambda} \\ \Phi_k^{y\mu} & \Phi_k^{y\lambda} \end{bmatrix} \tag{20}$$

where $\Phi_k^{vs} = \sum_{i=0}^{k} \alpha_i^{vs}$ for v=π,y and s=μ,λ.

Hence, the dynamic effect of each shock on output and inflation is given by:

$$\frac{\partial X_t}{\partial \varepsilon_{t-k}} = \sum_{j=0}^{k} a_j = \varphi_k \ . \tag{21}$$

And by letting k go to infinity, the long-run effect of each shock on each variable is obtained:

$$\lim_{k\to\infty}\left(\frac{\partial X_t}{\partial \varepsilon_{t-k}}\right) = \sum_{j=0}^{\infty} a_j = a(1)\ . \tag{22}$$

where the last equality comes from setting L=1 in a(L).

The structure can also be mapped into the VAR and so obtain a relationship for the VAR coefficients:

$$\beta(L) = a_0 a(L)^{-1} \tag{23}$$

the sum of VAR coefficients:

$$\beta(1) = a_0 a(1)^{-1} \tag{24}$$

and the VAR residuals:

$$e_t = a_0 \varepsilon_t \tag{25}$$

From the last equation and the assumption about variances and covariances for the structural shocks, the covariance matrix for residuals is:

$$\Sigma_e = a_0 a_0' \ . \tag{26}$$

Applying the equation for the sum of VAR coefficients to the last equation, one obtains:

$$a(1)a(1)' = \beta(1)^{-1}\Sigma_e\beta(1)'^{-1} \quad . \tag{27}$$

How is the Statistical Model Related to the Structure?

Now we will derive two important relationships between the statistical model and the structure. The first relationship comes from combining equations (14) and (27):

$$r(1)r(1)' = a(1)a(1)' \, . \tag{28}$$

This equation describes how the long-run properties of the statistical model are related to parameters that characterize structure in the long run. The second important equation comes from inserting (23) and (24) into (15) to obtain:

$$\begin{aligned} r(L) &= \beta(L)^{-1}\beta(1)r(1) = \left(a(L)a_0^{-1}\right)\left(a(1)a_0^{-1}\right)^{-1} r(1) \\ &= a(L)a(1)^{-1}r(1) \end{aligned} \tag{29}$$

or equivalently:

$$r_j = a_j a(1)^{-1} r(1) \qquad \text{for all j.} \tag{30}$$

Insert (30) into (4), the equation describing responses from the statistical model, and then use the definition of structural responses from (19) to obtain:

$$Q_k = \sum_{j=0}^{k} r_j = \sum_{j=0}^{k} a_j a(1)^{-1} r(1) = \varphi_k a(1)^{-1} r(1) \quad . \tag{31}$$

Equation (31) characterizes the relationship between the statistical model's impulse response function and the structure's dynamics.

The long-run effect matrix for the structure can be written as:

$$a(1) = \begin{bmatrix} \alpha_{\pi\mu} & \alpha_{\pi\lambda} \\ \alpha_{y\mu} & \alpha_{y\lambda} \end{bmatrix}. \tag{32}$$

Economic theory often tells is that some of these structural parameters satisfy certain inequality constraints. These constraints will help us determine the structural implications of the effects of permanent inflation shocks. For example, theory says that an exogenous increase in the growth rate of money will raise inflation in the long run:

$$\alpha_{\pi\mu} > 0.$$

In most modern macroeconomic theories this long-run parameter is equal to one because a one percentage point increase in money growth nearly always increases long-run inflation by one percentage point, holding all else constant. Theory also supports the assumption that a permanent beneficial technology shock raises the level of output in the long run:

$$\alpha_{y\lambda} > 0 .$$

If the central bank allows long-run money growth to respond to supply shocks, it will raise the growth rate of money when the economy experiences an adverse supply shock and lower money growth when there is a beneficial supply shock:

$$\alpha_{\pi\lambda} < 0 .$$

Otherwise $\alpha_{\pi\lambda} = 0$. A positive value for this structural parameter is ruled out because that would imply a beneficial supply shock leads to a permanently higher inflation and a negative supply shock causes permanent lower inflation. Such implications are not observed in the data and also a positive value for this parameter is inconsistent with the way monetary policy should behave.

(Later on I will prove that when $\alpha_{\pi\lambda} = 0$, the permanent shocks in the statistical model will identify the dynamic effects of the exogenous money growth shocks and the temporary shocks will identify the effects of exogenous technology shocks. As we saw previously, the temporary inflation shocks from the empirical models behave like supply shocks with output and price moving in opposite directions. Hence, this robust empirical finding is consistent with the simple theory's prediction.)

The long run effect of money growth on output, $\alpha_{y\mu}$, is what economists would like to estimate using the permanents inflation shocks. A primary motivation for empirical work on this issue is that economic theory gives a variety of predictions about this structural parameter. Sometimes theory yields superneutrality ($\alpha_{y\mu} = 0$), but often a theory implies non-superneutrality with either a Mundell-Tobin effect ($\alpha_{y\mu} > 0$) or a reverse Mundell-Tobin effect ($\alpha_{y\mu} < 0$). Thus we make no assumption about the value of this structural parameter.

