Purchasing Power Parity under a Taylor Rule Type Monetary Policy

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Abstract

Based on univariate estimation methods, the current Purchasing Power Parity (PPP, henceforth) literature suggests that PPP deviations tend to die out at a very slow rate of about 15% per year. Applying a system method based on a structural error correction model for the real exchange rate, Kim, Ogaki, and Yang (2003) find much shorter half-lives of the real exchange rates. This paper modifies their model by introducing a Taylor rule type monetary policy. Our model shows that the dynamics of the inflation and the real exchange rate can be quite different depending on the central bank's systematic response to the inflation rate. We also find shorter half-lives of real exchange rates than the current consensus of 3 to 5 years.

Keywords: Purchasing Power Parity, Taylor Rule, Convergence Rate, Half-Life

JEL Classification: C22, F31, F41

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1 Introduction

Since its revival by Cassel (1921), purchasing power parity (PPP, henceforth) has been one of the most useful building blocks in the monetary models (Frenkel, 1976) and in the neoclassical models of exchange rates (Lucas 1982) as well as in the Redux model (Obstfeld and Rogoff 1995) more recently. However, the profession is still as uncertain as ever about the empirical validity of PPP especially in the long-run.

Due to the frequent failure[1] of the law of one price in microeconomic data (Isard 1977), it is not surprising that few economists consider PPP as a short-run proposition. The empirical evidence on the validity of PPP in the long-run is mixed. If PPP holds in the long-run, PPP deviations should be only temporary and will die out eventually. If PPP does not hold even in the long-run as implied by the Balassa-Samuelson model, deviations from PPP would be permanent, and there should be a unit root in the log real exchange rate, which is a statistically testable hypothesis using either standard unit root tests or cointegration tests.

Univariate unit root tests for the post Bretton Woods period data typically do not reject the unit root null in the real exchange rate. However, the failure to reject the existence of the unit root in the real exchange rate may be simply due to a lack of power of the test. Such a conjecture has been supported by several recent studies using long-horizon data that includes both the fixed rate and the current floating data. Lothian and Taylor (1996), for example, found strong evidence of mean reversion in the US$/£ exchange rate and the Franc/£ exchange rate from almost two centuries of data. Similar evidence for PPP in the long-run can be found in researches that use panel unit root tests (Frankel and Rose 1995).

As pointed out by Rogoff (1996), one interesting consensus from these new researches is that estimates of the half-life for PPP deviations are roughly around 3 to 5 years, which seems too long to be explained by nominal rigidities. As in Dornbusch (1976), monetary shocks, with the existence of nominal rigidities, may have significant short-run effects on real exchange rates. If it is the entire story, to be consistent with the half-life estimates of 3 to 5 years, it should take about 3 to 5 years

[1] Frictions such as transportation costs, tariffs, nontariff barriers, nominal price stickiness, and pricing to markets may cause the failure.

for wages and prices to half way adjust to a shock, which is hard to believe.

Most aforementioned researches acquired half-life estimates using a single equation approach. For example, if the real exchange rate follows a stationary AR(1) process, the half-life can be approximately obtained by $-\ln 2/\ln \rho$, where ρ is the coefficient on the first lagged real exchange rate. However, Murray and Papell (2002) provide some evidence against such single equation approaches. They found that univariate methods provide virtually no useful information on the half-life due to wide confidence intervals.

One potential answer to this puzzle can be found in Kim, Ogaki, and Yang (2003). Rather than using a single equation, they suggest a system method that combines the single equation method with the restrictions implied by linear rational expectations models. One advantage of using a system method is that it will provide more efficient estimates than ones from a single equation method as long as the imposed restrictions are valid. Another advantage is that the validity of the restrictions is statistically testable by conventional Hansen's J-tests. They found much shorter half-lives for a broad range of exchange rate data sets over the current float.

This paper modifies Kim, Ogaki, and Yang (2003). They derive the restrictions from the money market equilibrium condition, identified by money demand functions. However, this paper attempts to identify the money market equilibrium from two types of interest rate rules: a central bank's reaction function with the inflation and output gap (Taylor 1993) and one with the additional exchange rate consideration (Clarida, Galí, and Gertler 1998, Engel and West 2002). Since seminal work by Taylor (1993), the Taylor rule has been one of the most popular approach in the monetary policy literature. The core implication of the Taylor rule is that the price level would become indeterminate unless the central bank responds to the inflation aggressively enough to raise the real interest rate.

One particularly interesting point was made in Clarida, Galí, and Gertler (1998, 2000). They provide strong empirical evidence of a structural break in the Fed's reaction function (Clarida, Galí, and Gertler, 2000)[2]. Putting it differently, they find that the estimate of the coefficient on the expected inflation in the near future becomes strictly greater than unity in Volker-Greenspan era, whereas the estimate from the pre-Volker era turned out to be strictly less than one, which is

[2] Unlike Taylor (1993), they used a forward looking version of the Taylor rule. In their models, the Fed is assumed to respond to the change in the inflation forecast rather than the current inflation.

consistent with the Taylor rule and the observed inflation dynamics[3]. Similar findings are made in Taylor (1999a) and Judd and Rudebusch (1998). Clarida, Galí, and Gertler (1998) also find similar international evidence for Germany and Japan.

This paper shows that the consideration of such a structural break may be critically important in using a system method, since the dynamics of prices and exchange rates can be shown quite different depending on the central bank's behavior. It turns out that the indeterminacy of the price may be transmitted to the exchange rate dynamics unless a certain condition is met. It also shows that the exchange rate determination may be affected by the present value of expected future fundamental variables. Engel and West (2002) also draw similar implication from the German data. However, they don't attempt to directly estimate the convergence rates or half-lives of exchange rates.

Once we identify reaction function coefficients of the central banks, we construct a system of difference equation for the real exchange rate with the Taylor rule consideration. Then, we implement a GMM estimation for the convergence rate of the real exchange rate and its implied half life using the current float data of the United States, Canada, France, Germany, Italy, Japan, and the United Kingdom. Unlike Kim, Ogaki, and Yang (2003) that introduce an exogenous instrumental variable in the system, a rather parsimonious estimation technique has been used as will be illustrated later. Using quarterly data sets from 1979:Q3 to 1998:Q4, this paper finds some evidence of shorter half-lives for real exchange rates, which may provide additional evidence for the link between nominal rigidities and PPP deviations.

The rest of the paper is organized as follows. In section 2, we construct a system of difference equations that combines the single equation approach with the restrictions implied by the Taylor rule type monetary policy under the assumption of nominal rigidities. In section 3, the data description and the estimation results will be discussed. Section 4 concludes.

[3] One can observe rising inflation rates in the pre-Volker era and declining inflation rates in the Volker-Greenspan era.

2 Model Specification

2.1 Gradual Adjustment Equation

We start from the conventional single equation method for the exchange rate in the presence of the nominal rigidities. Let p_t be the log domestic price level, p_t^* be the log foreign price level, and e_t be the log nominal exchange rate as the price of one unit of the foreign currency in terms of the home currency.

