TESTING CROSS-SECTION CORRELATION IN PANEL DATA USING SPACINGS

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Abstract

This paper provides tools to characterize the extent of cross-section correlation in panel data when we do not know a priori how many and which series are correlated. Our tests are based on the probability integral transformation of the ordered correlations. We first split the transformed correlations by their size into two groups, and then evaluate the variance-ratio of the two subsamples. The problem of testing cross-section correlation is thus turned into identifying mean shifts and testing non-stationarity. The tests can be applied to raw data and regression errors. We analyze data on industrial production amongst 12 OECD countries, as well as 21 real exchange rates. The evidence favors a common factor structure in European real exchange rates but not in industrial production.

Keywords: group effects, unit root, variance-ratio, PPP.
JEL Classification: C2, C3, C5.

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1 Introduction

Existing tests of cross-section correlation are primarily concerned with the null hypothesis that all units are uncorrelated against the alternative that the correlation is non-zero for some unit. Other statistics test group correlation in an error-component framework, and therefore maintain identical correlation within group as the null hypothesis, assuming group membership is known. But perfect and zero correlation are extreme hypotheses, and rejections by conventional tests do not always reveal much information about the extent of the correlation. It is often useful for estimation, inference, and economic interpretation to know if a rejection is due to, say, ten or eighty percent of the correlations being non-zero.

The present analysis is concerned with the situation when possibly some but not necessarily all of the units are correlated. The correlation is not prevalent enough to be judged common, but is extensive enough that testing the no correlation hypothesis will almost always lead to a rejection. Our objective is characterize the correlation without imposing a structure. This means finding out the number of units that are correlated, determining if the correlations are homogeneous, and evaluating the magnitude of the correlations.

We develop tools to assess the extent of cross-section correlation in a panel of data with N cross-section units and T times series observations. Our analysis is based on the $n = N \cdot (N-1)/2$ unique elements above the diagonal of the sample correlation coefficient matrix, and ordered from the smallest to the largest. We do not directly test if the sample correlations (jointly or individually) are zero. Instead, we test if the probability integral transformation of the ordered correlations, denoted $\bar{\phi}_j$, are uniformly distributed. If the underlying correlations are zero, then the 'uniform spacings', defined as $\bar{\phi}_j - \bar{\phi}_{j-1}$, is a stochastic process with well-defined properties, and it is these properties that we will test.

Exploiting the duality between uniformity and no correlation in hypothesis testing is not new. Durbin (1961) considered using uniformity as a test for serial correlation in independent normally distributed data. The idea is that if the periodogram is evenly distributed across frequencies, a suitably scaled periodogram is uniform on [0,1]. Here, we use uniformity to test cross-section correlation.

We partition the spacings into two groups, labeled S and L, with $\widehat{\theta} \in [0,1]$ being the estimated fraction of the sample in S, and which we will estimate using a breakpoint analysis. For each group, we test if the variance of $\bar{\phi}_j - \bar{\phi}_{j-h}$ is linear in h. Essentially, the problem of testing cross-section correlation is turned into a problem of testing uniformity and non-stationarity. If we reject the no correlation hypothesis in L but not in S, we can say that

a fraction $\widehat{\theta}$ of the correlation coefficients are not statistically different from zero. This is unlike existing tests when a rejection often reveal little about the extent of the correlation. Our procedures are valid when applied to the full or a subset of the correlations, a property conventional tests do not usually possess. As the identity of series generating non-zero correlations can always be recovered, we can also check if the assumed group membership is correct.

The treatment of cross-correlation in the errors have important implications for estimation and inference. In a recent paper, Andrews (2003) showed that OLS when applied to cross-section data can be inconsistent unless the errors conditional on the common shock are uncorrelated with the regressors. The usual t test will no longer be asymptotically normal when there is extreme correlation such as induced by a common shock. Knowledge of how pervasive and how large are the residuals correlations is thus important.

Omitting cross section correlation is known to create problems for inference. Richardson and Smith (1993) noted that evidence for cross-sectional kurtosis could be the result of omitted cross-section correlation in stock returns. Panel unit root tests developed by Levin et al. (2002), among others, are based on the assumption that the units in the panel are uncorrelated. In a large T small N setup, O'Connell (1998) found that the much emphasized power gain of panel over univariate root tests could be a consequence of omitted cross-section correlation, which leads the panel tests to be oversized. Tests of Moon and Perron (2003) and Bai and Ng (2004) are valid when the correlation is driven by a pervasive source. Which test is appropriate depends on which and how many units are correlated.

In recent years, there has been a good deal of interest in the use of approximate factor models in econometric modeling. These differ from classical factor models by allowing weak cross section correlation. When the correlation is strong, theoretical results developed in Stock and Watson (2002), Bai and Ng (2002), for example, will be invalid. The present analysis is largely motivated by the fact that the definition of weak correlation relevant for asymptotic theory is not useful in guiding practitioners as to how much residual correlation is in the data. To this end, this paper provides agnostic tools to identify and characterize correlation groups. For example, real exchange rates and output growth of some countries might be correlated. We can identify a strong and a weak correlation group without using spatial information, or country size, to classify the correlation pairs. The procedures can be applied to the raw data, or to the residuals of panel regressions. The paper proceeds with a review of tests for cross-section correlation in Section 2. The procedures will be developed in Section 3 and 4. Illustrative examples and simulations are reported in Section 5.

2 Related Literature

A formal framework is helpful in making precise the issues involved. Suppose we have N cross-section units each with T times series observations. Denote the $T \times N$ data matrix by z. This could be raw data, or in regression analysis, z would be the regression errors. We are interested in characterizing the cross-correlation structure of z_{it}. Write

$$z_{it} = \delta_i G_t + \Omega^{1/2} v_{it}, \tag{1}$$

where $v_{it} \sim iid(0,1)$ is uncorrelated across i and over t, $\Omega^{1/2}$ is a $N \times N$ symmetric positive definite matrix, while G_t (scalar, with $\text{var}(G_t)$ normalized to unity) is the common source of variation that loads onto z_{it} via the factor loading δ_i. The population covariance of z is

$$\Sigma_z = \delta\delta' + \Omega.$$

If $\delta_i = 0$ for all i and Ω is a diagonal matrix, the data are cross sectionally uncorrelated. If $\delta_i = \bar{\delta} \neq 0$ for every i and $\Omega = \sigma_v^2 I_n$, the error-component structure obtains. For a group or cluster with size less than N, let D be a $N \times 1$ vector with $D_j = 1$ if unit j belongs to the group. Then $\Sigma_z = DD'\bar{\delta} + \sigma_v^2 I_n$.

Let N_1 be the number of series with $\delta_i \neq 0$. The correlation is perfect when $N_1 = N$ and cross-section correlation is absent when $N_1 = 0$. If $\delta_i \neq 0$ for some i and $\frac{1}{N}\sum_{i=1}^N \delta_i^2 > 0$, the correlation is deemed pervasive in the sense that as $N_1/N \to 1$. If $N_1/N \to 0$ as N increases, $\frac{1}{N}\sum_{i=1}^N \delta_i^2$ is degenerate, and the data are approximately uncorrelated. Cross-section correlation can thus be local, it can be pervasive, the units need not have identical correlation coefficients.

Let $c_{ij} = \frac{\text{cov}(z_i, z_j)}{\sqrt{\text{var}(z_i)\text{var}(z_j)}}$ be the population correlation coefficient between two random variables, z_i and z_j. Given observations $\{z_{it}\}$ and $\{z_{jt}\}$, $t = 1, \ldots T$, the Pearson correlation coefficient is

$$\widehat{c}_{ij} = \frac{s_{ij}}{\sqrt{s_{ii}s_{jj}}}$$

with $s_{ij} = T^{-1}\sum_{t=1}^T (z_{it} - \bar{z}_i)(z_{jt} - \bar{z}_j)$. Throughout, we assume that T is large, so that

$$\sqrt{T}(\widehat{c}_{ij} - c_{ij}) \approx N(0, (1 - c_{ij}^2)^2).$$

Under the additional assumption that the data are normally distributed, tests have been developed to test equality of a set of coefficients. See Olkin and Finn (1990), Steiger (1980),

Meng et al. (1992). For testing all coefficients are jointly zero, Breusch and Pagan (1980) showed under normality and assuming N is fixed that

$$\text{LM} = \sum_{i \neq j}^{N} T\widehat{c}_{ij}^2 \rightarrow \chi_n^2 \tag{2}$$

with $n = N(N-1)/2$. When N is large, the normalized test $\frac{LM-n}{\sqrt{2n}}$ is asymptotically $N(0,1)$ as $T \rightarrow$ and then $N \rightarrow \infty$. A statistic that pools the p-values is also asymptotically valid.

The LM test is asymptotically nuisance parameter free when T is large, but it depends on the properties of the parent distribution even under the null hypothesis when T is fixed. Using rank correlations, Frees (1995) considered a non-parametric test of no cross-section correlation in panel data when N is large relative to T. A simulation based test was considered by Dufour and Khalf (2002) for the null hypothesis that the error matrix from estimating a system of seemingly unrelated regressions with normally distributed errors is diagonal. Like the test of Frees and the LM test, the null hypothesis is $c_{ij} = 0$ for all (i,j) pairs, and the test rejects when $c_{ij} \neq 0$ for some (i,j) pair. Moulton (1990) developed a test to detect group effects in panel data assuming group membership is known, see also Moulton (1987). But this amounts to testing perfect and identical correlation.

All the aforementioned tests amount to testing zero or perfect correlation, under the assumption of normality. Without normality, tests with limiting distributions that are nuisance parameter free are difficult to obtain. The need to impose a rigid correlation pattern (Σ_z is diagonal or has an error-component structure) in estimation and inference arises from that fact that unlike time series and spatial data, cross-section data have no natural ordering. Absent extraneous information, the correlation with the neighboring observation has no meaningful interpretation. One cannot impose some type of mixing condition to justify testing the correlation amongst just a few neighboring observations.

From a practical perspective, the shortcoming of these tests is that rejecting the null hypothesis does not reveal much information about how strong and how prevalent is the cross-section correlation. Whether the rejection is due to 10 or 80 percent of the correlations being non-zero makes a difference not just in the way we econometrically handle the correlation, but also the way we think about the economic reason behind the correlation.

3 The Econometric Framework

We start with the premise that the panel of data is neither perfectly correlated nor perfectly uncorrelated. As a matter of notation, we let $[x]$ denote the integer part of x. For a series

$x_j, j = 1, \ldots n$, let $x_{[1:n]}, x_{[2:n]}, \ldots, x_{[n:n]}$ denote the ordered series. That is, $x_{[1:n]} = \min_j(x_j)$ and $x_{[n:n]} = \max_j(x_j)$. We assume for now that the data are normally distributed and will discuss the case of non-normal data in Section 5. In the illustrations to follow, the Pearson correlation will be used. But since the analysis applies to other definitions of cross-section correlation coefficients, we will generically denote the vector of *unique* sample correlation coefficients by $\widehat{p}$ and the corresponding population coefficients by p. We also let $\bar{p} = |\widehat{p}|$. With use of Pearson coefficients, we have

$$\begin{aligned} \widehat{p} &= (\widehat{p}_1, \widehat{p}_2, \ldots, \widehat{p}_n) = \text{vech}(\widehat{c}), \qquad n = N(N-1)/2 \\ \bar{p} &= (|\widehat{p}_1|, |\widehat{p}_2|, \ldots |\widehat{p}_n|). \end{aligned}$$

We are interested in how severe is the correlation amongst the units. Our general strategy is to split the sample into a group of 'small' and a group of 'large' correlations, and then test if the small correlations are zero. The two steps allow us to understand how pervasive and how strong is the cross-section correlation.