Implications of Endogenous Policy for Empirical Findings from Bivariate Models

Combining equations (8) and (32) in equation (28) yields:

$$\begin{bmatrix} R_{\pi P} & 0 \\ R_{yP} & R_{yT} \end{bmatrix} \begin{bmatrix} R_{\pi P} & R_{yP} \\ 0 & R_{yT} \end{bmatrix} = \begin{bmatrix} \alpha_{\pi\mu} & \alpha_{\pi\lambda} \\ \alpha_{y\mu} & \alpha_{y\lambda} \end{bmatrix} \begin{bmatrix} \alpha_{\pi\mu} & \alpha_{y\mu} \\ \alpha_{\pi\lambda} & \alpha_{y\lambda} \end{bmatrix} \tag{33}$$

from which the statistical model's long-run coefficients are related to structural parameters:

$$R_{\pi P} = \sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2} \;,\quad R_{yP} = \frac{\alpha_{y\mu}\alpha_{\pi\mu} + \alpha_{y\lambda}\alpha_{\pi\lambda}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}} \;,\quad R_{yT} = \frac{\alpha_{y\lambda}\alpha_{\pi\mu} - \alpha_{y\mu}\alpha_{\pi\lambda}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}} \tag{34}$$

Note there are two solutions for each parameter in r(1), one is positive, the other negative value and each has the same absolute value. The positive square root for $R_{\pi P}$ is used because the empirical literature focuses on measuring the effects of a permanent 1 percentage point increase in inflation.

If $\alpha_{y\lambda}\alpha_{\pi\mu} > \alpha_{y\mu}\alpha_{\pi\lambda}$ then the solution above for R_{yT} is also positive. This inequality seems likely to hold. For example, it is guaranteed to hold if either $\alpha_{\pi\lambda}$ is equal to zero, or if $\alpha_{y\mu}$ is non-negative. For this inequality to be reversed, it is necessary for $\alpha_{y\mu}$ and $\alpha_{\pi\lambda}$ to both have relatively large negative values. However, if both are negative, a sufficient condition for the previous inequality to hold would be if $|\alpha_{y\lambda}| > |\alpha_{y\mu}|$ and $|\alpha_{\pi\mu}| > |\alpha_{\pi\lambda}|$. These inequalities hold if technology shocks explain a larger share of the long-run output variance and exogenous money growth shocks explain most of the long-run variance of inflation, respectively. While it is reasonable to believe that the value of R_{yT} given above is positive, whether or not this is true has no effect on results that we obtain for permanent inflation shocks.

Of course, the impulse responses for temporary shocks are affected when the previous inequality fails to hold. But since this paper puts no emphasis on results for temporary shocks this is irrelevant. If the inequality sign is flipped and we normalized the impulse responses so that the temporary inflation shock raises output in the long run, then that would flip the response of inflation to temporary shocks about its zero response line. The statistical model identifies temporary shocks that behave like adverse supply shocks, which is what the structural model implies will occur when $\alpha_{\pi\lambda}=0$. Hence, this evidence provides some support for the inequality, but whether or not the inequality holds is of no consequence for our results.

Interpreting the Long-run Effects on Output from Permanent Inflation Shocks

From equation (34) we can see how the ratio of parameters from the statistical model is related to the underlying structure:

$$\frac{R_{yP}}{R_{\pi P}} = \frac{\alpha_{y\mu}\alpha_{\pi\mu} + \alpha_{y\lambda}\alpha_{\pi\lambda}}{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2} \quad . \tag{35}$$

In general, the statistical model will not identify structural parameters, but first we investigate conditions under which it will.

Proposition 1: If the money growth rate does not permanently change in response to real shocks, then the permanent shock to inflation will identify the long-run effect on output of an exogenous change in money growth.

This result is easily seen by setting $\alpha_{\pi\lambda}$=0 in equation (35) and obtaining $\left(\frac{R_{yP}}{R_{\pi P}}\right) = \left(\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}\right)$. This result is expected because the statistical model's identification assumptions are equivalent to the restrictions implied by that structure. Not only are the long-run effects structural but also the dynamic responses of each variable to exogenous money growth and exogenous technology shocks are identified by this permanent-transitory decomposition of inflation. (This result illustrates a necessary condition for the empirical model to identify a structure.) (Somewhere, maybe here, mention the others). Hence, if the identification assumptions are valid structural assumptions, Bullard and Keating (1995) are correct in interpreting their empirical model as a means of testing for superneutrality. The statistical model will identify what we want when policy is endogenous as long as this endogeneity does not induce a permanent change in money growth following a supply shock.

The primary concern of the paper is that permanent changes in money growth may not be exogenous for a host of possible reasons. Such a policy may make inflation endogenous to aggregate supply shocks in the long run, and therefore the economic structure would be inconsistent with the statistical model's principal identifying assumption.

One possible concern is that exogenous changes in money growth are superneutral and endogenous monetary policy causes permanent inflation shocks to be associated with permanent output movements. The period of highest inflation for many countries coincides with the 1970's oil price shocks and that evidence seems to support this hypothesis. However, the following proposition rejects this explanation for significant positive estimates obtained for a number of low inflation countries.