Let's consider the following gradual adjustment equation of the exchange rate implied by the one-good version of Mussa (1982)'s model[4].

$$\Delta p_{t+1} = b\left[E(p_t - p_t^* - e_t) - (p_t - p_t^* - e_t)\right] + E_t \Delta p_{t+1}^* + E_t \Delta e_{t+1}, \tag{1}$$

where b is a positive constant less than unity, $E(\cdot)$ denotes the unconditional expectation operator, and $E_t(\cdot)$ is the conditional expectation operator on the economic agents' information set at time t, I_t. The main idea of this equation is that the domestic price level adjusts instantaneously to the expected change in its PPP level, while it adjusts to its unconditional PPP level, $E(p_t^* + e_t)$, only slowly with the rate b. Simply put, b is the parameter of the convergence rate to PPP that we are interested in. Note that $E(p_t - p_t^* - e_t)$ is not required to be zero even if PPP holds, either because of the difference in base years or due to the difference in basket components in price indices.

We assume that the purchasing power parity holds in the long-run so that the real exchange rate $s_t = p_t^* + e_t - p_t$ is stationary, whereas p_t^*, e_t, and p_t are assumed to be difference stationary. Put it differently, we are assuming that p_t, p_t^*, and e_t are cointegrated with a cointegrating vector $\begin{bmatrix} 1 & -1 & -1 \end{bmatrix}'$. Under this assumption, (1) can be rewritten as the following estimable equation of the real exchange rate.

$$s_{t+1} = d + (1-b)s_t + \varepsilon_{t+1}, \tag{2}$$

where,

$$\mu = E(p_t - p_t^* - e_t),\ d = -b\mu,$$

[4] We don't distinguish tradables from nontradables in this paper. For the model where PPP applies to tradables only, see Kim and Ogaki (2004) or Kim (2004).

$$\varepsilon_{t+1} = \varepsilon_{1,t+1} + \varepsilon_{2,t+1} = (e_{t+1} - E_t e_{t+1}) + (p^*_{t+1} - E_t p^*_{t+1}),$$

and,

$$E_t \varepsilon_{t+1} = 0$$

Note that, in the absence of measurement errors[5], we can estimate the gradual adjustment parameter, b, by the univariate ordinary least squares method for (2).

2.2 Model I: Taylor Rule Model without Exchange Rate Target

We assume that the uncovered interest parity holds. That is,

$$i_t = i^*_t + E_t \Delta e_{t+1} \tag{3}$$

The central bank in the home country is assumed to continuously set its optimal interest rate[6] by the following simple backward looking interest rate rule[7].

$$i_t = \iota + \gamma_\pi \Delta p_t + \gamma_x x_t \ , \ \gamma_\pi > 1, \tag{4}$$

where ι is a constant that includes a certain long-run equilibrium real interest rate along with a target inflation rate, and x_t is the output deviation. We assume that the central bank in the home country has a reaction function that is consistent with the Taylor rule[8]. That is, γ_π is assumed to be greater than unity. Note that the inflation rate Δp_t is defined as quarterly inflation rates. If we want to use more conventional inflation measures such as annualized inflation rates, we require γ_π to be greater than 0.25 rather than unity. Note also that we don't impose any assumption on γ_x, even

[5] In the presence of measurement errors, the real exchange rate constructed from available data will not be stationary. However, a similar model can still be constructed by the two stage method that starts with the estimation of the cointegrating vector as the first stage estimation. See Kim, Ogaki, and Yang (2003) for details.

[6] This can be a problematic assumption due to the existence of the interest rate inertia. However, since we use quarterly data sets, it would be less restrictive.

[7] If central banks adopt the interest rate smoothing behavior, lagged interest rate terms will be added.

[8] We don't impose any restrictions on the central bank's behavior in the foreign country. However, it is very straightforward to modify the current model to incorporate such consideration. See Engel and West (2003).

though strictly positive γ_x is a standard assumption. Combining (3) and (4), we obtain following.

$$E_t \Delta e_{t+1} = \gamma_\pi \Delta p_t + \gamma_x x_t - i_t^* + \iota \tag{5}$$

Now, let's rewrite (1) as the following equation of level variables.

$$p_{t+1} = b\mu + E_t e_{t+1} + (1-b)p_t - (1-b)e_t + E_t p_{t+1}^* - (1-b)p_t^*$$

Taking differences and rearranging it, (1) can be rewritten as follows.

$$\Delta p_{t+1} = E_t \Delta e_{t+1} + (1-b)\Delta p_t - (1-b)\Delta e_t + \left[E_t \Delta p_{t+1}^* - (1-b)\Delta p_t^* + \varepsilon_t\right], \tag{6}$$

where $\varepsilon_t = \varepsilon_{1t} + \varepsilon_{2t} = (e_t - E_{t-1}e_t) + (p_t^* - E_{t-1}p_t^*)$.

From (6) and (5), we can construct the following system of stochastic difference equations.

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_{t+1} \\ E_t \Delta e_{t+1} \end{bmatrix} = \begin{bmatrix} 1-b & -(1-b) \\ \gamma_\pi & 0 \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \end{bmatrix} + \begin{bmatrix} E_t \Delta p_{t+1}^* - (1-b)\Delta p_t^* + \varepsilon_t \\ \gamma_x x_t - i_t^* + \iota \end{bmatrix} \tag{7}$$

Or equivalently,

$$\begin{bmatrix} \Delta p_{t+1} \\ E_t \Delta e_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma_\pi + 1 - b & -(1-b) \\ \gamma_\pi & 0 \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \end{bmatrix} + \begin{bmatrix} E_t \Delta p_{t+1}^* - (1-b)\Delta p_t^* + \gamma_x x_t - i_t^* + \iota + \varepsilon_t \\ \gamma_x x_t - i_t^* + \iota \end{bmatrix} \tag{8}$$

For notational simplicity, let's rewrite (8) as the following matrix form.

$$\mathbf{E}_t \mathbf{y}_{t+1} = \mathbf{A}\mathbf{y}_t + \mathbf{c}_t, \tag{9}$$

where $\mathbf{y}_t = \begin{bmatrix} \Delta p_t & \Delta e_t \end{bmatrix}'$. Using the eigenvalue decomposition, we can rewrite (9) as follows.

$$\mathbf{E}_t \mathbf{y}_{t+1} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1}\mathbf{y}_t + \mathbf{c}_t,$$

where $\mathbf{V} = \begin{bmatrix} \frac{1-b}{\gamma_\pi} & 1 \\ 1 & 1 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 1-b & 0 \\ 0 & \gamma_\pi \end{bmatrix}$, and $\mathbf{V}^{-1} = \begin{bmatrix} \frac{-\gamma_\pi}{\gamma_\pi-(1-b)} & \frac{\gamma_\pi}{\gamma_\pi-(1-b)} \\ \frac{\gamma_\pi}{\gamma_\pi-(1-b)} & \frac{-(1-b)}{\gamma_\pi-(1-b)} \end{bmatrix}$. Pre-multiplying by $\mathbf{V}^{-1}$ on both sides, we get,

$$\mathbf{E}_t\mathbf{z}_{t+1} = \mathbf{D}\mathbf{z}_t + \mathbf{h}_t, \tag{10}$$

where $\mathbf{z}_t = \mathbf{V}^{-1}\mathbf{y}_t$ and $\mathbf{h}_t = \mathbf{V}^{-1}\mathbf{c}_t$.