The following lemma forms the basis of our methodology.

Lemma 1 *Let $(u_1, u_2, \ldots u_n)'$ be a $n \times 1$ vector of iid $U[0,1]$ variates and let $u_{[1:n]}, \ldots u_{[n:n]}$ denote the ordered data. Let $D_1 = u_1$, $D_j = u_j - u_{j-1}, j = 2, \ldots n$, and $D_{n+1} = 1 - u_n$ be the spacings. Then (i) $E(D_j) = \frac{1}{(n+1)}$, (ii) $var(D_j) = n(n+1)^{-2}(n+2)^{-1} \ \forall\ j$, and (iii) $cov(D_i, D_j) = \frac{-1}{n+1)^2(n+2)} \forall i \neq j$.*

If $u_i, i = 1, \ldots n$ is uniformly distributed on the unit interval, a plot of i against u_i should be a straight line with slope $1/(n+1)$. The variable $D_j = u_j - u_{j-1}$, known as the uniform 'spacings', has the property that $\sum_{j=1}^{n+1} D_j = 1$. Since $E(D_j)$ is the same for all j, $E(D_j) = 1/(n+1)$. Furthermore, the covariance between any two units is the same.[1] We will make use of the fact that if $u_i \sim U[.5, 1]$, Lemma 1 implies $E(D_i) = \frac{1}{2}\frac{1}{(n+1)}$.

3.1 Partitioning the Correlations into Two Sets

Under $H_0 : p_j = 0$, $\sqrt{T}\widehat{p}_j \sim N(0,1)$, we have

$$\sqrt{T} \cdot \bar{p}_j = |\sqrt{T} \cdot \widehat{p}_j| \sim \chi_1.$$

That is, $\bar{p}_j$ is asymptotically distributed as χ (not χ^2) with one degree of freedom.[2] Sort $\bar{p}$ from the smallest to the largest and denote the ordered series by $(\bar{p}_{[1:n]}, \ldots \bar{p}_{[n:n]})'$. Let $\Phi(\cdot)$

[1] See also Pyke (1965) for an excellent review on spacings.

[2] Equivalently, $\sqrt{T}\bar{p}_j$ is half-normally distributed (HN) and thus has mean .7979 and variance .3634.

denote the Gaussian distribution and define $\bar{\phi}_j$ as $\Phi(\sqrt{T}\bar{p}_{[j:n]})$. We have

$$\bar{\phi} = (\bar{\phi}_1, \bar{\phi}_2, \ldots \bar{\phi}_n)' \quad = \quad (\Phi(\sqrt{T}\bar{p}_{[1:n]})), \Phi(\sqrt{T}\bar{p}_{[2:n]})), \ldots, \Phi(\sqrt{T}\bar{p}_{[n:n]}))'.$$

Because $\bar{p}_{[j:n]}$ is ordered, ϕ_j is also ordered with $\bar{\phi}_j < \bar{\phi}_{j-1}$. The null hypothesis of $p_j = 0$ is now restated as $\bar{\phi}_j \sim U[.5, 1]$. Note that $\bar{\phi}_j$ is not uniform on [0,1] (even though $\widehat{\phi}_j$ is) under the no correlation assumption because $\bar{\phi}_j$ is transformed from the absolute values of the correlations.[3] This ensures that large negative correlations are treated symmetrically as large positive correlations.

The simplest test of no correlation is a t test on the mean of $\bar{\phi}_j$,

$$t_{\bar{\phi}_j} = \frac{\bar{\phi}_j - .75}{\sqrt{\widehat{\text{var}}(\bar{\phi}_j)}} \approx N(0,1). \tag{3}$$

Like the LM test, a rejection by the t test reveals little about the extent of the correlation.

Let $\Delta\bar{\phi}_j = \bar{\phi}_j - \bar{\phi}_{j-1}$ be the spacings. If $\bar{\phi}_j \approx U[.5, 1]$, then by Lemma 1, $E(\Delta\bar{\phi}_j) = \frac{1}{2(n+1)}$. Given $\bar{\phi}_j, j = 1, \ldots n$, consider estimating $E(\Delta\bar{\phi}_j)$ using subsamples partitioned arbitrarily at $1 < \widetilde{m} < n$:

$$\begin{aligned} \bar{\Delta}_S(\widetilde{m}) &= \frac{1}{\widetilde{m}} \sum_{j=1}^{\widetilde{m}} \Delta\bar{\phi}_j \\ \bar{\Delta}_L(\widetilde{m}) &= \frac{1}{n - \widetilde{m}} \sum_{j=\widetilde{m}+1}^{n} \Delta\bar{\phi}_j. \end{aligned}$$

If $\sqrt{T}\widehat{p}_j \approx N(0,1)$ we should have $\bar{\Delta}_S(\widetilde{m}) \approx \bar{\Delta}_L(\widetilde{m}) \approx \frac{1}{2(n+1)}$, $\forall \widetilde{m}$.

Now suppose only $m \leq n$ of the correlation coefficients are zero. We would expect 95 percent of these $\bar{\phi}_j$ to be less than .95. But since $p_{[j+1:n]} \neq 0$ for $j = m+1, \ldots n$, we would expect the corresponding $\bar{\phi}_j$ to be 1. In a qq-plot, one should expect $\bar{\phi}_j$ to be approximately linear in j until $j = m$ and then flattens out at the boundary of one. In terms of $\Delta\bar{\phi}_j$,

$$\begin{aligned} \bar{\Delta}_S(m) &= \frac{1}{2m} \\ \bar{\Delta}_L(m) &= 0. \end{aligned}$$

We can therefore make use of the difference between $\bar{\Delta}_S$ and $\bar{\Delta}_L$ to locate m. Define the total sum of squared residuals evaluated at breakpoint $\widetilde{m}$ as:

$$Q_n(\widetilde{m}) = \sum_{j=1}^{\widetilde{m}} (\Delta\bar{\phi}_j - \overline{\Delta}_S(\widetilde{m}))^2 + \sum_{j=\widetilde{m}+1}^{n} (\Delta\bar{\phi}_j - \bar{\Delta}_L(\widetilde{m}))^2. \tag{4}$$

[3]Because $\sum_{i=1}^{n+1} \widehat{\phi}_j = 1$, the Jacobian of the transformation from $\Phi(\widehat{p}_j)$ to $\widehat{\phi}_j$ is singular. This property applies to $\bar{\phi}_j$ as well. The one to one correspondence between $\bar{\phi}_j$ and $\Phi(\bar{p}_{[j]})$ exists if $\bar{\phi}_{n+1}$ is omitted.

Theorem 1 *Given jointly normally distributed data $z_{it}, t = 1, \dots T$, $i = 1, \dots N$, let $\widehat{p}_j$, $j = 1, \dots n$ be the vector of unique sample correlations such that for large T, $\sqrt{T}(\widehat{p}_j - p_j) \xrightarrow{d} N(0, (1-p_j^2)^2)$. Let $\bar{p}_{[j:n]}$ be formed by ordering $\bar{p}_j$ from smallest to largest, and let $\bar{\phi}_j = \Phi(\sqrt{T}\bar{p}_{[j:n]})$. Then for large n, $\widehat{m} = \text{argmin}_{\widetilde{m}} Q_n(\widetilde{m})$ is a global minimizer of (4).*

The procedure provides a data dependent way to separate the small from the large correlations. If the test was applied to an observed time series with T observations, then as discussed in Bai (1997), the break-point estimator is the global minimizer of the sum of squared residuals. As is known in the breakpoint literature, we can consistently estimate the break fraction at the accelerated rate of T. Our breakpoint analysis is applied to the n observations of $\bar{\phi}_j$, and so the convergence rate is n. Nonetheless, the breakpoint analysis here is non-standard because the observations being tested are transformations of the sample correlations $\bar{p}_j$ which are themselves estimates. But $\bar{p}_j$ is $\sqrt{T}$ consistent for p_j. As shown in the Appendix, having to replace p_j by $\bar{p}_j$ does not affect the location of the mean shift. Instead of identifying a mean shift in spacings, we could also have used the slope change in $\bar{\phi}_j$ to identify m. However, formal results are available for estimating mean shifts in stationary data, and as will be elaborated below, $\bar{\phi}_j$ is not stationary when appropriately scaled. Furthermore, we will use spacings again in the next section.

To learn more about the nature of the correlation, such as whether the correlation is homogeneous or heterogeneous, we need to go one step further. Before proceeding with such an analysis, We first illustrate the idea behind Theorem 1 with some examples.

3.2 Illustration

We simulate data as follows. For each $i = 1, \dots N$, $t = 1, \dots T$:

$$z_{it} = \delta_i G_t + e_{it}, \quad e_t = \Omega^{-1/2}\varepsilon_t, \quad \varepsilon_{it} \sim N(0,1)$$

where $G_t \sim N(0,1)$, where $\Omega^{1/2}$ is a $n \times n$ matrix. By varying the number of δ_i that are non-zero, or the structure of $\Omega^{1/2}$, we have ten configurations representing different degrees of correlation in the data. Define θ_0 as the number of zero elements above the diagonal of the $N \times N$ matrix Σ_z. Notice that θ_0 varies with N since the number of correlations is of order N^2 but the number of series with non-zero δ_i is of order N.

Table 1: DGP

	$\#\delta_i \neq 0$	δ_i	$\Omega^{1/2}$	$\theta_0(N)$			$\max(N)$		
DGP				10	20	30	10	20	30
1	0	0	-	1.0000	1.0000	1.0000	0.1707	0.1217	0.1048
2	N	N(0,1)	$.2I_n$	0.0000	0.0000	0.0000	0.7084	0.7110	0.4104
3	N	N(0,1)	I_n	0.0000	0.0000	0.0000	0.4884	0.4880	0.4886
4	0	0	$\mathrm{T}([1, .8, 0_{n-2}])$	0.6222	0.8053	0.8690	0.3509	0.2183	0.1723
5	0	0	$\mathrm{T}([1, -.5, .3, 0_{n-3}])$	0.3333	0.6316	0.7471	0.4312	0.2585	0.1995
6	$.4N$	1	I_n	0.8667	0.8526	0.8483	0.2977	0.2775	0.2709
7	$.8N$	1	I_n	0.3778	0.3684	0.3655	0.4935	0.4767	0.4733
8	$.4N$	N(0,1)	I_n	0.8667	0.8526	0.8483	0.2444	0.2293	0.2278
9	$.8N$	N(0,1)	I_n	0.3778	0.3684	0.3655	0.4019	0.4024	0.3990
10	$.8N$	N(0,1)	$.2I_n$	0.3778	0.3684	0.3655	0.5778	0.5795	0.5821

DGP 1 simulates a panel of uncorrelated data. DGPs 2 and 3 assume the presence of common factors but with different assumptions about the importance of the common to idiosyncratic component. DGP 4 and 5 assume that the errors have a Toeplitz structure but that the δ_is are all zero. DGP 6 and 7 assume equal correlation within group, but with varying group size. DGP 8 through 10 make different assumptions about the number of δ_i that are non-zero. Because δ_i varies across i, there is heterogeneity in the magnitude of the correlation. Since the null hypothesis that $p_j = 0 \; \forall j$ does not hold in DGP 2 to 10, an LM test should lead to rejections in every case, and this is confirmed in simulations.