Proposition 2: If exogenous money growth shocks are long-run superneutral with respect to output and the central bank permanently raises the money growth rate in response to adverse supply shocks, then a permanent shock to inflation will cause the level of output to fall in the long run.

By setting $\alpha_{y\mu} = 0$ in equation (35), we get $\frac{R_{yP}}{R_{\pi P}} = \frac{\alpha_{y\lambda}\alpha_{\pi\lambda}}{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}$. The denominator is positive and the structural assumptions make the numerator negative. Thus $\left(\frac{R_{yP}}{R_{\pi P}}\right) < 0$ when $\alpha_{y\mu} = 0$ and $\alpha_{\pi\lambda} < 0$. Someone who believes in superneutrality and that permanent inflation shocks are primarily the result of monetary policy being endogenous to real disturbances would expect to find mostly negative estimates of $\left(\frac{R_{yP}}{R_{\pi P}}\right)$, instead of the positive estimates found for nearly all the low inflation countries in Bullard and Keating's sample of countries for which they can apply their method. Proposition 2 is important because it shows that endogenous monetary policy, by itself, is unable to explain the positive estimates. However, this hypothesis might explain negative estimates obtained for most of the high inflation countries and for one low inflation country.

This leads directly to a second question about the statistical model: Do positive estimates for this ratio of parameters have structural implications?

Proposition 3: Positive values for $\frac{R_{yP}}{R_{\pi P}}$ imply $\alpha_{y\mu} > 0$.

If $\alpha_{y\mu} < 0$ then $\frac{R_{yP}}{R_{\pi P}}$ is negative because the numerator in (35) is negative (and the denominator is clearly positive). The only way for the ratio of parameters to be positive is if there is a long-run positive effect of money growth on output that outweighs the negative effect resulting from long-run money growth being endogenous. Hence a positive long-run output response to a permanent increase in inflation implies a Mundell-Tobin effect.

Proposition 2 says that the estimate of $\frac{R_{yP}}{R_{\pi P}}$ is downward biased when there is superneutrality and the central bank makes money growth respond in the long run to supply. In fact, downward bias is not limited to structures that have superneutrality.

Proposition 4: For all non-negative values and a potentially wide range of negative values for $\alpha_{y\mu}$, if long-run money growth responds countercyclically to supply shocks, then the long-run output effect of a permanent inflation shock will be smaller than the long-run effect on output of an exogenous permanent shock to money growth.

To prove this proposition we need to determine the condition for $\alpha_{y\mu}$ such that:

$$\left(\frac{R_{yP}}{R_{\pi P}}\right) - \left(\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}\right) < 0 .$$

Inserting equation (35) into this inequality yields:

$$\frac{\left(\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}\right) + \left(\frac{\alpha_{y\lambda}\alpha_{\pi\lambda}}{\alpha_{\pi\mu}^2}\right)}{1 + \left(\frac{\alpha_{\pi\lambda}}{\alpha_{\pi\mu}}\right)^2} - \left(\frac{\alpha_{y\mu}}{\alpha_{\pi\mu}}\right) < 0 .$$

After some algebraic manipulation this inequality can be written as:

$$\alpha_{\pi\lambda}^2 \left(\frac{\alpha_{y\lambda}\alpha_{\pi\mu}}{\alpha_{\pi\lambda}} - \alpha_{y\mu}\right) < 0 \quad ,$$

and since the first term is positive, it implies

$$\alpha_{y\mu} > \frac{\alpha_{y\lambda}\alpha_{\pi\mu}}{\alpha_{\pi\lambda}}.$$

This inequality sets a non-positive lower bound for the long-run effect of money growth on output such that in the long run the output response to a permanent inflation shock is smaller than the effect on output from an exogenous shock to money growth.

For economies with Mundell-Tobin effects or where superneutrality holds, the estimate is clearly downward biased. Thus Bullard and Keating's conclusion that some low inflation countries experienced Mundell-Tobin effects is reinforced by endogenous policy. Proposition 3 shows that the positive estimates imply Mundell-Tobin effects are present and Proposition 4 shows that when Mundell-Tobin effects are present, endogenous money growth forces the long-run output effect from permanent inflation shocks to be smaller than the effect one would like to measure using permanent inflation shocks.

But the estimate may be downward biased even when reverse Mundell-Tobin effects are present. To get a rough measure of what this lower bound is, assume the long-run variances of output and inflation explained by technology shocks are equal to one another: $|\alpha_{y\lambda}| = |\alpha_{\pi\lambda}|$. Economic theory nearly always yields $\alpha_{\pi\mu} = 1$, and combining these two conditions with the previous inequality, we get that the estimate of $\frac{R_{yP}}{R_{\pi P}}$ is downward biased for any value of $\alpha_{y\mu} > -1$. This range encompasses most, if not all, of the values that macroeconomic theory provides for this structural parameter. And if technology shocks explain more of the long-run variance of output than of inflation, that would serve to increase the range of parameters over which this bias occurs. The inequality implies that the estimates of long-run output response to permanent inflation shocks will typically, if not always, be downward biased estimates of the intended effect.