Note that the system has two eigenvalues $1-b$ and γ_π. Since $1-b$ is less than unity in norm, and γ_π is assumed to be strictly greater than 1, the system of difference equations in (10) has a saddle path equilibrium. It should be noted that the system would have an indeterminate solution if γ_π is also strictly less than one.

It turns out that the solution to (7) or (8) satisfies the following relation (see Appendix A for the derivation).

$$\Delta e_{t+1} = \bar{\iota} + \frac{\gamma_\pi}{1-b}\Delta p_{t+1} - \Delta p^*_{t+1} + \frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)} E_t\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+j+1}\right) + \varpi_{t+1}, \tag{11}$$

where

$$\bar{\iota} = \frac{\gamma_\pi - (1-b)}{(\gamma_\pi - 1)(1-b)}\iota,$$

$$f_t = -(i^*_t - \Delta p^*_{t+1}) + \gamma_x x_t = -r^*_t + \gamma_x x_t,$$

$$\varpi_{t+1} = \left[\frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)} E_{t+1}\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+1+j}\right) - \frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)} E_t\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+j+1}\right)\right] + \frac{1}{1-b}\varepsilon_{t+1},$$

and, by the law of iterated expectations,

$$E_t\varpi_{t+1} = 0$$

Here, f_t is a proxy variable that summarizes the fundamentals (foreign real interest rate and domestic output deviation) in the economy.

As briefly mentioned before, if γ_π is strictly less than unity, the restriction in (10) will not be valid, since the system would have a sunspot equilibrium rather than a saddle path equilibrium. Put it differently, the validity of the model critically depends on the size of γ_π. Fortunately, we have some supporting evidence, as mentioned in the introduction, at least for the data sample period we choose[9]. It should be also noted that the existence of a structural break regarding γ_π may provide some warnings to an attempt to estimate the half-life of the exchange rate using the current float data.

2.3 Model II: Taylor Rule with Exchange Rate Target

2.3.1 Model II-1: Endogenous Exchange Rate

In the Model II, we include the exchange rate in the reaction function of the central bank in the home country. Viewing the exchange rate as an endogenous variable, we start with the following Taylor rule type reaction function[10].

$$i_t = \iota + \gamma_\pi \Delta p_t + \gamma_x x_t + \gamma_e \Delta e_t \ , \ \gamma_\pi + \gamma_e > 1, \tag{12}$$

where ι is, now, a constant that includes both a target inflation rate and a target exchange rate differential as well as a certain real interest rate. In contrast to the model I, the sum of the inflation coefficient and the exchange rate coefficient is assumed to be greater than unity[11]. Combining the uncovered interest parity in (3) with (12), we obtain the following difference equation.

$$E_t \Delta e_{t+1} = \gamma_\pi \Delta p_t + \gamma_x x_t + \gamma_e \Delta e_t - i_t^* + \iota \tag{13}$$

From (6) and (13), we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_{t+1} \\ E_t \Delta e_{t+1} \end{bmatrix} = \begin{bmatrix} 1-b & -(1-b) \\ \gamma_\pi & \gamma_e \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \end{bmatrix} + \begin{bmatrix} E_t \Delta p_{t+1}^* - (1-b)\Delta p_t^* + \varepsilon_t \\ \gamma_x x_t - i_t^* + \iota \end{bmatrix} \tag{14}$$

[9] In contrast to Clarida, Gali, and Gertler (1998, 2000), Taylor (1999a), and Judd and Rudebusch (1998), Orphanides (2002) found the estimates of the inflation coefficient γ_π to be consistently greater than unity in both pre- and post-Volker regimes. Therefore, our model specification would still be valid.

[10] Similar specification can be found in Clarida, Gali, and Gertler (1998) or Engle and West (2002).

[11] Again, if we want to use annualized measures, the sum of two coefficients needs to be greater than 0.25.

Or equivalently,

$$\mathbf{E}_t\mathbf{y}_{t+1} = \tilde{\mathbf{A}}\mathbf{y}_t + \mathbf{c}_t, \tag{15}$$

where $\mathbf{y}_t$ and $\mathbf{c}_t$ are defined as before, and $\tilde{\mathbf{A}} = \begin{bmatrix} \gamma_\pi + 1 - b & \gamma_e - (1-b) \\ \gamma_\pi & \gamma_e \end{bmatrix}$. Using the eigenvalue decomposition again, (15) can be rewritten as follows. Denoting $\tilde{\mathbf{A}} = \tilde{\mathbf{V}}\tilde{\mathbf{D}}\tilde{\mathbf{V}}^{-1}$, $\tilde{\mathbf{z}}_t = \tilde{\mathbf{V}}^{-1}\mathbf{y}_t$, and $\tilde{\mathbf{h}}_t = \tilde{\mathbf{V}}^{-1}\mathbf{c}_t$ as before,

$$\mathbf{E}_t\tilde{\mathbf{z}}_{t+1} = \tilde{\mathbf{D}}\tilde{\mathbf{z}}_t + \tilde{\mathbf{h}}_t \tag{16}$$

, where $\tilde{\mathbf{V}} = \begin{bmatrix} \frac{1-b-\gamma_e}{\gamma_\pi} & 1 \\ 1 & 1 \end{bmatrix}$, $\tilde{\mathbf{D}} = \begin{bmatrix} 1-b & 0 \\ 0 & \gamma_\pi + \gamma_e \end{bmatrix}$, and $\tilde{\mathbf{V}}^{-1} = \begin{bmatrix} \frac{-\gamma_\pi}{(\gamma_\pi+\gamma_e)-(1-b)} & \frac{\gamma_\pi}{(\gamma_\pi+\gamma_e)-(1-b)} \\ \frac{\gamma_\pi}{(\gamma_\pi+\gamma_e)-(1-b)} & \frac{\gamma_e-(1-b)}{(\gamma_\pi+\gamma_e)-(1-b)} \end{bmatrix}$.