In Table 1, θ_0 is the fraction of correlation coefficients that are zero in the population (over 1000 replications) computed for three values of N. The column labeled 'max' reports $\max_i \frac{1}{N}\sum_{j=1}^{N}|\widehat{c}_{ij}|$. This statistic is an upper bound on the average correlation in the data because from matrix theory, the largest eigenvalue of Σ_z is bounded by $\max_j \sum_{i=1}^{n}|c_{ij}|$. The bound is often used as a condition on permissible cross-section correlation in approximate factor models. See, for example, Bai and Ng (2002), Stock and Watson (2002). However, the statistic has shortcomings as a measure of pervasiveness in the correlations. For one thing, 'max' is not monotonic in the number of non-zero correlations. For example, DGP 3 has more non-zero correlations than DGP 10, yet 'max' is larger in DGP 10 than in DGP 3. For another, two similar values of 'max' can be consistent with very different degrees of pervasiveness. For example, DGP 5 and 6 both have similar 'max' values when $N = 20$, but θ_0 is much higher under DGP 6. The point to highlight is that when there is heterogeneity in the correlation, it will be difficult to characterize the severity of the correlation with a single statistic. This is unlike time series data when the largest root of the autocorrelation matrix is informative about the extent of serial correlation.

The top panel of Figure 1 considers DGP1 (uncorrelated data) for $T = 200, N = 20$ (or $n = 190$). The bottom panel considers $T = 400, N = 30$ (or $n = 435$). Depicted in Figure 1 is the line $.5 + .5j/n$ on which all $\bar{\phi}_j$ should lie if they are exactly uniformly distributed. As the data are uncorrelated, the quantile function does not exhibit any abrupt change in slope and the average of $\Delta\bar{\phi}_j$ is approximately $\frac{1}{2n}$.

Now consider the case of correlated data generated by DGP 9 and 10. In both cases, about one-third of the correlations are zero, but the correlations are much smaller (and hence $\bar{\phi}_j$) under DGP 9 because of the larger idiosyncratic variance. Because many correlations are non-zero, Figure 2a shows that $\bar{\phi}_j$ no longer evolves around the 45 degree line drawn for $n = 190$. Instead, a subset of them vary around a 45 degree line if the x-axis is appropriately truncated to the number of correlation coefficients that are zero.

For the sample of data simulated using DGP 9, 82 of the $\bar{\phi}_j s$ are less than .95. This means that if we had used the asymptotic standard error under no correlation of $\frac{1}{\sqrt{T}}$ to test the 190 correlations one by one, we would end up with a group S consisting of 82 correlations deemed insignificant at the 5% level. A different sigifnicance level will produce a different group size. Our breakpoint analysis does not depend on the choice of the significance level. It lets the data determine the sharpest difference between the two groups.

Figure 3 presents $\bar{\phi}_j$ for DGP 2 (small idiosyncratic error) and 3 (large idiosyncratic error). Since DGP 2 has a stronger factor structure than DGP3, there are more $\bar{\phi}_j$ at unity under DGP 2. In both cases, the minimum $\bar{\phi}_j$ is above .5, indicating even the small correlations are non-zero. The feature to highlight in Figure 3 is that p_j can be non-zero and yet $\bar{\phi}_j < 1$. We cannot always use the boundary of one to split the sample. Theorem 1 proposes to use the least squares criterion to separate the small from the large correlations.

Figures 1, 2 and 3 show that the qq-plot indeed reveals a lot of information about the extent of cross-correlation in the data. If all correlations are non-zero, the qq-plot will be shifted upward with an intercept exceeding 0.5. If there is homogeneity in the correlations, the slope of $\bar{\phi}_j$ will be flatter than implied by uniformity. The more prevalent and the stronger is cross-correlation, the sooner will $\bar{\phi}_j$ hit the boundary of one. This is the basis of Theorem 1.

4 Testing the Correlation in the Sub-Samples

Theorem 1 merely splits the sample of n observations into two groups, one of size $\widehat{m}$, and one of size $n - \widehat{m}$. It is possible that the $n - \widehat{m}$ correlations in L are in fact not statistically different from the $\widehat{m}$ correlations in S. Testing this hypothesis by testing the null of no break

is uninformative because an error-component structure would be consistent with the no break hypothesis as all correlations are identical, and yet incompatible with the no correlation assumption.

More importantly, we want to test if the subset of $\widehat{m}$ correlations are zero because if we reject, the correlations in L must also reject and we can conclude that there is extensive correlation. Using order statistics to test the first $\widehat{m}$ correlations is problematic because some of the correlations may not be large enough to render $\bar{\phi}_j$ exactly one.[4] For order statistics to work well, all non-zero correlations, when multiplied by $\sqrt{T}$, would have to be 'large' with probability one. Similarly, applying an LM test to the subsamples is also problematic because $\bar{p}_{[j:n]}, j = 1, \ldots \widehat{m}$ is a censored sample whenever $\widehat{m} < n$, and the test will no longer be asymptotically χ^2. Here, we consider a different approach.

Lemma 2 *Let $u_j \sim U[0,1]$ with $D_j = u_j - u_{j-1}$ being uniform spacings. Then (i) $nD_j \sim \Gamma(1,1)$ and (ii) $corr(nD_j, nD_k) = \frac{-1}{n}\ \forall j \neq k$. Let $D_j^q = u_j - u_{j-q}$ be q-order spacings. Then (iii) $D_j^q \sim beta(q, n-q+1)$ and (iv) $nD_j^q \sim \Gamma(q,1)$.*

The simple spacings, multiplied by n, are distributed $\Gamma(1,1)$. It is thus a random variable with well defined mean and variance. Although the spacings are not independent, they are asymptotically uncorrelated. Furthermore, the structure of dependence is the same between D_j and D_k for any j and k. Spacings are thus 'exchangeable'. The difference between $\bar{\phi}_j$ and $\bar{\phi}_{j-q}$ are referred to in the statistics literature as q-order spacings. Properties of q-order spacings are given in Holst (1979). See also Arnold and Nagaraja (1992).

The feature that motivates the test to follow is (iv), ie. the mean and variance of q-order spacings are $\frac{q}{n+1}$ and $\frac{q(n+1-q)}{(n+1)^2(n+2)}$, respectively. This implies that the mean and variance of q order spacings are q times the mean and variance of the scaled first order spacings. Specifically, if $\bar{\phi}_j \sim U[.5, 1]$, then upon defining

$$\bar{\phi}_j^n = n \cdot \bar{\phi}_j,$$

we can represent the scaled spacings process $\bar{\phi}_j^n$ as

$$\bar{\phi}_j^n - \bar{\phi}_{j-1}^n = .5 + \varepsilon_j, \quad \varepsilon_j \sim (0, .25), \qquad \frac{\text{cov}(\varepsilon_j, \varepsilon_k)}{\text{var}(\varepsilon_j)} = \frac{-1}{n}.$$

Viewed in this light, $\bar{\phi}_j^n$ is a unit root process with a non-zero drift of .5. Like a process that is integrated of order one, the variance of the q-th order spacings relative to a first order

[4] Such an order statistic approach was used in Chang (2002) in panel unit root tests.

process is linear in q. Figure 1 shows that the deviations of $\bar{\phi}_j$ from the benchmark line of $.5 + .5j/n$ (with scale on the right hand side) do not appear mean reverting. The bottom panel shows that the random walk like property of $\bar{\phi}_j$ (and thus $\bar{\phi}_j^n = n\bar{\phi}_j$) remains even when the sample size increases.

The feature that $\bar{\phi}_j^n$ is differenced-stationary originates from the property that $\bar{\phi}_j$ is uniformly distributed under the no correlation assumption. Evidence against non-stationarity of $\bar{\phi}_j^n$ is thus evidence against a set of coefficients being uncorrelated. We can therefore use methods for analyzing integrated data to test cross-section correlation. Given a sample of $\bar{\phi}_k$ of size η, consider the following:

$$\begin{aligned}
\widehat{\mu}_1 &= \frac{1}{\eta - 1}\sum_{k=1}^{\eta}(\bar{\phi}_k^n - \bar{\phi}_{k-1}^n) \\
\widehat{\mu}_q &= \frac{1}{\eta - q}\sum_{k=q+1}^{\eta}(\bar{\phi}_k^n - \bar{\phi}_{k-q}^n) \\
\widehat{\sigma}_1^2 &= \frac{1}{\eta}\sum_{k=1}^{\eta}(\bar{\phi}_k^n - \bar{\phi}_{k-1}^n - \widehat{\mu}_1)^2 \\
\widehat{\sigma}_q^2 &= \frac{1}{q \cdot m_q}\sum_{k=q+1}^{\eta}(\bar{\phi}_k^n - \bar{\phi}_{k-q}^n - \widehat{\mu}_q)^2,
\end{aligned}$$

where $m_q = (\eta - q)$, or $m_q = (\eta - q)(1 - q/\eta)$ for unbiased estimates of the variance. Define

$$SVR(\eta) = (\widehat{\sigma}_q^2/\widehat{\sigma}_1^2 - 1).$$

To gain more intuition, let $q = 2$ and consider

$$\widehat{\sigma}_q^2/\widehat{\sigma}_1^2 - 1 \approx \frac{\mathrm{cov}(\Delta\bar{\phi}_j^n, \Delta\bar{\phi}_{j-1}^n) + (2\widehat{\mu}_1 - \widehat{\mu}_2)^2}{\mathrm{var}(\Delta\bar{\phi}_j^n)}.$$

The first ratio on the right hand side is the first order autocorrelation coefficient for $\Delta\bar{\phi}_j^n$, while the second term in the numerator is the squared difference between the mean of q-order and simple spacings. But by (ii) of Lemma 2, $\Delta\bar{\phi}_j^n$ is asymptotically uncorrelated and by (i) of Lemma 2, the mean of q order spacings is linear in q. Both terms on the right hand side should be close to zero. A comparison of $\widehat{\sigma}_q^2$ to $\widehat{\sigma}_1^2$ thus allows us to test both properties of uniform spacings that should hold if the underlying correlations are indeed zero.

Theorem 2 *Consider the null hypothesis that* $\sqrt{T}\widehat{p}_{[\eta-j+1:\eta]} \sim N(0,1)$ *for all* $j = 1, \ldots \eta$. *Then as* $\eta \to \infty$, $\sqrt{\eta} \cdot SVR(\eta) \xrightarrow{d} N(0, \omega_q^2)$ *where* $\omega_q^2 = \frac{2(2q-1)(q-1)}{3q}$.