The next issue is to determine what can explain the finding that almost all the low inflation countries in Bullard and Keating had positive estimates and none of the higher inflation countries had positive estimates. Can this cross-country relationship be explained by variation in the long-run effect of exogenous money growth on output?

Proposition 5: Holding all other structural parameters constant, the long-run response of output to a permanent inflation shock varies with the long-run effect of exogenous money growth on output.

The partial derivative of $\frac{R_{yP}}{R_{\pi P}}$ with respect to $\alpha_{y\mu}$ is:

$$\frac{\alpha_{\pi\mu}}{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2} .$$

Since that derivative is positive, the ratio of parameters will vary with the long-run effect of money growth on output, and so as $\alpha_{y\mu}$ falls so will $\frac{R_{yP}}{R_{\pi P}}$. Bullard and Keating's finding of different effects of permanent inflation shocks on output for low and high inflation countries is consistent that $\alpha_{y\mu}$ depending on average inflation. For nearly all of the low inflation countries we know $\alpha_{y\mu} > 0$ given Propositions 3 and 4. The evidence in combination with Proposition 5 suggests that this parameter is typically smaller for higher inflation countries. Therefore, Bullard and Keating's interpretation of this cross-country relationship as evidence that the Mundell-Tobin effects decline as inflation gets higher is not rejected by endogenous money growth.

The previous result is obtained by holding fixed the long-run response of money growth to supply shocks. However, $\alpha_{\pi\lambda}$ could be negatively related to the rate of inflation. Central banks in higher inflation countries tend to be less independent, and so these policymakers may experience more pressure to stimulate the economy when a negative supply shock strikes. Also, countries that have experienced higher inflation are typically small open economies that usually have relatively less-developed financial markets. Such countries often have difficulty issuing a large quantity of public debt to the private sector, and therefore have been forced to use the inflation tax to finance a deficit. Can endogenous money growth explain the difference between low and high inflation countries in terms of the long-run output effect from a permanent inflation shock?

Proposition 6: The long-run response of output to a permanent inflation shock is not clearly related to $\alpha_{\pi\lambda}$..

Taking the derivative of $\frac{R_{yP}}{R_{\pi P}}$ with respect to $\alpha_{\pi\lambda}$ and simplifying, we obtain:

$$\frac{\partial\left(\frac{R_{yP}}{R_{\pi P}}\right)}{\partial\alpha_{\pi\lambda}} = \frac{\alpha_{\pi\mu}^2\alpha_{y\lambda} - \alpha_{\pi\lambda}^2\alpha_{y\lambda} - 2\alpha_{y\mu}\alpha_{\pi\mu}\alpha_{\pi\lambda}}{\left(\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2\right)^2}.$$

The sign of this derivative is ambiguous; The denominator is positive, but the numerator consists of a positive term, a negative term and one with the sign determined by the sign of $\alpha_{y\mu}$, respectively. Therefore we can't say that the smaller estimates from high inflation countries obtain because higher inflation countries have more endogenous long-run money growth. In fact, the opposite effect is quite possible. If the values for $\alpha_{\pi\lambda}$ and $\alpha_{y\mu}$ become increasingly negative as inflation rises, then this derivative is more likely to be negative for high inflation countries. And such were the case, a more endogenous monetary policy would make the estimate of $\frac{R_{yP}}{R_{\pi P}}$ rise with inflation which, of course, would be inconsistent with the evidence.

Interpreting the Impulse Responses of Output to Permanent Inflation Shocks

Interesting findings were also observed for the impulse responses to permanent inflation shocks. The impulse responses are obtained by first calculating:

$$a(1)^{-1} r(1) = \frac{\begin{bmatrix} \alpha_{\pi\mu} & -\alpha_{\pi\lambda} \\ \alpha_{\pi\lambda} & \alpha_{\pi\mu} \end{bmatrix}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}} \tag{36}$$

and then combining this equation and equation (20) in (31) to obtain:

$$\frac{\partial X_t}{\partial u_{t-k}} = Q_k = \frac{\begin{bmatrix} \Phi_k^{\pi\mu} & \Phi_k^{\pi\lambda} \\ \Phi_k^{y\mu} & \Phi_k^{y\lambda} \end{bmatrix} \begin{bmatrix} \alpha_{\pi\mu} & -\alpha_{\pi\lambda} \\ \alpha_{\pi\lambda} & \alpha_{\pi\mu} \end{bmatrix}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}} \ . \tag{37}$$

From this equation the response of output to a permanent inflation shock is:

$$\frac{\partial y_t}{\partial u_{t-k}^P} = \frac{\Phi_k^{y\mu} \alpha_{\pi\mu} + \Phi_k^{y\lambda} \alpha_{\pi\lambda}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}} \ . \tag{38}$$

To interpret this impulse response we must make assumptions about the dynamic responses of output to the two structural shocks:

A1: Output rises for some time following an exogenous increase in money growth:

$$\frac{\partial y_t}{\partial \mu_{t-k}} = \Phi_k^{y\mu} > 0 \quad \text{for} \quad 0<k<K \quad \text{with} \quad K>0;$$

A2: Output responds positively to a beneficial supply shock:

$$\frac{\partial y_t}{\partial \lambda_{t-k}} = \Phi_k^{y\lambda} > 0 \quad \forall k.$$

Assumption A1 does not restrict the sign or magnitude of $\alpha_{y\mu}$. Assumptions about the dynamic responses of inflation to these shocks are unnecessary. (However, to answer questions about dynamic responses of inflation one would need to make assumptions about the dynamic effects of structural shocks on inflation. Most theories imply that an exogenous permanent increase in money growth will cause inflation to rise and that an exogenous permanent increase in productivity will cause inflation to fall although this decline would only be temporary if $\alpha_{\pi\lambda} = 0$.)