Note that the system has two eigenvalues $1-b$ and $\gamma_\pi + \gamma_e$. Since $\gamma_\pi + \gamma_e$ is assumed to be greater than unity, whereas $1-b$ is less than one as before, the system has a saddle path equilibrium. It turns out that the solution to (14) or (15) satisfies the following relation (see Appendix B for the derivation).

$$\begin{aligned} \Delta e_{t+1} &= \tilde{\iota} + \frac{\gamma_\pi}{1-b-\gamma_e}\Delta p_{t+1} - \frac{1-b}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\Delta p^*_{t+1} \\ &\quad + \frac{(\gamma_\pi+\gamma_e)-(1-b)}{(\gamma_\pi+\gamma_e)(1-b-\gamma_e)}E_t\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j \tilde{f}_{t+1+j}\right) + \tilde{\varpi}_{t+1} \end{aligned} \tag{17}$$

, where

$$\tilde{\iota} = \frac{(\gamma_\pi+\gamma_e)-(1-b)}{(1-b-\gamma_e)(\gamma_\pi+\gamma_e-1)}\iota,$$

$$\tilde{f}_t = -(i^*_t - \frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\Delta p^*_{t+1}) + \gamma_x x_t,$$

$$\begin{aligned} \tilde{\varpi}_{t+1} &= \frac{(\gamma_\pi+\gamma_e)-(1-b)}{(\gamma_\pi+\gamma_e)(1-b-\gamma_e)}\left[\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j E_{t+1}\tilde{f}_{t+1+j} - \sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j E_t\tilde{f}_{t+1+j}\right] \\ &\quad + \frac{1}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\varepsilon_{t+1}, \end{aligned}$$

and, by the law of iterated expectations,

$$E_t\tilde{\varpi}_{t+1} = 0$$

Here, $\tilde{f}_t$ is a similar proxy as before for the fundamentals in the economy. We can easily check the solutions in (11) and (17) are identical if γ_e is zero.

Note that this model will be relevant only to the cases of the DM/US$ and ¥/US$ exchange rate dynamics when either Germany or Japan is the home country whereas the US is the foreign country.

2.3.2 Model II-2: Exogenous Exchange Rate

In previous section, we assume that the dynamics of the exchange rate would be determined from inside the system. Now we consider a case that a certain exchange rate is included in the Taylor rule as a pure forcing variable. For example, the Bundesbank may consider the DM/US$ exchange rate, but not necessarily the DM/¥ exchange rate, in seeking for the target interest rate. Then, in our model, the dynamics of the DM/¥ exchange rate should incorporate the DM/US$ exchange rate as a forcing variable as will be shown in this section.

Let's consider the following interest rate rule.

$$i_t = \iota + \gamma_\pi \Delta p_t + \gamma_x x_t + \gamma_e^u \Delta e_t^u \ , \ \gamma_\pi > 1, \tag{18}$$

where e_t^u is an exogenous exchange rate that is different from endogenous exchange rate e_t, whereas γ_e^u is the corresponding reaction coefficient. Note that the saddle point equilibrium requires $\gamma_\pi > 1$ as in the Model I, because the exchange rate e_t^u is a forcing variable in this system. Combining (18) with (3) as before, we get,

$$E_t \Delta e_{t+1} = \gamma_\pi \Delta p_t + \gamma_x x_t + \gamma_e^u \Delta e_t^u \ - i_t^* + \iota \tag{19}$$

From (6) and (19), we obtain the following system of difference equation.

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_{t+1} \\ E_t \Delta e_{t+1} \end{bmatrix} = \begin{bmatrix} 1-b & -(1-b) \\ \gamma_\pi & 0 \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta e_t \end{bmatrix} + \begin{bmatrix} E_t \Delta p^*_{t+1} - (1-b)\Delta p^*_t + \varepsilon_t \\ \gamma_x x_t + \gamma_e^u \Delta e_t^u - i_t^* + \iota \end{bmatrix} \quad (20)$$

Note that the only difference between (20) and (7) is the forcing variables in the second equation. Therefore, it is easy to show that the solution satisfies the following.

$$\Delta e_{t+1} = \bar{\iota} + \frac{\gamma_\pi}{1-b}\Delta p_{t+1} - \Delta p^*_{t+1} + \frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)} E_t \left(\sum_{j=0}^{\infty} \left(\frac{1}{\gamma_\pi} \right)^j \breve{f}_{t+j+1} \right) + \varpi_{t+1}, \quad (21)$$

where

$$\breve{f}_t = -r_t^* + \gamma_x x_t + \gamma_e^u \Delta e_t^u$$

The constant $\bar{\iota}$ remains the same and the same orthogonality condition holds. And the exchange rate with the third party country Δe_t^u is simply added to the fundamental term $\breve{f}_t$ due to (18).

Note that this model would be relevant to the cases of all Deutch Mark based and Japanese Yen based exchange rates except DM/US$ and ¥/US$.

2.4 GMM Estimation

We start with the model I. The first equation in our estimable system is from the single equation approach. That is, from (1) we get the following.

$$\Delta p_{t+1} = c + b(p_t^* + e_t - p_t) + \Delta p^*_{t+1} + \Delta e_{t+1} - \varepsilon_{t+1}, \quad (22)$$

where

$$c = b\mu,\ \varepsilon_{t+1} = \varepsilon_{1t+1} + \varepsilon_{2t+1} = (e_{t+1} - E_t e_{t+1}) + (p^*_{t+1} - E_t p^*_{t+1}),$$

and,

$$E_t \varepsilon_{t+1} = 0 \quad (23)$$

Or (2) can be used under the cointegration assumption.

The estimation of the equation (11) in our system is a challenging task, since it has an infinite sum of rationally expected discounted future fundamentals.

Following Hansen and Sargent (1980, 1982), we will linearly project $E_t(\cdot)$ onto the econometrician's information set at time t, Ω_t, which is a subset of I_t. Denoting $\hat{E}_t(\cdot)$ as the linear projection operator onto Ω_t, let's rewrite (11) as follows.

$$\Delta e_{t+1} = \bar{\iota} + \frac{\gamma_\pi}{1-b}\Delta p_{t+1} - \Delta p^*_{t+1} + \frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)}\hat{E}_t\left[\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+j+1}\right] + \omega_{t+1}, \tag{24}$$

where,

$$\omega_{t+1} = \varpi_{t+1} + \frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)}E_t\left[\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+j+1}\right] - \frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)}\hat{E}_t\left[\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+j+1}\right],$$

and,

$$\hat{E}_t\omega_{t+1} = 0, \tag{25}$$

by the law of iterated expectations.

It is not an easy task to pick appropriate variables that belongs to Ω_t[12]. Thus, we are going to follow a rather parsimonious way by assuming $\Omega_t = \{f_t, f_{t-1}, f_{t-2}, \cdots\}$ instead of introducing other exogenous instrumental variables. This assumption will be an innocent one under the stationarity assumption of the fundamental term, f_t, and it can greatly lessen the burden in our GMM estimation process by reducing the number of coefficients to be estimated significantly.

Let f_t be a zero mean covariance stationary, linearly deterministic stochastic process so that it has a Wold representation as follows.

$$f_t = \alpha(L)e_t, \tag{26}$$

where $e_t = f_t - \hat{E}_{t-1}f_t$ and $\alpha(L)$ is square summable. Assuming that $\alpha(L) = 1 + \alpha_1 L + \alpha_2 L^2 + \cdots$ is invertible, (26) can be rewritten as the following autoregressive representation.

$$\zeta(L)f_t = e_t \tag{27}$$

[12] Kim, Ogaki, and Yang (2003) use the foreign inflation rate as a scalar instrument variable.