Various authors have used functions of simple and $q > 1$ order spacings to test uniformity. In the statistics literature, $\widehat{\sigma}_1^2$ and $q \cdot \widehat{\sigma}_q^2$ are referred to as simple and generalized Greenwood spacings test for uniformity. The original Greenwood statistic was developed to test whether the spread of disease is random in time. Rao and Kuo (1984), among others, showed optimality of $q = 2$, i.e. using squares of the spacings.

We combine the simple Greenwood test with a q-order test. The limiting distribution of the spacings variance-ratio test (SVR) is obtained by noting that $\widehat{\sigma}_1^2$ and $\widehat{\sigma}_q^2$ are both consistent estimators for σ_ε^2, but $\widehat{\sigma}_1^2$ is efficient. Thus the Hausman principle applies, and $\text{var}(\widehat{\sigma}_q^2 - \widehat{\sigma}_1^2) = \text{var}(\widehat{\sigma}_q^2) - \text{var}(\widehat{\sigma}_1^2)$, from which $\text{var}(\widehat{\sigma}_q^2/\widehat{\sigma}_1^2 - 1)$ is obtained. Because the test is based on estimated correlations, we need $\frac{N}{\sqrt{T}} \to c$ to avoid excess variability in $\widehat{\rho}_j$ to influence the limiting distribution of the test. The method is well suited for analyzing macroeconomic panels with about 20 units and 200 quarterly observations.

Some might observe that the SVR has the same limiting distribution as the variance ratio (VR) test of Lo and MacKinlay (1988) for testing the null hypothesis of a random walk, i.e. with $\varepsilon_j \sim N(0, \sigma^2)$. The ε_js considered here are, however, non-normal. Although the kurtosis of $\bar{\phi}_j$ appears in the variance of $\widehat{\sigma}_1^2$ and $\widehat{\sigma}_q^2$, the two terms offset one another since we consider the variance of the difference. At an intuitive level, the reason why the SVR has the same distribution as the VR is that we are testing if the variance of $\Delta^q \bar{\phi}_j^n$ is linear in q. This is a property of an integrated process, not just that of a random walk with Gaussian innovations.

Note that the proposed SVR is constructed slightly different from the Greenwood and the VR tests. The former is constructed with $\widehat{\mu}_1$ set to the population mean, which amounts to .5 in our setup. The latter sets $\widehat{\mu}_q$ to $q\widehat{\mu}_1$. We freely estimating $\widehat{\mu}_q$ instead. Because $\widehat{\mu}_1$ consistently estimates the mean of .5, and $\widehat{\mu}_q$ and $q\widehat{\mu}_1$ are both consistent for the mean of q-spacings under the null, the limiting distribution of SVR is the same for all implementations. By freely estimating the mean of q-spacings, we may expect to gain some power.

The SVR can be applied to the full sample, to S, or to L, with $\eta = n$, $\eta = [\widehat{\theta} n]$ and $[(1-\widehat{\theta})n]$ respectively. An important appeal of the SVR test is that it is based upon spacings, and spacings are exchangeable.[5] As seen from Figure 1 for uncorrelated data, any subsample of $\bar{\phi}_j$ has the same slope on average. This permits us to test any partition of the full sample provided there are enough observations to make the test meaningful. We can thus repeat the breakpoint analysis to partition S into subsamples, say, SS and SL. The SVR can then

[5] That is, if $D_j = u_j - u_{j-1}$ are the spacings, a statistic such as $\sum_{i=1}^n D_j^2$ remains valid when constructed with a subset of the n spacings between adjacent order statistics.

be performed on these two subsamples. In contrast, the LM test looses its asymptotic χ^2 property once the sample is censored. Of course the power of the SVR is a function of the size of the subsample. Thus if $\widehat{\theta}$ is close to ε, the test applied to the S subsample will not be precise. Obviously, if we reject the uniformity hypothesis in S, testing the random walk hypothesis in L is not too interesting as by construction, S is the sample with smaller correlations.

Since the identity of the two series underlying any given $\bar{p}_j$ can always be recovered, the procedure can be used as a test of group membership. For example, one might expect residual correlation amongst sibling observations. One can check if the pairs found correlated are in fact siblings. The identity of the correlated pairs can also bear interesting economic information. For example, one might expect behavioral correlation amongst those who are liquidity constrained, but the econometrician cannot know with certainty who is or is not liquidity constrained. Practitioners often rely on cut-off points in wealth or income to split the sample into constrained and unconstrained groups. Our analysis provides an agnostic way to group the data. One can then check if low income is the common denominator amongst those in the group.

The setup also provides a convenient way to test patterned correlation. An error component structure implies $\text{var}(\bar{\phi}_j) = 0$. To test this hypothesis, Let $\widehat{\beta}$ be obtained by regressing $(\bar{\phi}_j - \frac{1}{n}\sum_{k=1}^{n}\bar{\phi}_k)^2$ on a constant. Under the assumption of equal correlation, β is zero. Thus,

$$t_{\widehat{\beta}} = \frac{\widehat{\beta}}{se(\widehat{\beta})} \approx N(0,1). \tag{5}$$

To test an error component structure when group membership is known, we can use only those observations belonging to the group. A test for unknown group membership can likewise be constructed, as we can use the sample split procedure to determine $\widehat{m}$. Instead of variance, we can also construct other measures of dispersion, such as the Gini coefficient.

5 Finite Sample Properties

For each of the 10 models given in Table 1, we consider (N,T) = (10,100), (20,100) (20, 200), and (30,200). The sample correlations are more precisely estimated the larger is T, though the sample splitting procedure is more accurate the larger is N. We use $q = 2$. The two-tailed SVR test has critical value of 1.96. The results are based on 1000 replications. Because it is difficult to identify mean shifts occurring at the two ends of the sample, it is common practice to locate the break fraction in the $[\varepsilon, 1-\varepsilon]$ range of the sample. Trimming

is not strictly speaking necessary to obtain $\widehat{\theta}$, but the SVR is not meaningful without a reasonable number of observations in a partition. We use $\varepsilon = .1$.

In addition to the Pearson correlation, we also use other measures for $\widehat{p}_j$. Let R_i be a $T \times 1$ vector of rankings of z_{it}. The Spearman rank correlation coefficient is defined as

$$\widehat{r}_{ij} = \frac{\mathcal{R}_{ij}}{\sqrt{\mathcal{R}_{ii}\mathcal{R}_{jj}}},$$

where

$$\mathcal{R}_{ij} = \text{cov}(R_i, R_j) = \frac{1}{T}\sum_{t=1}^{T}(R_{it} - (T+1)/2)(R_{jt} - (T+1)/2)$$

is the sample covariance in the rankings of two series, z_{it} and z_{jt}. It is well known that

$$\frac{\sqrt{(T-2)} \cdot \widehat{r}_{ij}}{\sqrt{1-\widehat{r}_{ij}^2}} \sim t_{T-2}.$$

The statistic is mean zero with variance $\frac{T-3}{T-5}$ which becomes approximately unity as $T \to \infty$. We also consider the Fisher's z-transformation of the Pearson coefficient:

$$\widehat{c}_{ij}(z) = \frac{1}{2}\ln\left(\frac{1+\widehat{c}_{ij}}{1-\widehat{c}_{ij}}\right).$$

If $c_{ij} = 0$, $\widehat{c}_{ij}(z) \approx N(0, \frac{1}{T-3})$. The transformation is especially useful in stabilizing variance when T is small. Analogously, a transformation can also be applied to the Spearman rank correlations and Kendall's τ.

To summarize, given N cross-section units over T time periods with $n = N(N-1)/2$, the following variables are being constructed:

$\widehat{c}_{ij}, \widehat{c}_{ij}(z)$	sample cross-section correlations;
$\widehat{p}_j$	vech(x), where x is one of $\widehat{c}$, $\widehat{c}(z)$, $\widehat{r}$, or $\widehat{r}(z)$;
$\bar{p}_j$	$\lvert\widehat{p}_j\rvert$;
$\bar{p}_{[j:n]}$	$\bar{p}_j$ ordered from smallest to largest;
$\bar{\phi}_j$	$\Phi(\sqrt{T}\bar{p}_{[j:n]})$;
$\bar{\phi}_j^n$	$n \cdot \bar{\phi}_j$;
$\Delta^q\bar{\phi}_j^n$	$\bar{\phi}_j^n - \bar{\phi}_{j-q}^n$.

Table 2 reports results for Pearson and Spearman rank correlations assuming ε_{it} is normally distributed. We begin by conducting two full sample tests. The results for these are given in the last two columns of Tables 2. The first is the t test for the mean of $\bar{\phi}_j$ being 0.75, see (3). The test has a rejection rate of around 5% for uncorrelated data, but for DGP 2

to 10, the hypothesis is rejected with probability close to one. The second test is for the null hypothesis of perfect correlation and is conducted as a t test for $\text{var}(\bar{\phi}_j) = 0$. The equal variance hypothesis is correctly rejected in all cases as none of the DGPs considered have equal correlation. Nonetheless, rejections by these full-sample tests reveal little information about the severity of the correlation.

We proceed to determine the fraction of the sample in S. Although θ_0 need not coincide with $\widehat{\theta}$, for the models considered, $\widehat{\theta}$ is generally closer to θ_0. As expected $\widehat{\theta}$ is more precisely estimated the larger is N, recalling that the effective sample size from the point of view of sample splitting is $n = N(N-1)/2$. When $N = 10$, $n = 45$. Since the estimated break fraction converges at rate n, the sample split procedure can be quite precise even when applied to panels with as few as 10 units.

Our procedure yields an average $\widehat{\theta}$ of around .1 for DGP2. As we use $\varepsilon = .1$ trimming, there is clear evidence of correlation. The average $\widehat{\theta}$ is around .3 for DGP 3. The reason why $\widehat{\theta}$ is different even though θ_0 is the same for DGP 2 and 3 is that DGP 3 has higher idiosyncratic variance and more correlations are non-zero but small. This again highlights the distinction between θ_0 and $\widehat{\theta}$ (see also Figure 3).

Next, we apply the SVR to the two subsamples. The rejection rates are labeled SVR_S and SVR_L, respectively. The rejection rate should be .05 in both subsamples of DGP 1 since the data are uncorrelated. The test is oversized when $(N,T) = (10,100)$ but is close to the nominal size of 5% for larger (N,T). The size distortion when $N = 10$ is due to the small size of the subsamples (which have fewer than 25 observations given $\widehat{\theta}$ is, on average, around .5). For DGP 2 to 10, the SVR_L tends to reject uniformity DGP 2 to 10 with probability close to one when $N = 20$, showing that the test has power. As the correlations in the L subsample are large by selection, being able to reject uniformity in L is not surprising. More difficult is to correctly reject uniformity in S when the underlying correlations are small but not necessarily zero. The size of the test generally accurate, especially considering the average size of the subsamples. We also split S into two samples, SS and SL and then apply the SVR to SS. The second panel of Table 2 shows that the rejection rates are generally similar to those using the larger sample, S, showing that censoring posts no problem for the spacings based SVR.