Bullard and Keating observe that a permanent increase in inflation never has a positive effect on output in countries with the highest inflation rates. Under our assumptions about the dynamic responses of output to exogenous money growth and technology shocks, this finding implies that the long-run effect of a technology shock on money growth is negative for these countries.

Proposition 7: If the impulse response of output to a permanent inflation shock is never positive, then $\alpha_{\pi\lambda} < 0$.

When the response of output to a permanent inflation shock is non-positive for all k<K, equation (38) implies:

$$\alpha_{\pi\lambda} \leq \frac{-\Phi_k^{y\mu}\alpha_{\pi\mu}}{\Phi_k^{y\lambda}}.$$

The structural assumptions guarantee that the right side is negative, and therefore non-positive impulse responses imply that money growth was endogenous to real shocks in the countries from Bullard and Keating's sample with very high rates of inflation. The finding that high inflation countries have $\alpha_{\pi\lambda} < 0$ is consistent with the fact that most run away inflations have been caused by out of control fiscal policy, resulting in deficits that had to be financed by the printing press. The two high inflation countries, Chile and Argentina, ran huge budget deficits in the 70s and 80s, in part a consequence of adverse oil price shocks, and they monetized much of their exploding debt load. One would expect $\alpha_{\pi\lambda}$ to be significantly less than zero for these two countries.

Another finding is that the impulse responses of output to permanent inflation shocks are related to inflation. This can be seen from Figure ? in Bullard and Keating (1995), where the dynamic response of output to a permanent shock tends to fall as average inflation rises. Can the hypothesis that money growth becomes more endogenous as inflation rises explain this finding?

Proposition 8: The dynamic response of output to a permanent inflation shock shifts down as $\alpha_{\pi\lambda}$ becomes more negative.

The partial derivative of Q_k^{yP} with respect to $\alpha_{\pi\lambda}$ is:

$$\frac{\alpha_{\pi\mu}^2 \Phi_k^{y\lambda} - \alpha_{\pi\mu}\alpha_{\pi\lambda}\Phi_k^{y\mu}}{(\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2)^{3/2}} \; .$$

Since this is positive, the dynamic response of output to a permanent shock to inflation shifts lower as $\alpha_{\pi\lambda}$ becomes more negative. Therefore more endogenous monetary policy can explain the finding that impulse responses become lower as inflation gets higher.

Alesina and Summers (1992) and others have shown an inverse empirical relationship between central bank independence and inflation. Central bank independence should be positively correlated with $\alpha_{\pi\lambda}$ because less independent central banks will be under greater pressure to respond to adverse supply shocks. This cross-country relationship between independence and inflation provides further support for the hypothesis that $\alpha_{\pi\lambda}$ tends to be more negative for high inflation countries.

But cross country variation in the endogeneity of monetary policy may not be the only reason why impulse responses vary with the rate of inflation. Another possibility is that as the average rate of inflation rises, inflation responds faster to a permanent change in the growth rate of money while output becomes less responsive. Ball, Mankiw and Romer (1989) derive such a relationship from a theory of sticky price adjustment and then subject this theory to empirical tests. They investigate the relationship between inflation and the responsiveness of inflation and output to an aggregate demand shock, and find that their theoretical prediction is supported: The dynamic effect of an aggregate demand shock on output becomes smaller as inflation rises. Can this idea explain the relationship between inflation and dynamic impulse responses?

Proposition 9: When the dynamic response of output to an exogenous permanent money growth shock is smaller, the dynamic response of output to a permanent shock shifts downward.

The partial derivative with respect to $\Phi_k^{y\mu}$ of output's response to a permanent shock, $\dfrac{\alpha_{\pi\mu}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}}$, is positive. Thus if $\Phi_k^{y\mu}$ falls with inflation, consistent with the prediction of Ball, Mankiw and Romer, then the dynamic response function of output to a permanent shock will shift downward with inflation, holding all other structural parameters fixed. Hence, the cross country variation of impulse responses with inflation can be explained by dynamic responses of output to permanent inflation shocks falling with inflation and/or by central banks in higher inflation counties being less independent.