, where $\zeta(L) = \alpha^{-1}(L) = 1 - \zeta_1 L - \zeta_2 L^2 - \cdots$. Linearly projecting $\hat{E}_t \left[\sum_{j=0}^{\infty} \left(\frac{1}{\gamma_\pi} \right)^j f_{t+j+1} \right]$ onto Ω_t, Hansen and Sargent (1980) show that the following relation holds.

$$\hat{E}_t \left[\sum_{j=0}^{\infty} \left(\frac{1}{\gamma_\pi} \right)^j f_{t+j+1} \right] = \psi(L) f_t = \left[\frac{1 - \left(\gamma_\pi \zeta(\gamma_\pi^{-1}) \right)^{-1} \zeta(L) L^{-1}}{1 - (\gamma_\pi L)^{-1}} \right] f_t \tag{28}$$

Let's assume that f_t can be represented by a finite order AR(r) process[13], which implies $\zeta(L) = 1 - \sum_{j=1}^{r} \zeta_j L^j$, where $r < \infty$. Then, it can be shown that the coefficients of $\psi(L)$ can be computed recursively as follows.

$$\psi_0 = \left(1 - \left(\frac{1}{\gamma_\pi} \right) \zeta_1 - \cdots - \left(\frac{1}{\gamma_\pi} \right)^q \zeta_q \right)^{-1}, \tag{29}$$

$$\psi_r = 0 \tag{30}$$

$$\psi_{j-1} = \left(\frac{1}{\gamma_\pi} \right) \psi_j + \left(\frac{1}{\gamma_\pi} \right) \psi_0 \zeta_j, \tag{31}$$

where $j = 1, 2, \cdots, r$. Then, the system of estimable equations can be constructed as follows.

$$\Delta p_{t+1} = c + \Delta p_{t+1}^* + \Delta e_{t+1} + b(p_t^* + e_t - p_t) + \varepsilon_{1,t+1} \tag{32}$$

$$\Delta e_{t+1} = \bar{\iota} + \frac{\gamma_\pi}{1-b} \Delta p_{t+1} - \Delta p_{t+1}^* + \frac{\gamma_\pi - (1-b)}{\gamma_\pi (1-b)} \left(\psi_0 f_t + \psi_1 f_{t-1} + \cdots + \psi_{r-1} f_{t-r+1} \right) + \varepsilon_{2,t+1} \tag{33}$$

$$f_{t+1} = k + \zeta_1 f_t + \zeta_2 f_{t-1} + \cdots \zeta_r f_{t-r+1} + \varepsilon_{3,t+1}, \tag{34}$$

where k is a constant scalar[14].

A GMM estimation, then, can be implemented by the following set of orthogonality conditions.

$$E z_{1,t} \left(s_{t+1} - d - (1-b) s_t \right) = 0 \tag{35}$$

[13] We can use conventional Akaike Information criteria or Bayesian Information criteria in order to choose the degree of such autoregressive processes.

[14] Recall that Hansen and Sargent (1980) assumed zero-mean covariance stationary processes. If variables of interest have non-zero unconditional means, we can either demean them prior to estimation or include them but leave their coefficients unconstrained. West (1989) showed that the efficiency can be obtained by adding additional restrictions on deterministic terms. However, the imposition of such additional restrictions is quite burdensome, so we simply add the constants here.

$$Ez_{2,t-\tau}\left(\Delta e_{t+1} - \bar{\imath} - \frac{\gamma_\pi}{1-b}\Delta p_{t+1} + \Delta p^*_{t+1} - \frac{\gamma_\pi - (1-b)}{\gamma_\pi(1-b)}\left(\psi_0 f_t + \cdots + \psi_{r-1} f_{t-r+1}\right)\right) = 0 \quad (36)$$

$$Ez_{2,t-\tau}\left(f_{t+1} - k - \zeta_1 f_t - \zeta_2 f_{t-1} - \cdots - \zeta_r f_{t-r+1}\right) = 0, \quad (37)$$

where $z_{1,t} = \{1, s_t\}$, $z_{2,t} = \{1, f_t\}$, and $\tau = 0, 1, \cdots, p$[15].

Note that the convergence rate parameter b can be estimated separately either by a univariate least squares estimation to (35) or by Hansen and Sargent method to (36) along with (37). Our system method combines them by estimating (35) through (37) simultaneously.

Similarly, a GMM estimation for the model II-1 can be implemented by redefining the fundamental term as in (17) and replacing (36) by the relevant orthogonality condition. A GMM estimation for the model II-2 is also straightforward.

3 Empirical Results

In this section, we report estimates of the convergence rate parameter b and its implied half-lives. I choose Germany, Japan, and the US as a home country, since we have some empirical evidence on their reaction functions that are consistent with our models. We use quarterly data from 1979:III to 1998:IV for all countries we consider. For the United States, the chosen sample period seems appropriate, because it coincides with the on-going Volker-Greenspan era. The evidence of a structural break at the beginning of the Volker-Greenspan era (1979:III, see Clarida, Galí, and Gertler 2000) may give some rationale to this choice especially for our model that requires aggressive policy responses to the inflation. One can also find similar justification for the sample period of Germany and Japan from Clarida, Galí, and Gertler (1998). One complication comes from the German reunification issue. However, we will ignore it for now to keep a reasonable number of observations.

I use CPIs, PPIs, and GDP deflators for price variables in order to construct real exchange rates. The aggregate industry selling price (Canada), WPI (Italy, Japan), and Industrial Output Prices (the U.K.) are used for PPIs. Short-term Treasury bill rates were used for interest rates with an exception of Japan[16]. For output deviations, I use the quadratically detrended real GDP series[17].

[15] p does not necessarily coincide with r.

[16] I use the call money rate and the discount rate for Japan, since the Treasury bill rate was not available.

[17] I also tried the cyclical component of the real GDP using HP-filter with 1600 of smooting parameter. The results

All data are obtained from IFS CD Rom. Degrees of autoregressive processes of fundamentals were chosen by Bayesian Information criteria.

I start with the estimation of the central bank's reaction functions by ordinary least squares. One related issue is what price based inflation should be included in the reaction function. CPI inflations will definitely qualify because a central bank may seek for domestic price stability and CPI data are available on a monthly basis[18] (Clarida, Galí, and Gertler 1998, 2000). GDP deflator inflations seem also a suitable choice, since it measures overall price movements (Taylor 1999a). It is hard to find a very convincing rationale to the use of PPI inflations separately, even though it may deliver some additional information on the future inflation to the central bank. Another issue is that whether the Model II provides an appropriate specification of the Fed's reaction function. The current literature seems to be pessimistic on that matter. So I try only Model I for the US, while both models apply to Germany and Japan.

The estimates of the reaction function coefficients for the US, Germany, and Japan are reported in Table 1. The ordinary least squares estimation was used for all countries. The estimation results for the Model II didn't change the estimates for the inflation and output deviation coefficients much, and coefficient estimates of the exchange rate were very small, and thus are not reported here.