Strictly speaking, Lemmas 1 and 2 apply to iid uniform variables. Except when the data are normally distributed, the sample correlations are not independent. A theory for spacings when the data are not independent is required to rigorously justify application of Theorems 1 and 2 (or the LM and other tests cited earlier) to non-normal data. Nonetheless,

we can use simulations to check robustness of the results against departures from normality. Tables 3a report results for Pearson correlations assuming ε_{it} is χ^2. Table 3b assumes ε_{it} a ARCH(1) process (with parameter .5). As we can see, the breakpoint analysis and the SVR test have properties similar to those reported in Tables 2a,b. This robustness to departures from normality is likely due to the fact that the breakpoint analysis is valid under very general assumptions, and the SVR tests a generic feature of integrated data. Thus, both the breakpoint analysis and the SVR continue to have satisfactory finite sample properties.

5.1 Applications

We apply the procedures to two applications. The first considers the correlation in real economic activity in the Euro area. The second considers a panel of real exchange rate data. To isolate cross-section from serial correlation, the correlation coefficients are computed for residuals from a regression of each variable of interest on a constant and its own lags.

Industrial Production Let $z_{it}, i = 1, 12$, t=1982:1 to 1997:8 be monthly data on industrial production for Germany, Italy, Spain, France, Austria, Luxembourg, the Netherlands, Finland, Portugal, Belgium, Ireland, and the US, respectively.[6]. With twelve countries, we have $n = 66$ for $T = 186$ observations. The variable of interest is the growth rate of industrial production. Figure 4 plots $\bar{\phi}_j$. While most of the $\bar{\phi}_j$ are far from unity, the observations do not lie along a straight line, indicating substantial heterogeneity in the correlations. The breakpoint analysis yields $\widehat{\theta} = .468$, 31 correlations in S, and 35 in L. A listing of the correlations in the two groups is given in Table 4. The largest correlations is the France/Germany pair ($\bar{p}_j = .33$), and the weakest is between Austria and Luxembourg ($\bar{p}_j = .006$). The SVR tests are -.234 and 2.673 respectively. Industrial production in 31 the 66 countries appear to contemporaneously uncorrelated. A common factor structure does not appear to be a suitable characterization of output of these 12 countries.

Real Exchange Rates Quarterly data for nominal exchange rates and the consumer price indicies are obtained from the International Finance Statistics. We use data from 1974:1-1997:4 for 21 countries:- Canada, Australia, New Zealand, Australia, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Spain, Sweden, Switzerland, UK, Japan, Korea, Singapore, and Thailand. The U.S. is used as the numeraire country. The variable of interest is the log real exchange rates.

[6]The data are the same as in Stock et al. (2001)

A plot of $\bar{\phi}_j$ is presented in Figure 5. The algorithm finds $\widehat{\theta} = .142$, so that .85 of the sample belongs to L. As there are a total of $n = 210$ correlations in this example, Table 5 only gives the 30 correlations in S, and the 30 largest correlations in L. Evidently, the real exchange rate for Canada and Korea do not display much correlation with those of the European countries. However, the European real exchange rates are strongly correlated with each other. The results suggest one cannot simply attribute the source of the correlations to the use of a common numeraire (the US dollar). If this was the sole source of the correlation, all real exchange rates should be equally correlated.

The SVR test is 1.884 for the S sample of size 30, and 8.574 for L. Because L constitutes .85 of the sample, the cross section correlation is pervasive. Not accounting for these correlations can indeed lead to misleading inference about persistence and measures of half-lives.

Putting the two sets of results together, the evidence favors a factor structure for the European real exchange rates but not for industrial production. Indeed, the first principal component explains over 60 percent of the variation in real exchange rates. Since a common factor exists in European real exchange rates but not in real output, the evidence points to the presence of a nominal or monetary factor. That we can identify such a clear structure without any assumptions imposed is quite striking.

6 Conclusion

The present paper provides an agnostic way to test cross-section correlation when possibly a subset of the sample is correlated. This is a situation in which traditional testing of the no correlation assumption in the full sample will be rejected, but for which the rejection provides little understanding about the extent of the correlation. The approach also provides a way to identify group membership and test patterned correlations. An extension, which is now being considered, is to test if the identity of those pairs in S and L changed over time.

An immediate application of the procedures developed here is factor analysis, where the model is $x_{it} = \lambda_i F_t + e_{it}$. While strict factor model assumes e_{it} is iid across i and t, approximate models permit some correlation in the errors and thus does not require the $N \times N$ matrix $\Omega = \text{cov}(e_t)$ to be diagonal. Which model and which estimator is more appropriate thus depend on Ω. The present analysis provides a way to identify those elements of Ω that are zero. The next step is to use this information to improve estimation and inference.

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Appendix

We need to show that (i) the breakpoint analysis and (ii) spacings tests based on transformations of the sample correlations are unaffected by the sampling variability of correlation coefficients. Without loss of generality, we consider $\sqrt{T}\widehat{p}_j$ instead of $\sqrt{T}\bar{p}_j$. The former will give us transformation of standard normal variates to uniform $[0,1]$ variates, while the latter transform them to uniform $[.5,1]$ variates. Let $\Phi(\cdot)$ denote the standard normal distribution. It is known that if a statistic $\widehat{\tau}$ is $\sqrt{T}$ consistent and asymptotically normal, and $\text{prob}(\widehat{\tau} \leq q_\alpha) = \alpha$, the quantile q_α admits the expansion $q_\alpha = x_\alpha + T^{-1/2}\Phi(x_\alpha) + \ldots$, where $\Phi(x_\alpha) = \alpha$. Thus, by mean value theorem, $\Phi(q_\alpha) = \Phi(x_\alpha) + \Phi'(\bar{q}_\alpha)(q_\alpha - x_\alpha)$ for $\bar{q}_\alpha \in [q_\alpha, x_\alpha]$, and $\Phi(x_\alpha) \sim U[0,1]$, for $\alpha = j/n$, $j = 1, 2, \ldots n$.

Consider the test statistic

$$\widehat{\tau}_j(\widehat{p}_j, p_j) = \frac{\sqrt{T}(\widehat{p}_j - p_j)}{(1 - p_j^2)}.$$

Under $p_j = 0$, $\Phi(\sqrt{T}\widehat{p}_{[j:n]}) = \Phi(x_{[j:n]}) + O_p(T^{-1/2})$, where $x_{[j:n]}$ is such that $\Phi(x_{[j:n]}) = j/(n+1)$. Define $\phi_j = \Phi(x_{[j:n]})$. We may write

$$\widehat{\phi}_j = \phi_j + \widehat{u}_j$$

where $\phi_j \sim U[0,1]$ and $\widehat{u}_j = O_p(T^{-1/2})$. When $p_j \neq 0$, $\widehat{\tau}(\widehat{p}_j, 0)$ is $O_p(\sqrt{T})$ and $\Phi(\widehat{\tau}(\widehat{p}_j, 0)) \to 1$ as $T \to \infty$.

Let $\Delta\widehat{\phi}_j = \widehat{\phi}_j - \widehat{\phi}_{j-1}$. Under $p_j = 0$, $\Delta\widehat{\phi}_j = \Delta\phi_j + \Delta\widehat{u}_j$. Recall that $\Delta\phi_j$ is a uniform spacing, and $n \cdot \Delta\phi_j \sim \Gamma(1,1)$. Let $g(x)$ be a continuous function such $g'(x) = \partial g/\partial x$ exists. By Taylor series expansion of $g(n\Delta\widehat{\phi}_j)$ around $\widehat{p}_j = p_j$,

$$\begin{aligned} g(n\Delta\widehat{\phi}_j) &= g(n\Delta\phi_j) + \left[\frac{\partial g(n\Delta\widehat{\phi}_j)}{\partial \Delta\widehat{\phi}_j}\right]_{p_j} \left[\frac{\partial[\Phi(\sqrt{T}\widehat{p}_{[j:n]}) - \Phi(\sqrt{T}\widehat{p}_{[j-1:n]})}{\partial \widehat{p}_j}\right]_{p_j} (\widehat{p}_j - p_j) + o_p(1) \\ &= g(n\Delta\phi_j) + n \cdot g_j'(n\Delta\widehat{\phi}_j) \cdot h_j'(p_j)(\widehat{p}_j - p_j) + o_p(1) \end{aligned}$$

where g_j' and h_j' are the two terms in big square brackets, and for notation simplicity, we write $\partial\Phi/\partial\widehat{p}_j$ as Φ' so that

$$h_j'(p_j) = \Phi'(\sqrt{T}\widehat{p}_{[j:n]}) - \Phi'(\sqrt{T}\widehat{p}_{[j-1:n]}).$$

Using $\Phi_j''(\widetilde{p}_{[j:n]}) \approx \frac{\Delta\Phi'(\widetilde{p}_{[j:n]})}{\Delta p_{[j:n]}}$ for $\widetilde{p}_{[j:n]} \in [p_{[j:n]}, p_{[j-1:n]}]$, we have

$$h_j'(p_{[j:n]}) \approx \Phi_j''(\widetilde{p}_{[j:n]})\Delta p_{[j:n]}.$$

Thus, for Theorem 2, we consider.

$$\frac{1}{\sqrt{n}}\sum_{j=1}^{n} g(n\Delta\widehat{\phi}_j) = \frac{1}{\sqrt{n}}\sum_{j=1}^{n} g(n\Delta\phi_j) + \frac{\sqrt{n}}{\sqrt{T}}\frac{1}{n}\sum_{j=1}^{n} n\Delta p_{[j:n]} g'(n\Delta\phi_j)\Phi''(\widetilde{p}_j)\sqrt{T}(\widehat{p}_j - p_j) + o_p(1).$$

Now $\sqrt{T}(\widehat{p}_j - p_j) = O_p(1)$, $\frac{1}{n}\sum_{j=1}^{n} g_j'(n\phi_j) \xrightarrow{p} E(g'(X))$ where $X \sim \Gamma(1,1)$, and $\frac{1}{n}\sum_{j=1}^{n} \Phi''(\widetilde{p}_{[j:n]}) = \int_0^1 \Phi''(u)du = 0$ since Φ is the normal distribution function. Finally, $\sum_{j=1}^{n} p_{[j:n]} - p_{[j-1:n]} = p_{[1:n]} - p_{[n:n]}$ and so $\frac{1}{n}\sum_{j=1}^{n} n(p_{[j:n]} - p_{[j-1:n]}) = 0_p(1)$. The null distribution of $\frac{1}{\sqrt{n}}\sum_{j=1}^{n} g(n\widehat{\phi}_j)$ is the same as $\frac{1}{\sqrt{n}}\sum_{j=1}^{n} g(n\phi_j)$ if $\frac{\sqrt{n}}{\sqrt{T}} = O(1)$, or $\frac{N}{\sqrt{T}} = O(1)$. For Theorem 1, we consider $\sum_{j=1}^{\eta} g(\Delta\widehat{\phi}_j)$, $g(\cdot)$ being the sum of squared residuals function

$$\sum_{j=1}^{\eta} g(\Delta\widehat{\phi}_j) = \sum_{j=1}^{n} g(\Delta\phi_j) + \frac{1}{\sqrt{T}}\sum_{j=1}^{n} \Delta p_{[j:n]} g'(\Delta\phi_j) n\frac{1}{n}\Phi''(\widetilde{p}_j)\sqrt{T}(\widehat{p}_j - p_j) + o_p(1).$$

Using analogous arguments, $\sum_{j=1}^{n} g(\Delta\widehat{\phi}_j) = \sum_{j=1}^{n} g(\Delta\phi_j) + o_p(T^{-1/2})$. By the results of Bai (1997), least squares provides a super-consistent estimate of the break fraction.