Extending the Results to Systems with more than Two Variables

The relationship between the statistical model and to a structure can be extended to an arbitrary number of variables. The Appendix derives the general results. Here I present these results in a somewhat simpler form, but they are equivalent. Assume we have n variables, $v_t^1, v_t^2, \ldots, v_t^n$, each of which is difference-stationary and we have estimated a VAR with all variables in first differences. (It is conceptually straight-forward to modify the analysis for stationary variables and cointegration). Suppose we are interested in the effects on all variables of a permanent shock to one of these variables, and call that variable v_t^p with p∈(1,2,...,n), and assume that this shock explains all of the long-run variance of v_t^p. If there are as many structural shocks, $\varepsilon_t^1, \varepsilon_t^2, \ldots, \varepsilon_t^n$, as variables, the impulse response of variable i to a permanent shock to variable p is given by:

$$\frac{\sum_{j=1}^{n} \Phi_k^{v_i \varepsilon_j} \alpha_{v_p \varepsilon_j}}{\sqrt{\sum_{j=1}^{n} \alpha_{v_p \varepsilon_j}^2}} \quad \text{for } i = 1, 2, \ldots n. \tag{39}$$

Once can use this equation to interpret permanent changes in any variable in a system, no matter how many variables there are in the empirical model.

Letting $k \to \infty$ in the last equation yields the long run effect of this permanent shock to v_t^p on each variable:

$$\frac{\sum_{j=1}^{n} \alpha_{v_i \varepsilon_j} \alpha_{v_p \varepsilon_j}}{\sqrt{\sum_{j=1}^{n} \alpha_{v_p \varepsilon_j}^2}} \quad \text{for } i = 1, 2, \ldots n. \tag{40}$$

And when i=p, this equation becomes:

$$\sqrt{\sum_{j=1}^{n} \alpha^2_{v_P \varepsilon_j}} \tag{41}$$

Clearly the number of inequalities describing economic structure needed to interpret the effects of this permanent shock on a particular variable will increase with the number of variables in the model.

Rapach (2002) uses a trivariate VAR model and data from a number of countries to investigate the long-run effects on output and interest rates from permanent inflation shocks. This framework allows him to jointly address two hypotheses in a single empirical model for each country: (1) Long-run superneutrality with respect to output; and (2) the Fisher hypothesis that in the long run a permanent change in inflation of a given percentage point change will yield precisely the same change in nominal interest rates. He finds some evidence against both hypotheses. The evidence against superneutrality points to Mundell-Tobin effects. The evidence against Fisher's hypothesis finds that the long-run nominal rates response to a permanent change in inflation is significantly less than one, both statistically and in quantitative terms.

Using our notation, the variables in Rapach are given by $X_t = (\pi_t, i_t, y_t)'$ where i_t is the nominal interest rate, the shocks are given by $u_t = (u_t^P, u_t^2, u_t^3)'$ where I use 2 and 3 to index the second and third shocks, respectively, in his model. He restricts the second and third shocks to have only temporary effects on inflation, and writes the long-run effects of shocks on variables as:

$$r(1) = \begin{bmatrix} r_{\pi P} & 0 & 0 \\ r_{iP} & r_{i2} & 0 \\ r_{yP} & r_{y2} & r_{y3} \end{bmatrix}.$$

Rapach argues the second shock is to preferences and the third one is a shock to technology. If the actual structure is lower triangular, the identification restrictions are valid and his model can be used to obtain consistent estimates of all structural parameters.

A key identification assumption is that permanent shocks to inflation explain all the long-run variance of inflation, similar to Bullard and Keating (1995) and others. I modify Rapach's structural assumptions by allowing money growth to be endogenous to supply shocks, similar to the previous analysis in this paper. In this case we can write the vector of structural shocks as $\varepsilon_t = (\mu_t, \rho_t, \lambda_t)'$ where ρ is the shock to preferences and the structural parameter matrix is written as:

$$a(1) = \begin{bmatrix} \alpha_{\pi\mu} & 0 & \alpha_{\pi\lambda} \\ \alpha_{i\mu} & \alpha_{i\rho} & 0 \\ \alpha_{y\mu} & \alpha_{y\rho} & \alpha_{y\lambda} \end{bmatrix}.$$

The general results from above make it simple to solve for the relationship between parameters in r(1) and parameters in a(1). Interestingly, the solutions for $r_{\pi P}$ and r_{yP} in terms of structural parameters are precisely the same as we obtained earlier for the bivariate case, and so the ratio of these two parameters reported in Rapach is subject to the same analysis that was already done in this paper. In particular, his evidence of Mundell-Tobin effects is not refuted by endogenous monetary policy.

...... More on what he finds ....

The long-run effect of permanent inflation shocks on the nominal interest rate is:

$$r_{iP} = \frac{\alpha_{\pi\mu}\alpha_{i\mu}}{\sqrt{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2}}$$

Dividing by $r_{\pi P}$ allows us to calculate the effect of a 1 percentage point increase in the rate of inflation on the nominal interest rate:

$$\frac{r_{iP}}{r_{\pi P}} = \frac{\alpha_{\pi\mu}\alpha_{i\mu}}{\alpha_{\pi\mu}^2 + \alpha_{\pi\lambda}^2} = \frac{\left(\frac{\alpha_{i\mu}}{\alpha_{\pi\mu}}\right)}{1 + \left(\frac{\alpha_{\pi\lambda}^2}{\alpha_{\pi\mu}^2}\right)}.$$

If an exogenous permanent increase in money growth of 1 percentage point ultimately raises inflation by

1 percentage point and the Fisher hypothesis is true, then $\frac{\alpha_{i\mu}}{\alpha_{\pi\mu}}$ is equal to one. Consequently, $\frac{r_{iP}}{r_{\pi P}}$ will

be less than one for all non-zero values of $\alpha_{\pi\lambda}$. In general, when money growth is endogenous, the long-run effect on the nominal interest rates of a permanent shock to inflation is biased downward from the effect of an exogenous permanent increase in money growth when money growth is endogenous.