Estimates of the Fed are consistent with our model. The coefficient of the GDP deflator inflation is highly significant and is about 1.32 with annualized inflation rates. Coefficients on output deviations are problematic, because estimates were either insignificant or had wrong sign. Estimates of the Bundesbank's reaction function are consistent with our model. Inflation coefficients are always greater than unity and highly significant. Output deviation coefficient estimates are also reasonable and highly significant. All coefficient estimates in the Bank of Japan's reaction function are highly significant and consistent with our Model with the exception of the PPI inflation. It should be noted that the OLS estimation may not yield precise estimates for the reaction function coefficients for several reasons such as serially correlated errors. However, since our focus does not lie on estimating a reaction function, we will take those estimates as given at this stage. Furthermore, the half-life

are quantitatively similar.

[18] The FOMC in the US, for example, has 8 regular meetings in a year as well as unscheduled meetings, monetary data such as CPI may greatly affect the Fed's decision process.

estimation results were quantitatively similar over a range of coefficient values.

Half-life estimates are reported in Table 2, 3, and 4 for the Model I. Model II estimates are not reported here, since estimates were quantitatively similar due to the similarity of reaction function estimates. Overall, we find the shorter half-life estimates of the real exchange rate than the current consensus of 3 to 5 years[19]. The average half-life estimates were 2.025, 2.812, and 3.527 when the base country was Germany, Japan, and the US, respectively.

It should be noted that the half-life estimates of a certain real exchange rate might be different depending on which country is chosen as a base country. Interestingly, the half-life estimates of the real exchange rate between Germany and Japan were quite similar irrespective of the choice of base country. Similarly, the half-life estimates between Germany and the US were less than 2 years for the CPI based real exchange rate, and less than 3 years for the PPI and GDP deflator based real exchange rates.

4 Conclusion and Future Research

As Rogoff (1996) points out, the current consensus of 3 to 5 years of half lives are too long for the PPP deviations to be explained by nominal rigidities, since the current business cycle literature suggests that such frictions would die out about in two years. This paper notes that such 3 to 5 years of half life estimates are mostly obtained from a single equation approach of the real exchange rate. Even though it is easy to implement and robust to misspecification, it may result in less efficient estimates by not utilizing economic theories.

This paper adopts a system method by Kim, Ogaki, and Yang (2003). In a system method, we construct a system of difference equation that incorporates economic theories or models in order to derive economically meaningful restrictions from the solution, which will be imposed to the estimation procedures. If the theory or model is correct, the resulted estimates will be more efficient compared with the estimates from a single equation approach. Using a Taylor rule type monetary policy as a relevant model, this paper builds a estimable system of the real exchange rate and inflation rates.

[19] Since our results are based on the Taylor rule estimations, we need to be careful in interpreting the results due to second stage estimation bias.

The resulted half life estimates are overall shorter than the current consensus of 3 to 5 years. If the imposed theory of interest rate rules of a central bank is valid for Germany, Japan, the US, the restrictions I derived here may be appropriate, and thus the resulted shorter half life estimates might be more efficient than the 3 to 5 years of consensus. And we may have some added evidence for the argument that the PPP deviations are mostly caused by nominal rigidities.

However, the results in this paper were not fully satisfactory considering low p-values of J-statistics. I believe that this is partly due to the use of a single instrument. Therefore, I will try the similar analysis by using a set of instruments, possibly including all variables in the system.

It is also possible to extend the model to the two goods (tradables and nontradables) model of the exchange rate as in Kim (2004) or Kim and Ogaki (2004). We may assume that a central bank targets more general inflation indices such as the GDP deflator inflation, whereas the long-run PPP holds only for the tradable goods price, which may be proxied by PPI inflation.

Table 1. Taylor Rule Estimation (Model I)

		γ_π	γ_x	R^2
Germany	CPI	1.438 (0.301)	0.311 (0.049)	0.944
	PPI	1.306 (0.226)	0.320 (0.045)	0.949
	Def	1.356 (0.403)	0.293 (0.057)	0.936
Japan	CPI	1.510 (0.405)	0.275 (0.120)	0.799
	PPI	0.836 (0.298)	0.261 (0.131)	0.785
	Def	2.485 (0.397)	0.285 (0.103)	0.844
US	CPI	3.553 (0.393)	-0.128 (0.137)	0.919
	PPI	1.336 (0.319)	0.026 (0.176)	0.863
	Def	5.290 (0.426)	0.070 (0.110)	0.945

† Standard errors are reported in parentheses.

Table 2. Germany as the Home Country (Model I)

		Half Life	b	s.e.	J-stat	p-value	d.f.
	CPI	2.753	0.061	0.037	1.724	0.422	2
Canada	PPI	1.951	0.085	0.039	5.011	0.286	4
	Def	5.010	0.034	0.028	1.050	0.592	2
	CPI	0.578	0.259	0.071	9.087	0.003	1
France	PPI	0.785	0.198	0.069	6.662	0.010	1
	Def	0.716	0.215	0.069	4.499	0.034	1
	CPI	3.182	0.053	0.062	8.221	0.004	1
Italy	PPI	0.912	0.173	0.062	4.981	0.026	1
	Def	3.378	0.050	0.057	4.510	0.034	1
	CPI	1.380	0.118	0.048	6.616	0.158	4
Japan	PPI	1.562	0.105	0.060	4.333	0.363	4
	Def	1.320	0.123	0.054	4.101	0.392	4
	CPI	1.881	0.088	0.054	4.588	0.332	4
UK	PPI	1.816	0.091	0.060	8.772	0.067	4
	Def	2.106	0.079	0.048	4.161	0.385	4
	CPI	1.796	0.092	0.042	3.672	0.452	4
US	PPI	2.663	0.063	0.037	3.679	0.451	4
	Def	2.663	0.063	0.029	2.945	0.567	4

Table 3. Japan as the Home Country (Model I)

		Half Life	b	s.e.	J-stat	p-value	d.f.
Canada	CPI	4.245	0.040	0.033	0.791	0.852	3
	Def	5.164	0.033	0.030	0.213	0.899	2
France	CPI	1.405	0.116	0.044	1.573	0.456	2
	Def	1.473	0.111	0.047	2.953	0.399	3
Germany	CPI	1.276	0.127	0.045	1.631	0.803	4
	Def	1.265	0.128	0.049	2.624	0.623	4
Italy	CPI	4.726	0.036	0.049	1.948	0.745	4
	Def	2.133	0.078	0.061	0.001	0.977	1
UK	CPI	2.663	0.063	0.038	0.577	0.966	4
	Def	2.707	0.062	0.039	1.309	0.860	4
US	CPI	3.378	0.050	0.035	0.421	0.517	1
	Def	3.310	0.051	0.031	1.639	0.441	2

Note: Estimates with PPI based real exchange rate not reported due to the $\gamma_\pi < 1$.