Proof of Theorem 2 Let D_j be a first order uniform spacing and let D_j^q be a q-order spacing. The statistic $G_{1n} = \frac{1}{n}\sum_{j=1}^{n}(nD_j)^2$ is referred to as the first order Greenwood statistic, and $G_{3n} = \sum_{j=1}^{n}(n\Delta^q D_j)^2$ is a generalized Greenwood statistic based on overlapping spacings. As summarized in Rao and Kuo (1984),

$$\begin{aligned} \sqrt{n}(G_{1n} - 2) &\xrightarrow{d} N(0,4) \\ \sqrt{n}(G_{3n} - q(q+1)) &\xrightarrow{d} N(0, 2q(q+1)(2q+1)/3). \end{aligned}$$

Now $E(D_j) = 1/(n+1)$. Thus, $\text{var}(nD_j) = \frac{1}{n}\sum_{j=1}^{n}(nD_j - 1)^2 = \frac{1}{n}\sum_{j=1}^{n}(nD_j)^2 - 1$ has the same limiting variance as G_{1n}. Likewise, $\text{var}(D_i^q) = \frac{1}{n}\sum_{i=1}^{n}(nD_i^q - q)^2$ has the same limiting variance as G_{3n}.

As defined, $\widehat{\sigma}_1^2 = \text{var}(nD_j)$ and $\sigma_q^2 = \text{var}(nD_j^q/q)$. Thus,

$$\begin{aligned} \text{var}(\widehat{\sigma}_1^2) &= 4 \\ \text{var}(\widehat{\sigma}_q^2) &= \frac{1}{q^2}\text{var}(nD_j^q) = \frac{2(q+1)(2q+1)}{3q}. \end{aligned}$$

While $\widehat{\sigma}_1^2$ and $\widehat{\sigma}_q^2$ are both consistent estimates of σ_ε^2, $\widehat{\sigma}_1^2$ is asymptotically efficient. Thus, following the argument as in Lo and MacKinlay (1988), we have

$$\begin{aligned} \text{var}(\widehat{\sigma}_q^2 - \widehat{\sigma}_1^2) &= \text{var}(\widehat{\sigma}_q^2) - \text{var}(\sigma_1^2) \\ &= \frac{2(q+1)(2q+1)}{3q} - 4 = \frac{2(2q-1)(q-1)}{3}. \end{aligned}$$

This is precisely the asymptotic variance of the SVR statistic of Lo and MacKinlay (1988).

Table 2a: Pearson Correlation Coefficients: Normal Errors

dgp	θ_0	$\widehat{\theta}$	std$(\widehat{\theta})$	SVR_S	SVR_L	$\widehat{\theta}$	std$(\widehat{\theta})$	SVR_{SS}	$t_{\bar{\phi}}$	$t_{\text{var}(\Delta\bar{\phi})}$
		first split				second split				
$N = 10, T = 100$										
1	1.000	0.475	0.252	0.091	0.095	0.549	0.264	0.048	0.066	0.944
2	0.000	0.171	0.092	0.272	0.938	0.392	0.233	0.021	0.999	0.257
3	0.000	0.279	0.166	0.095	0.764	0.452	0.247	0.043	0.989	0.605
4	0.622	0.524	0.161	0.059	0.837	0.539	0.271	0.038	0.924	0.852
5	0.333	0.416	0.158	0.088	0.883	0.480	0.260	0.045	0.993	0.841
6	0.867	0.728	0.212	0.066	0.767	0.538	0.278	0.057	0.308	0.949
7	0.378	0.327	0.080	0.074	0.953	0.553	0.267	0.052	1.000	0.734
8	0.867	0.539	0.261	0.092	0.263	0.557	0.264	0.049	0.188	0.936
9	0.378	0.489	0.182	0.064	0.725	0.497	0.266	0.054	0.894	0.870
10	0.378	0.385	0.147	0.079	0.877	0.524	0.284	0.052	0.988	0.748
$N = 20, T = 100$										
1	1.000	0.483	0.271	0.062	0.068	0.562	0.272	0.049	0.050	1.000
2	0.000	0.150	0.076	0.200	0.998	0.355	0.251	0.049	1.000	0.618
3	0.000	0.302	0.138	0.060	0.990	0.418	0.236	0.053	1.000	0.950
4	0.805	0.762	0.115	0.051	0.947	0.493	0.261	0.043	0.923	0.999
5	0.632	0.692	0.131	0.063	0.975	0.465	0.248	0.063	0.991	0.999
6	0.853	0.831	0.062	0.046	0.984	0.505	0.273	0.055	0.890	0.999
7	0.368	0.345	0.046	0.071	1.000	0.522	0.278	0.061	1.000	0.994
8	0.853	0.760	0.192	0.058	0.603	0.485	0.272	0.056	0.591	0.998
9	0.368	0.540	0.129	0.047	0.963	0.445	0.243	0.057	1.000	1.000
10	0.368	0.419	0.112	0.093	0.998	0.483	0.264	0.060	1.000	0.991
$N = 20, T = 200$										
1	1.000	0.482	0.275	0.065	0.059	0.571	0.271	0.062	0.052	1.000
2	0.000	0.128	0.051	0.285	0.999	0.314	0.243	0.030	1.000	0.488
3	0.000	0.237	0.117	0.096	0.997	0.385	0.229	0.054	1.000	0.901
4	0.805	0.774	0.086	0.059	0.983	0.534	0.269	0.051	0.938	0.998
5	0.632	0.666	0.121	0.058	0.977	0.483	0.256	0.053	0.993	0.999
6	0.853	0.827	0.073	0.048	0.980	0.509	0.273	0.057	0.883	1.000
7	0.368	0.349	0.042	0.079	1.000	0.554	0.277	0.062	1.000	0.983
8	0.853	0.796	0.169	0.042	0.774	0.490	0.269	0.052	0.656	0.999
9	0.368	0.507	0.119	0.058	0.990	0.454	0.246	0.060	1.000	0.999
10	0.368	0.400	0.100	0.087	0.998	0.492	0.259	0.051	1.000	0.983
$N = 30, T = 200$										
1	1.000	0.501	0.270	0.055	0.055	0.571	0.279	0.055	0.051	1.000
2	0.000	0.127	0.046	0.332	1.000	0.342	0.246	0.061	1.000	0.726
3	0.000	0.240	0.103	0.094	1.000	0.435	0.216	0.056	1.000	0.988
4	0.869	0.857	0.037	0.063	0.997	0.543	0.260	0.057	0.937	1.000
5	0.747	0.790	0.071	0.051	0.999	0.513	0.253	0.044	0.996	1.000
6	0.848	0.841	0.024	0.067	0.999	0.537	0.267	0.056	0.997	1.000
7	0.366	0.355	0.026	0.081	1.000	0.564	0.270	0.057	1.000	0.999
8	0.848	0.858	0.075	0.060	0.938	0.488	0.264	0.055	0.907	1.000
9	0.366	0.525	0.090	0.072	1.000	0.510	0.239	0.060	1.000	1.000
10	0.366	0.417	0.081	0.081	1.000	0.495	0.257	0.058	1.000	0.999

$\widehat{\theta}$ is the estimated fraction of the sample in S (small correlations). SVA_S and SVR_L are the rejection rates of spacings variance-ratio test applied to the subsamples S and L respectively. $t_{\bar{\phi}}$ tests if the mean of $\bar{\phi}_j$ is .75, and $t_{\text{var}(\bar{\phi})}$ tests if var$(\bar{\phi}_j) = 0$.

Table 2b: Fisher Transformed Pearson Correlation Coefficients: Normal Errors

dgp	θ_0	$\widehat{\theta}$	std($\widehat{\theta}$)	SVR_S	SVR_L	$\widehat{\theta}$	std($\widehat{\theta}$)	SVR_{SS}	$t_{\bar{\phi}}$	$t_{\text{var}(\Delta\bar{\phi})}$
		first split				second split				
$N = 10, T = 100$										
1	1.000	0.478	0.253	0.091	0.097	0.546	0.264	0.048	0.066	0.944
2	0.000	0.170	0.092	0.271	0.941	0.391	0.233	0.022	0.999	0.255
3	0.000	0.280	0.166	0.095	0.776	0.452	0.247	0.044	0.989	0.605
4	0.622	0.525	0.160	0.060	0.841	0.540	0.272	0.038	0.924	0.857
5	0.333	0.416	0.157	0.087	0.882	0.481	0.259	0.046	0.993	0.840
6	0.867	0.729	0.210	0.064	0.770	0.538	0.277	0.059	0.315	0.950
7	0.378	0.327	0.079	0.074	0.954	0.552	0.267	0.054	1.000	0.733
8	0.867	0.545	0.261	0.092	0.276	0.556	0.263	0.048	0.194	0.936
9	0.378	0.489	0.180	0.065	0.736	0.497	0.267	0.054	0.898	0.871
10	0.378	0.385	0.146	0.080	0.884	0.525	0.283	0.053	0.989	0.746
$N = 20, T = 100$										
1	1.000	0.485	0.272	0.064	0.071	0.560	0.272	0.049	0.052	0.999
2	0.000	0.150	0.076	0.199	0.999	0.355	0.251	0.049	1.000	0.616
3	0.000	0.302	0.138	0.060	0.990	0.420	0.238	0.053	1.000	0.951
4	0.805	0.762	0.112	0.051	0.951	0.490	0.258	0.041	0.929	0.999
5	0.632	0.691	0.128	0.064	0.976	0.463	0.248	0.061	0.991	0.999
6	0.853	0.829	0.062	0.046	0.986	0.506	0.272	0.054	0.895	0.999
7	0.368	0.344	0.046	0.069	1.000	0.517	0.277	0.060	1.000	0.994
8	0.853	0.767	0.186	0.059	0.633	0.481	0.271	0.056	0.604	0.998
9	0.368	0.539	0.129	0.048	0.966	0.446	0.244	0.057	1.000	1.000
10	0.368	0.418	0.111	0.093	0.998	0.484	0.264	0.062	1.000	0.991
$N = 20, T = 200$										
1	1.000	0.485	0.275	0.064	0.059	0.568	0.272	0.063	0.054	1.000
2	0.000	0.128	0.051	0.285	0.999	0.314	0.243	0.030	1.000	0.488
3	0.000	0.237	0.117	0.094	0.997	0.385	0.228	0.054	1.000	0.900
4	0.805	0.773	0.086	0.060	0.984	0.535	0.269	0.052	0.938	0.998
5	0.632	0.665	0.121	0.058	0.978	0.486	0.256	0.054	0.993	0.999
6	0.853	0.826	0.073	0.050	0.982	0.510	0.274	0.058	0.886	1.000
7	0.368	0.349	0.042	0.077	1.000	0.551	0.276	0.062	1.000	0.984
8	0.853	0.798	0.166	0.042	0.786	0.489	0.269	0.052	0.665	0.999
9	0.368	0.507	0.118	0.059	0.990	0.454	0.247	0.058	1.000	0.999
10	0.368	0.400	0.099	0.089	0.998	0.493	0.259	0.049	1.000	0.984
$N = 30, T = 200$										
1	1.000	0.505	0.268	0.053	0.056	0.574	0.279	0.055	0.053	1.000
2	0.000	0.127	0.046	0.332	1.000	0.343	0.247	0.061	1.000	0.727
3	0.000	0.240	0.103	0.093	1.000	0.435	0.216	0.055	1.000	0.989
4	0.869	0.857	0.037	0.065	0.997	0.543	0.260	0.058	0.939	1.000
5	0.747	0.788	0.071	0.050	0.999	0.510	0.253	0.046	0.996	1.000
6	0.848	0.840	0.025	0.067	0.999	0.537	0.266	0.054	0.997	1.000
7	0.366	0.355	0.025	0.080	1.000	0.560	0.269	0.057	1.000	1.000
8	0.848	0.859	0.072	0.062	0.947	0.488	0.264	0.054	0.912	1.000
9	0.366	0.526	0.089	0.072	1.000	0.510	0.240	0.061	1.000	1.000
10	0.366	0.417	0.081	0.079	1.000	0.497	0.257	0.058	1.000	0.999