Endogenous monetary policy does not interfere with the interpretation of permanent positive output effects from permanent changes in inflation as evidence of Mundell-Tobin effects. This conclusion is a consequence of the propositions and discussions about Mundell-Tobin effects made previously in the paper. However, Rapach's (2002) finding that in the long run nominal interest rates move less than one for one following a permanent inflation shock (and what about others with a similar finding?) does not necessarily mean that Fisher's hypothesis is rejected. We can't rule out the hypothesis that endogenous money growth is forcing the long-run response of interest rates to a permanent inflation shock to be smaller than 1. Future empirical studies of Fisher's theory should control for endogenous money growth so that empirical tests won't be biased toward incorrectly rejecting the hypothesis.

Conclusion

The results of this paper are briefly summarized. When money growth is not permanently affected by supply shocks, permanent inflation shocks may be used to test hypotheses about superneutrality. If money growth is superneutral with respect to output and monetary policy allows long-run money growth to be endogenous to supply shocks, the long-run output effect of a permanent inflation shock will tend to be negative. Hence, the positive estimates of long-run output effects from permanent increase in inflation for low inflation countries can not be explained by this simple reverse causation story. (An interesting question is whether high inflation countries would still obtain zero or negative point estimates if we explicitly allowed for endogenous monetary policy.) This evidence can be interpreted as Mundell-Tobin effects. If long run money growth is affected by supply shocks, then the long-run effect on output of a permanent exogenous increase in the growth rate of money exceeds the long-run output effect of an permanent shocks to inflation for all non-negative values and a potentially large range of negative values for the parameter describing the long-run output effect from an exogenous money growth shock. This result raises the possibility that Mundell-Tobin effects may exist in virtually all countries, and it is the bias from using permanent inflation shocks combined with the possibility that the Mundell-Tobin effect becomes smaller with inflation explain why the estimates are zero or negative for high inflation countries. The fact that higher inflation countries have smaller long-run output responses to permanent movements in inflation can be explained by long-run output responses to exogenous money growth shocks that fall with inflation. It is not true, in general, that this finding can be explained by long-run money growth that tends to be more endogenous as inflation rises. But since endogenous policy can not be ruled out, this suggests the need to estimate models that allow for endogenous monetary policy in order to get consistent estimates of the this effect in the low inflation countries and also to determine whether or not high inflation countries may actually be experiencing Mundell-Tobin effects. Another motivation for more elaborate models of policy behavior is that the dynamic responses suggest endogenous monetary policy has been a factor. In particular, the finding that the dynamic response of output to a permanent shock is always negative implies that long-run money growth was endogenous to aggregate supply in the very high inflation countries that Bullard and Keating method could be applied to. Another finding is that the dynamic responses of output tend to be lower as inflation rises. This result can be explained by long-run money growth tending to become more endogenous as inflation increases or by dynamic responses of output to money growth shocks that tend to become smaller as the rate of inflation increases.

Appendix: The General Relationship between the Statistical Model and the Structure

Equations in the text can be written in terms of n-vectors and n×n matrices. For example, let X_t, e_t, u_t be n-vectors, and I, β_j, a_j, r_j and all functions of these matrices be n×n matrices.

Recall that (28) is a key relationship and rewrite it here:

$$r(1)r(1)' = a(1)a(1)' .$$

Partition matrices such that:

$$r(1) = \begin{bmatrix} R_{11} & 0_{12} \\ r_{21} & r_{22} \end{bmatrix} \quad \text{and} \quad a(1) = \begin{bmatrix} \alpha_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} .$$

This Appendix uses two numerical superscripts or subscripts to indicate the position as well as the dimension of each component in a partitioned matrix. Assume n>1 and let n_1=1 and n_2 =n-1. Hence, R_{11} and α_{11} are scalars, r_{21}, r_{12}, a_{21} and a_{12} are vectors of length n-1, and r_{22} and a_{22} are (n-1)×(n-1) matrices. This partitioning allows us to study the general n variable case for any statistical model that assumes the permanent shock to a single variable explains all of the long-run variance of that variable. Using the partitioned matrices in equation (28):

$$\begin{bmatrix} R_{11} & 0_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} R_{11} & r'_{21} \\ 0_{21} & r'_{22} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \alpha_{11} & a'_{21} \\ a'_{12} & a'_{22} \end{bmatrix}$$

it is easy to calculate:

$$R_{11} = \sqrt{\alpha_{11}^2 + a_{12}a'_{12}}$$

$$r_{21} = \frac{a_{21}\alpha_{11} + a_{22}a'_{12}}{\sqrt{\alpha_{11}^2 + a_{12}a'_{12}}}$$

Given that we want to measure the effects of a permanent increase in the first variable (e.g. inflation, or equivalently money growth) the positive square root for R_{11} is used. These two equations provide the long-run effect on each variable of a permanent shock to the first variable. One way to obtain a permanent shock to a variable is to place it first in a long-run recursively ordered system. More generally, such a shock would obtain from any model that is long-run partially recursive, in the sense of Keating (2002), for which the first block of equations consists of a single variable. Hence, this assumption applies to most structural VAR model specifications where long-run identification assumptions have been used.