Table 4. US as the Home Country (Model I)

		Half Life	b	s.e.	J-stat	p-value	d.f.
Canada	CPI	n.a.	-0.027	0.028	4.599	0.032	1
	PPI	n.a.	-0.090	0.036	16.82	0.002	4
	Def	n.a.	-0.014	0.032	0.156	0.693	1
France	CPI	4.139	0.041	0.041	8.182	0.042	3
	PPI	3.182	0.053	0.040	1.656	0.437	2
	Def	4.139	0.041	0.037	2.873	0.238	2
Germany	CPI	2.000	0.083	0.046	0.018	0.894	1
	PPI	2.663	0.063	0.040	2.369	0.124	1
	Def	3.063	0.055	0.034	4.514	0.341	4
Italy	CPI	2.192	0.076	0.044	4.320	0.364	4
	PPI	2.578	0.065	0.041	2.096	0.351	2
	Def	3.680	0.046	0.034	1.137	0.566	2
Japan	CPI	5.164	0.033	0.037	1.613	0.656	3
	PPI	6.331	0.027	0.035	1.280	0.527	2
	Def	4.139	0.041	0.030	0.137	0.711	1
UK	CPI	1.298	0.125	0.063	4.657	0.324	4
	PPI	5.328	0.032	0.043	14.05	0.003	3
	Def	3.007	0.056	0.057	1.103	0.576	2

Appendix

A Derivation of (11)

Since $\mathbf{D}$ is diagonal, (10) can be decomposed to the following two scalar difference equations.

$$E_t z_{1t+1} = (1-b)z_{1t} + h_{1t} \tag{38}$$

$$E_t z_{2t+1} = \gamma_\pi z_{2t} + h_{2t} \tag{39}$$

Solving (38) backward while solving (39) forward, we get,

$$z_{1t} = \sum_{j=0}^{\infty}(1-b)^j h_{1t-1-j} + \sum_{j=0}^{\infty}(1-b)^j u_{t-j} \tag{40}$$

$$z_{2t} = -\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j+1} E_t h_{2t+j} \tag{41}$$

, where u_t is any white noise. Since $\mathbf{y}_t = \mathbf{V}\mathbf{z}_t$,

$$\begin{bmatrix} \Delta p_t \\ \Delta e_t \end{bmatrix} = \begin{bmatrix} \frac{1-b}{\gamma_\pi} & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \sum_{j=0}^{\infty}(1-b)^j h_{1t-1-j} + \sum_{j=0}^{\infty}(1-b)^j u_{t-j} \\ -\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j+1} E_t h_{2t+j} \end{bmatrix} \tag{42}$$

Therefore,

$$\Delta p_t = \frac{1-b}{\gamma_\pi}\sum_{j=0}^{\infty}(1-b)^j h_{1t-1-j} - \sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j+1} E_t h_{2t+j} + \frac{1-b}{\gamma_\pi}\sum_{j=0}^{\infty}(1-b)^j u_{t-j} \tag{43}$$

$$\Delta e_t = \sum_{j=0}^{\infty}(1-b)^j h_{1t-1-j} - \sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j+1} E_t h_{2t+j} + \sum_{j=0}^{\infty}(1-b)^j u_{t-j} \tag{44}$$

, where

$$\mathbf{h}_t = \frac{1}{\gamma_\pi - (1-b)} \begin{bmatrix} -\gamma_\pi(E_t\Delta p^*_{t+1} - (1-b)\Delta p^*_t + \varepsilon_t) \\ \gamma_\pi\left(E_t\Delta p^*_{t+1} - (1-b)\Delta p^*_t + \varepsilon_t\right) + (\gamma_\pi - (1-b))(\gamma_x x_t - i^*_t + \iota) \end{bmatrix} \tag{45}$$

From (43) and (44), we obtain following.

$$\Delta e_t = \frac{\gamma_\pi}{1-b}\Delta p_t + \frac{\gamma_\pi - (1-b)}{1-b}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j+1} E_t h_{2t+j} \tag{46}$$

Applying law of iterated expectations and rearranging the second term in (46),

$$\begin{aligned}
& \frac{\gamma_\pi - (1-b)}{1-b}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j+1} E_t h_{2t+j} \\
= \; & \frac{1}{1-b}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j+1} [\gamma_\pi \left(E_t\Delta p^*_{t+1+j} - (1-b)E_t\Delta p^*_{t+j} + E_t\varepsilon_{t+j}\right) \\
& +(\gamma_\pi - (1-b))(\gamma_x E_t x_{t+j} - E_t i^*_{t+j} + \iota)] \\
= \; & \frac{1}{1-b}\left[E_t\Delta p^*_{t+1} - (1-b)\Delta p^*_t + \varepsilon_t + \left(\frac{1}{\gamma_\pi}\right)E_t\Delta p^*_{t+2} - \left(\frac{1}{\gamma_\pi}\right)(1-b)E_t\Delta p^*_{t+1} + \left(\frac{1}{\gamma_\pi}\right)E_t\varepsilon_{t+1} + \cdots\right] \\
& + \frac{\gamma_\pi - (1-b)}{1-b}\left[\left(\frac{1}{\gamma_\pi}\right)\gamma_x x_t - \left(\frac{1}{\gamma_\pi}\right) i^*_t + \left(\frac{1}{\gamma_\pi}\right)\iota + \left(\frac{1}{\gamma_\pi}\right)^2 \gamma_x E_t x_{t+1} - \left(\frac{1}{\gamma_\pi}\right)^2 E_t i^*_{t+1} + \left(\frac{1}{\gamma_\pi}\right)^2 \iota + \cdots\right] \\
= \; & -\Delta p^*_t + \frac{1}{1-b}\varepsilon_t + \frac{1}{1-b}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j}\left[\left(\frac{\gamma_\pi-(1-b)}{\gamma_\pi}\right)\left(E_t\Delta p^*_{t+1+j} + \gamma_x E_t x_{t+j} - E_t i^*_{t+j} + \iota\right)\right] \\
= \; & \frac{\gamma_\pi-(1-b)}{(\gamma_\pi-1)(1-b)}\iota - \Delta p^*_t + \frac{1}{1-b}\varepsilon_t + \frac{\gamma_\pi-(1-b)}{\gamma_\pi(1-b)}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j} E_t\left[-(i^*_{t+j} - \Delta p^*_{t+1+j}) + \gamma_x x_{t+j}\right] \\
= \; & \bar{\iota} - \Delta p^*_t + \frac{1}{1-b}\varepsilon_t + \frac{\gamma_\pi-(1-b)}{\gamma_\pi(1-b)}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j} E_t f_{t+j}
\end{aligned}$$

, therefore we obtain the following[20].

$$\Delta e_t = \bar{\iota} + \frac{\gamma_\pi}{1-b}\Delta p_t - \Delta p^*_t + \frac{1}{1-b}\varepsilon_t + \frac{\gamma_\pi-(1-b)}{\gamma_\pi(1-b)}E_t\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j} f_{t+j}\right) \tag{47}$$

, where $\bar{\iota} = \frac{\gamma_\pi-(1-b)}{(\gamma_\pi-1)(1-b)}\iota$ and $f_t = -(i^*_t - \Delta p^*_{t+1}) + \gamma_x x_t = -r^*_t + \gamma_x x_t$, which is a proxy that summarizes fundamentals (foreign real interest rate and domestic output deviation) in a economy of interest.