$\widehat{\theta}$ is the estimated fraction of the sample in S. SVR_S and SVR_L are the rejection rates of the SVR applied to subsamples S and L. $t_{\bar{\phi}}$ tests if the mean of $\bar{\phi}_j$ is .75, and $t_{\text{var}(\bar{\phi})}$ tests if $\text{var}(\bar{\phi}_j) = 0$.

Table 3a: Pearson Correlation Coefficients: ARCH(1) errors

dgp	θ_0	$\widehat{\theta}$	std$(\widehat{\theta})$	SVR_S	SVR_L	$\widehat{\theta}$	std$(\widehat{\theta})$	SVR_{SS}	$t_{\bar{\phi}}$	$t_{\text{var}(\Delta\bar{\phi})}$
		first split				second split				
$N = 10, T = 100$										
1	1.000	0.487	0.255	0.106	0.084	0.533	0.260	0.045	0.072	0.942
2	0.000	0.170	0.091	0.272	0.939	0.399	0.239	0.017	0.999	0.275
3	0.000	0.278	0.164	0.091	0.793	0.441	0.248	0.031	0.990	0.626
4	0.622	0.511	0.174	0.075	0.822	0.515	0.268	0.058	0.925	0.867
5	0.333	0.420	0.159	0.071	0.887	0.496	0.260	0.059	0.994	0.829
6	0.867	0.726	0.211	0.057	0.764	0.572	0.269	0.059	0.304	0.944
7	0.378	0.330	0.078	0.078	0.956	0.540	0.269	0.061	1.000	0.738
8	0.867	0.556	0.255	0.075	0.305	0.559	0.264	0.036	0.183	0.947
9	0.378	0.484	0.184	0.074	0.733	0.494	0.263	0.044	0.900	0.886
10	0.378	0.377	0.149	0.089	0.880	0.519	0.270	0.043	0.991	0.771
$N = 20, T = 100$										
1	1.000	0.512	0.273	0.072	0.083	0.580	0.272	0.062	0.060	0.999
2	0.000	0.150	0.076	0.212	0.995	0.355	0.252	0.043	1.000	0.624
3	0.000	0.299	0.137	0.066	0.984	0.409	0.237	0.056	1.000	0.964
4	0.805	0.736	0.133	0.071	0.939	0.497	0.258	0.056	0.937	0.999
5	0.632	0.689	0.132	0.052	0.976	0.485	0.256	0.057	0.990	0.997
6	0.853	0.830	0.052	0.056	0.987	0.519	0.272	0.055	0.892	1.000
7	0.368	0.344	0.049	0.081	0.999	0.528	0.276	0.073	1.000	0.990
8	0.853	0.765	0.189	0.053	0.650	0.511	0.271	0.040	0.573	0.998
9	0.368	0.537	0.132	0.057	0.956	0.443	0.246	0.068	1.000	0.997
10	0.368	0.418	0.111	0.082	0.995	0.476	0.258	0.049	1.000	0.986
$N = 20, T = 200$										
1	1.000	0.513	0.267	0.069	0.067	0.559	0.276	0.058	0.054	1.000
2	0.000	0.128	0.052	0.255	0.999	0.316	0.247	0.032	1.000	0.486
3	0.000	0.239	0.123	0.094	0.996	0.396	0.229	0.058	1.000	0.910
4	0.805	0.756	0.103	0.048	0.974	0.521	0.269	0.059	0.943	1.000
5	0.632	0.671	0.120	0.055	0.991	0.489	0.253	0.045	0.993	0.997
6	0.853	0.829	0.063	0.056	0.990	0.521	0.270	0.056	0.875	0.999
7	0.368	0.348	0.045	0.079	0.999	0.548	0.276	0.066	1.000	0.981
8	0.853	0.796	0.168	0.060	0.758	0.495	0.269	0.054	0.658	1.000
9	0.368	0.510	0.117	0.060	0.985	0.462	0.245	0.076	1.000	0.999
10	0.368	0.401	0.103	0.078	1.000	0.511	0.269	0.047	1.000	0.985
$N = 30, T = 200$										
1	1.000	0.496	0.274	0.059	0.066	0.576	0.276	0.058	0.062	1.000
2	0.000	0.127	0.048	0.335	1.000	0.347	0.243	0.066	1.000	0.731
3	0.000	0.239	0.100	0.097	1.000	0.418	0.222	0.060	1.000	0.989
4	0.869	0.842	0.068	0.050	0.992	0.537	0.258	0.042	0.946	1.000
5	0.747	0.788	0.073	0.058	0.999	0.516	0.248	0.045	0.998	1.000
6	0.848	0.840	0.019	0.046	1.000	0.538	0.271	0.047	0.993	1.000
7	0.366	0.355	0.028	0.079	1.000	0.577	0.271	0.058	1.000	0.999
8	0.848	0.859	0.064	0.061	0.941	0.500	0.267	0.056	0.900	1.000
9	0.366	0.524	0.088	0.068	1.000	0.517	0.241	0.049	1.000	1.000
10	0.366	0.416	0.084	0.098	1.000	0.505	0.255	0.054	1.000	1.000

$\widehat{\theta}$ is the estimated fraction of the sample in S (small correlations). SVR_S and SVR_L are the rejection rates in the subsamples S and L respectively. $t_{\bar{\phi}}$ tests if the mean of $\bar{\phi}_j$ is .75, and $t_{\text{var}(\bar{\phi})}$ tests if $\text{var}(\bar{\phi}_j) = 0$.

Table 3b: Pearson Correlation Coefficients: χ^2_1 Errors

dgp	θ_0	$\widehat{\theta}$	std$(\widehat{\theta})$	SVR_S	SVR_L	$\widehat{\theta}$	std$(\widehat{\theta})$	SVR_{SS}	$t_{\bar{\phi}}$	$t_{\text{var}(\Delta\bar{\phi})}$
		first split				second split				
$N = 10, T = 100$										
1	1.000	0.500	0.263	0.094	0.092	0.539	0.260	0.033	0.065	0.966
2	0.000	0.200	0.114	0.187	0.883	0.412	0.241	0.015	1.000	0.411
3	0.000	0.358	0.188	0.071	0.681	0.460	0.255	0.046	0.942	0.746
4	0.622	0.523	0.178	0.087	0.845	0.523	0.272	0.036	0.902	0.857
5	0.333	0.424	0.157	0.070	0.887	0.500	0.260	0.060	0.998	0.828
6	0.867	0.680	0.236	0.074	0.633	0.561	0.274	0.052	0.239	0.945
7	0.378	0.321	0.090	0.080	0.853	0.505	0.257	0.047	1.000	0.767
8	0.867	0.515	0.264	0.084	0.211	0.556	0.258	0.052	0.120	0.950
9	0.378	0.511	0.212	0.072	0.565	0.494	0.266	0.046	0.757	0.899
10	0.378	0.418	0.164	0.074	0.841	0.518	0.267	0.041	0.982	0.806
$N = 20, T = 100$										
1	1.000	0.529	0.289	0.055	0.066	0.561	0.272	0.052	0.066	1.000
2	0.000	0.186	0.102	0.138	0.995	0.383	0.237	0.061	1.000	0.780
3	0.000	0.416	0.162	0.064	0.928	0.413	0.229	0.046	1.000	0.989
4	0.805	0.761	0.130	0.049	0.937	0.504	0.263	0.052	0.912	0.999
5	0.632	0.697	0.121	0.061	0.978	0.486	0.256	0.058	0.982	0.999
6	0.853	0.824	0.074	0.068	0.945	0.509	0.278	0.060	0.848	1.000
7	0.368	0.346	0.053	0.047	0.996	0.467	0.254	0.059	1.000	0.996
8	0.853	0.675	0.261	0.050	0.441	0.547	0.280	0.061	0.390	1.000
9	0.368	0.613	0.140	0.057	0.853	0.450	0.242	0.065	0.994	1.000
10	0.368	0.473	0.111	0.095	0.994	0.463	0.255	0.066	1.000	0.997
$N = 20, T = 200$										
1	1.000	0.516	0.282	0.057	0.065	0.551	0.267	0.059	0.047	1.000
2	0.000	0.155	0.076	0.169	0.998	0.372	0.239	0.047	1.000	0.693
3	0.000	0.341	0.145	0.065	0.978	0.407	0.235	0.072	1.000	0.974
4	0.805	0.770	0.090	0.061	0.983	0.527	0.269	0.041	0.929	1.000
5	0.632	0.672	0.117	0.051	0.985	0.494	0.257	0.044	0.997	1.000
6	0.853	0.837	0.037	0.056	0.996	0.530	0.277	0.059	0.896	1.000
7	0.368	0.353	0.034	0.061	1.000	0.533	0.275	0.071	1.000	0.994
8	0.853	0.749	0.220	0.052	0.626	0.535	0.281	0.050	0.472	1.000
9	0.368	0.581	0.121	0.058	0.943	0.440	0.241	0.051	1.000	1.000
10	0.368	0.445	0.100	0.076	0.998	0.475	0.255	0.073	1.000	0.997
$N = 30, T = 200$										
1	1.000	0.508	0.280	0.055	0.065	0.565	0.280	0.053	0.068	1.000
2	0.000	0.156	0.073	0.187	1.000	0.390	0.233	0.069	1.000	0.885
3	0.000	0.354	0.121	0.075	0.998	0.459	0.217	0.063	1.000	1.000
4	0.869	0.857	0.046	0.061	0.996	0.545	0.263	0.052	0.946	1.000
5	0.747	0.796	0.061	0.068	0.996	0.539	0.252	0.053	0.995	1.000
6	0.848	0.841	0.011	0.054	1.000	0.538	0.276	0.059	1.000	1.000
7	0.366	0.357	0.019	0.072	1.000	0.543	0.270	0.048	1.000	1.000
8	0.848	0.843	0.127	0.054	0.807	0.487	0.278	0.050	0.763	1.000
9	0.366	0.601	0.095	0.061	0.995	0.508	0.228	0.042	1.000	1.000
10	0.366	0.458	0.081	0.068	1.000	0.505	0.244	0.046	1.000	0.999

$\widehat{\theta}$ is the estimated fraction of the sample in S (small correlations). SVR_S and SVR_L are the rejection rates for subsamples S and L respectively. $t_{\bar{\phi}}$ tests if the mean of $\bar{\phi}_j$ is .75, and $t_{\text{var}(\bar{\phi})}$ tests if $\text{var}(\bar{\phi}_j) = 0$.