In general, r_{22} will be a function of every parameter in the structure. While an expression for each coefficient in r_{22} can be derived, that is unnecessary for our purposes.

The key matrix for mapping the structure into the coefficients from the statistical model is $a(1)^{-1}r(1)$, and for the case of arbitrary n this can be written as:

$$\begin{bmatrix} \dfrac{1}{\alpha_{11} - a_{12}a_{22}^{-1}a_{21}} & \dfrac{-a_{12}a_{22}^{-1}}{\alpha_{11} - a_{12}a_{22}^{-1}a_{21}} \\ -\left(\dfrac{1}{\alpha_{11}}\right)\left(a_{22} - \dfrac{a_{21}a_{12}}{\alpha_{11}}\right)^{-1} a_{21} & \left(a_{22} - \dfrac{a_{21}a_{12}}{\alpha_{11}}\right)^{-1} \end{bmatrix} \begin{bmatrix} \sqrt{\alpha_{11}^2 + a_{12}a_{12}'} & 0_{12} \\ \dfrac{a_{21}\alpha_{11} + a_{22}a_{12}'}{\sqrt{\alpha_{11}^2 + a_{12}a_{12}'}} & r_{22} \end{bmatrix}.$$

Multiplying and simplifying we obtain:

$$a(1)^{-1}r(1) = \begin{bmatrix} \dfrac{\alpha_{11}}{\sqrt{\alpha_{11}^2 + a_{12}a_{12}'}} & \dfrac{-a_{12}a_{22}^{-1}r_{22}}{\alpha_{11} - a_{12}a_{22}^{-1}a_{21}} \\ \dfrac{a_{12}'}{\sqrt{\alpha_{11}^2 + a_{12}a_{12}'}} & \left(a_{22} - \dfrac{a_{21}a_{12}}{\alpha_{11}}\right)^{-1} r_{22} \end{bmatrix}.$$

The first column is used to derive the relationship between structure and the impulse responses to a permanent shock to a particular variable. (Actually the basic idea can be extended to cases where a variable is placed first in a Cholesky decomposition of the residuals' covariance matrix. Of course, this is a short-run recursive empirical model, and so a shock would not necessarily have a permanent effect on any variable. Some of the analysis would also be a bit different, but basically a(1) could be replaced by a_0 and r(1) could be replaced by r_0 and the basic points remain valid.)

Another key relationship is:

$$Q_k = \sum_{j=0}^{k} r_j = \sum_{j=0}^{k} a_j a(1)^{-1} r(1) = \varphi_k a(1)^{-1} r(1)$$

The cumulative structural impulse responses can be written as:

$$\varphi_k(L) = \begin{bmatrix} \Phi_k^{11}(L) & \varphi_k^{12}(L) \\ \varphi_k^{21}(L) & \varphi_k^{22}(L) \end{bmatrix}$$

where Φ_k^{11} is a scalar, φ_k^{21} and φ_k^{12} are (n-1)vectors, and φ_k^{22} is an (n-1)×(n-1) matrix, and these lag polynomials represent sums of structural parameters:

$$\Phi_k^{11} = \sum_{j=0}^{k} \alpha_j^{11} \quad \text{and}$$

$$\varphi_k^{vw} = \sum_{j=0}^{k} a_j^{vw} \quad \text{for} \quad vw = (12), (21), (22)$$

The responses of variables in levels for the statistical model are given by:

$$\begin{bmatrix} \Phi_k^{11}(L) & \varphi_k^{12}(L) \\ \varphi_k^{21}(L) & \varphi_k^{22}(L) \end{bmatrix} \begin{bmatrix} \dfrac{\alpha_{11}}{\sqrt{\alpha_{11}^2 + a_{12}a'_{12}}} & \dfrac{-a_{12}a_{22}^{-1}r_{22}}{\alpha_{11} - a_{12}a_{22}^{-1}a_{21}} \\ \dfrac{a'_{12}}{\sqrt{\alpha_{11}^2 + a_{12}a'_{12}}} & \left(a_{22} - \dfrac{a_{21}a_{12}}{\alpha_{11}}\right)^{-1} r_{22} \end{bmatrix}$$

As a result the response of the first variable to its own permanent shock is:

$$\frac{\Phi_k^{11}(L)\alpha_{11} + \varphi_k^{12}(L)a'_{12}}{\sqrt{\alpha_{11}^2 + a_{12}a'_{12}}}$$

and the response of all other variables to a permanent shock to the first variable is given by:

$$\frac{\varphi_k^{21}(L)\alpha_{11} + \varphi_k^{22}(L)a'_{12}}{\sqrt{\alpha_{11}^2 + a_{12}a'_{12}}}$$

If we take the limit as k goes to infinity for each of the last two expressions we obtain the long run effects R_{11} and r_{21}, respectively, that were calculated previously.

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