Updating (47) once and applying the law of iterated expectations, we get

$$\Delta e_{t+1} = \bar{\iota} + \frac{\gamma_\pi}{1-b}\Delta p_{t+1} - \Delta p^*_{t+1} + \frac{\gamma_\pi-(1-b)}{\gamma_\pi(1-b)}E_t\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^{j} f_{t+j+1}\right) + \varpi_{t+1} \tag{48}$$

[20] We also used the fact $E_t\varepsilon_{t+j} = 0$, $j = 1, 2, \cdots$.

, where $\varpi_{t+1} = \frac{\gamma_\pi-(1-b)}{\gamma_\pi(1-b)}\left[E_{t+1}\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+1+j}\right) - E_t\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi}\right)^j f_{t+j+1}\right)\right] + \frac{1}{1-b}\varepsilon_{t+1}$, and $E_t\varpi_{t+1} = 0$.

B Derivation of (17)

Since $\tilde{\mathbf{D}}$ is diagonal, (16) can be decomposed to the following two scalar difference equations.

$$E_t\tilde{z}_{1t+1} = (1-b)\tilde{z}_{1t} + \tilde{h}_{1t} \tag{49}$$

$$E_t\tilde{z}_{2t+1} = (\gamma_\pi + \gamma_e)\tilde{z}_{2t} + \tilde{h}_{2t} \tag{50}$$

After solving (49) backward and (50) forward, and recognizing $\mathbf{y}_t = \tilde{\mathbf{V}}\tilde{\mathbf{z}}_t$,

$$\begin{bmatrix} \Delta p_t \\ \Delta e_t \end{bmatrix} = \begin{bmatrix} \frac{1-b-\gamma_e}{\gamma_\pi} & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \sum_{j=0}^{\infty}(1-b)^j\tilde{h}_{1t-1-j} + \sum_{j=0}^{\infty}(1-b)^j u_{t-j} \\ -\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^{j+1} E_t\tilde{h}_{2t+j} \end{bmatrix} \tag{51}$$

From (51), we obtain,

$$\Delta e_t = \frac{\gamma_\pi}{1-b-\gamma_e}\Delta p_t + \frac{(\gamma_\pi+\gamma_e)-(1-b)}{1-b-\gamma_e}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^{j+1} E_t\tilde{h}_{2t+j} \tag{52}$$

, where $\tilde{h}_{2t}$ is the second row element in,

$$\tilde{\mathbf{h}}_t = \frac{1}{(\gamma_\pi+\gamma_e)-(1-b)}\begin{bmatrix} -\gamma_\pi(E_t\Delta p^*_{t+1} - (1-b)\Delta p^*_t + \varepsilon_t) \\ \gamma_\pi\left(E_t\Delta p^*_{t+1} - (1-b)\Delta p^*_t + \varepsilon_t\right) + ((\gamma_\pi+\gamma_e)-(1-b))(\gamma_x x_t - i^*_t + \iota) \end{bmatrix} \tag{53}$$

Applying law of iterated expectations and rearranging the second term in (52),

$$
\begin{aligned}
& \frac{(\gamma_\pi+\gamma_e)-(1-b)}{1-b-\gamma_e}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^{j+1} E_t\tilde{h}_{2t+j} \\
= \quad & \frac{1}{1-b-\gamma_e}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^{j+1}\left[\gamma_\pi\left(E_t\Delta p^*_{t+1}-(1-b)\Delta p^*_t+\varepsilon_t\right)+((\gamma_\pi+\gamma_e)-(1-b))(\gamma_x x_t-i^*_t+\iota)\right] \\
= \quad & \frac{1}{1-b-\gamma_e}[\left(\frac{1}{\gamma_\pi+\gamma_e}\right)(\gamma_\pi E_t\Delta p^*_{t+1}-\gamma_\pi(1-b)\Delta p^*_t+\gamma_\pi\varepsilon_t) \\
& +\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^2(\gamma_\pi E_t\Delta p^*_{t+2}-\gamma_\pi(1-b)E_t\Delta p^*_{t+1})+\cdots] \\
& +\frac{(\gamma_\pi+\gamma_e)-(1-b)}{1-b-\gamma_e}[\left(\frac{1}{\gamma_\pi+\gamma_e}\right)(\gamma_x x_t-i^*_t+\iota)+\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^2(\gamma_x E_t x_{t+1}-E_t i^*_{t+1}+\iota)+\cdots] \\
= \quad & \frac{(\gamma_\pi+\gamma_e)-(1-b)}{(1-b-\gamma_e)(\gamma_\pi+\gamma_e-1)}\iota-\frac{1-b}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\Delta p^*_t+\frac{1}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\varepsilon_t \\
& +\frac{(\gamma_\pi+\gamma_e)-(1-b)}{(\gamma_\pi+\gamma_e)(1-b-\gamma_e)}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j E_t[-(i^*_{t+j}-\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\Delta p^*_{t+1+j})+\gamma_x x_{t+j}]
\end{aligned}
$$

Denoting $\tilde{f}_t$ as $-(i^*_t-\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\Delta p^*_{t+1})+\gamma_x x_t$, which is a proxy that summarizes the fundamentals in this economy, we can rewrite (52) as follows.

$$
\begin{aligned}
\Delta e_t \quad = \quad & \tilde{\iota}+\frac{\gamma_\pi}{1-b-\gamma_e}\Delta p_t-\frac{1-b}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\Delta p^*_t+\frac{1}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\varepsilon_t \\
& +\frac{(\gamma_\pi+\gamma_e)-(1-b)}{(\gamma_\pi+\gamma_e)(1-b-\gamma_e)}\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j E_t\tilde{f}_{t+j}
\end{aligned} \tag{54}
$$

Updating (54) once and applying law of iterated expectations, we get,

$$
\begin{aligned}
\Delta e_{t+1} \quad = \quad & \tilde{\iota}+\frac{\gamma_\pi}{1-b-\gamma_e}\Delta p_{t+1}-\frac{1-b}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\Delta p^*_{t+1} \\
& +\frac{(\gamma_\pi+\gamma_e)-(1-b)}{(\gamma_\pi+\gamma_e)(1-b-\gamma_e)}E_t\left(\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j\tilde{f}_{t+1+j}\right)+\tilde{\varpi}_{t+1}
\end{aligned} \tag{55}
$$

, where $\tilde{\varpi}_{t+1}=\frac{(\gamma_\pi+\gamma_e)-(1-b)}{(\gamma_\pi+\gamma_e)(1-b-\gamma_e)}\left[\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j E_{t+1}\tilde{f}_{t+1+j}-\sum_{j=0}^{\infty}\left(\frac{1}{\gamma_\pi+\gamma_e}\right)^j E_t\tilde{f}_{t+1+j}\right]+\frac{1}{1-b-\gamma_e}\frac{\gamma_\pi}{\gamma_\pi+\gamma_e}\varepsilon_{t+1}$, and $E_t\tilde{\varpi}_{t+1}=0$.

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