Table 4: Correlations in Industrial Production, 12 OECD Countries

Group S			Group L		
Country		$\widehat{p}_j$	Country		$\widehat{p}_j$
FRA	BEL	0.003	FRA	FIN	0.116
GER	PORT	-0.003	AUS	FIN	0.117
NETH	BEL	0.004	GER	LUX	0.118
BEL	US	-0.008	LUX	PORT	0.123
AUS	IRE	0.012	IRE	US	0.124
FIN	IRE	-0.013	ITA	AUS	0.125
PORT	BEL	-0.013	ITA	BEL	0.130
ITA	FIN	-0.018	FRA	PORT	0.135
FRA	IRE	0.021	BEL	IRE	0.144
AUS	PORT	-0.027	FRA	LUX	0.145
AUS	LUX	0.032	GER	NETH	0.147
SPA	FIN	0.036	GER	BEL	0.148
FRA	AUS	0.036	LUX	US	0.149
SPA	BEL	0.037	LUX	BEL	0.160
LUX	NETH	0.038	GER	SPA	0.166
NETH	FIN	-0.038	FRA	NETH	0.169
NETH	US	0.039	SPA	US	0.170
GER	US	0.043	FIN	US	0.173
GER	AUS	-0.044	FIN	BEL	0.175
LUX	IRE	0.051	ITA	PORT	0.177
GER	FIN	0.051	GER	IRE	0.178
PORT	IRE	0.067	SPA	IRE	0.180
PORT	US	0.068	SPA	LUX	0.182
NETH	IRE	0.071	AUS	NETH	0.185
GER	ITA	0.078	SPA	PORT	0.186
LUX	FIN	0.089	ITA	NETH	0.191
NETH	PORT	0.091	SPA	NETH	0.205
SPA	AUS	0.094	ITA	IRE	0.205
FIN	PORT	0.094	ITA	SPA	0.226
AUS	US	0.097	ITA	US	0.233
ITA	FRA	0.103	FRA	US	0.250
			AUS	BEL	0.293
			ITA	LUX	0.299
			SPA	FRA	0.364
			GER	FRA	0.372

The industrial production data are for Germany, Italy, Spain, France, Austria, Luxembourg, the Netherlands, Finland, Portugal, Belgium, Ireland, and the US, respectively from 1982:1 to 1997:8.

Table 5: Real Exchange Rates, 21 Countries

Group S			Largest 30 Correlations in Group L		
Country		$\widehat{p}_j$	Country		$\widehat{p}_j$
CAN	FR	0.008	NETH	NOR	0.847
CAN	NETH	0.009	AUS	IRE	0.851
CAN	SPA	0.016	FR	SWTZD	0.857
CAN	GER	0.019	GER	IRE	0.860
CAN	ITA	0.020	BEL	SWTZD	0.862
CAN	BEL	0.021	BEL	IRE	0.867
CAN	NOR	0.025	DEN	SWTZD	0.867
UK	KOR	-0.026	DEN	IRE	0.869
CAN	AUS	0.030	IRE	NETH	0.869
SWTZD	KOR	0.045	FR	IRE	0.870
CAN	SWED	0.050	GER	NOR	0.871
FIN	KOR	0.050	AUS	NOR	0.872
DEN	KOR	0.052	AUS	SWTZD	0.876
CAN	SWTZD	0.052	NETH	SWTZD	0.877
CAN	SG	0.052	GER	SWTZD	0.879
NETH	KOR	0.053	DEN	FR	0.913
SWED	KOR	0.056	AUS	FR	0.921
CAN	IRE	0.056	BEL	FR	0.922
CAN	THAI	0.058	FR	GER	0.922
ITA	KOR	0.058	FR	NETH	0.927
FR	KOR	0.059	AUS	DEN	0.950
BEL	KOR	0.064	DEN	GER	0.956
CAN	JP	0.064	DEN	NETH	0.959
GER	KOR	0.065	BEL	DEN	0.963
AUS	KOR	0.068	AUS	BEL	0.963
CAN	DEN	0.070	BEL	GER	0.964
IRE	KOR	0.076	BEL	NETH	0.970
NOR	KOR	0.078	AUS	NETH	0.978
CAN	UK	0.081	GER	NETH	0.982
SPA	KOR	0.085	AUS	GER	0.985

The real exchange rates are for Canada, Australia, New Zealand, Australia, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Spain, Sweden, Switzerland, UK, Japan, Korea, Singapore, and Thailand. The U.S. is used as the numeraire country. The sample is 1974:1-1997:4

Figure 1: $\bar{\phi}(j)$: DGP 1

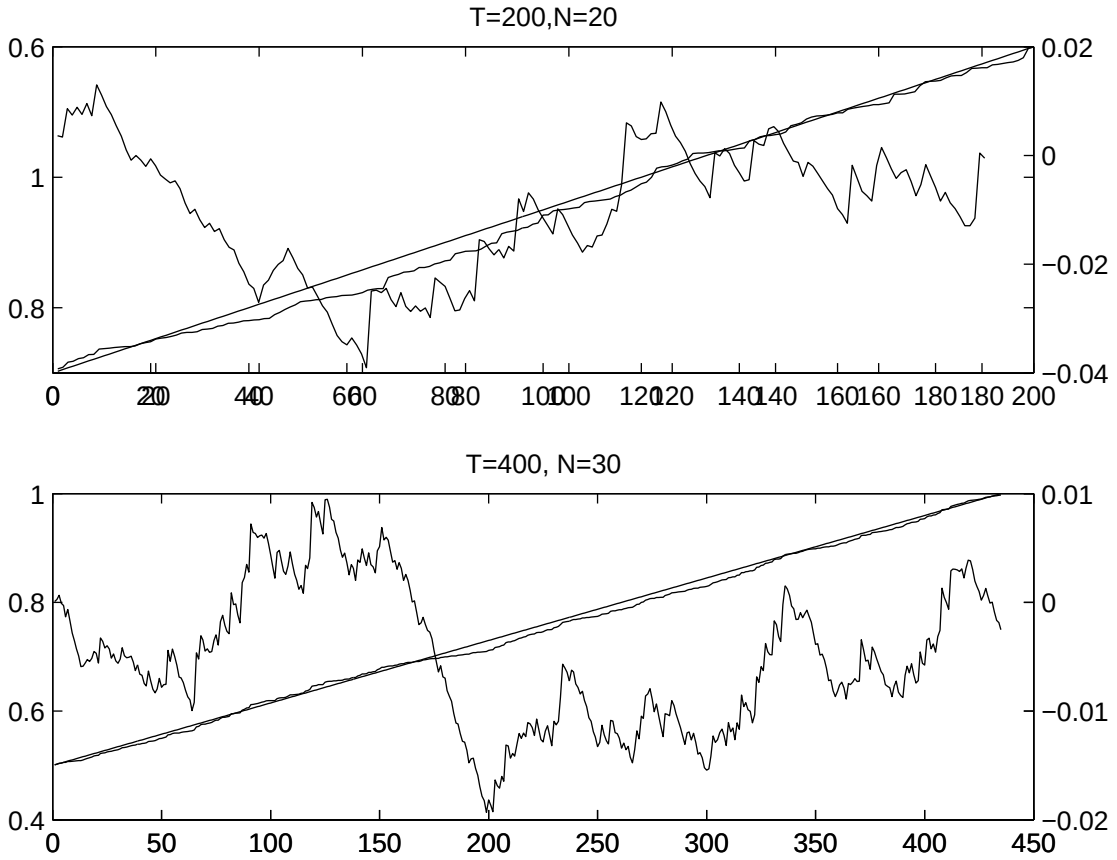
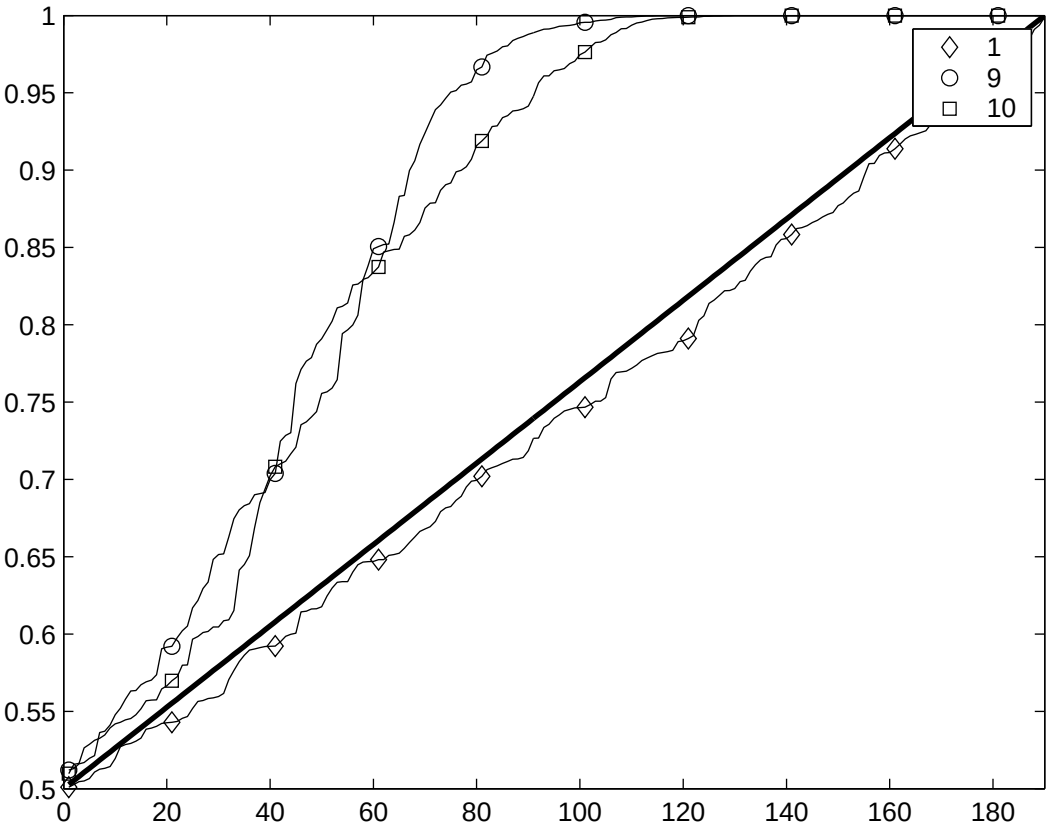
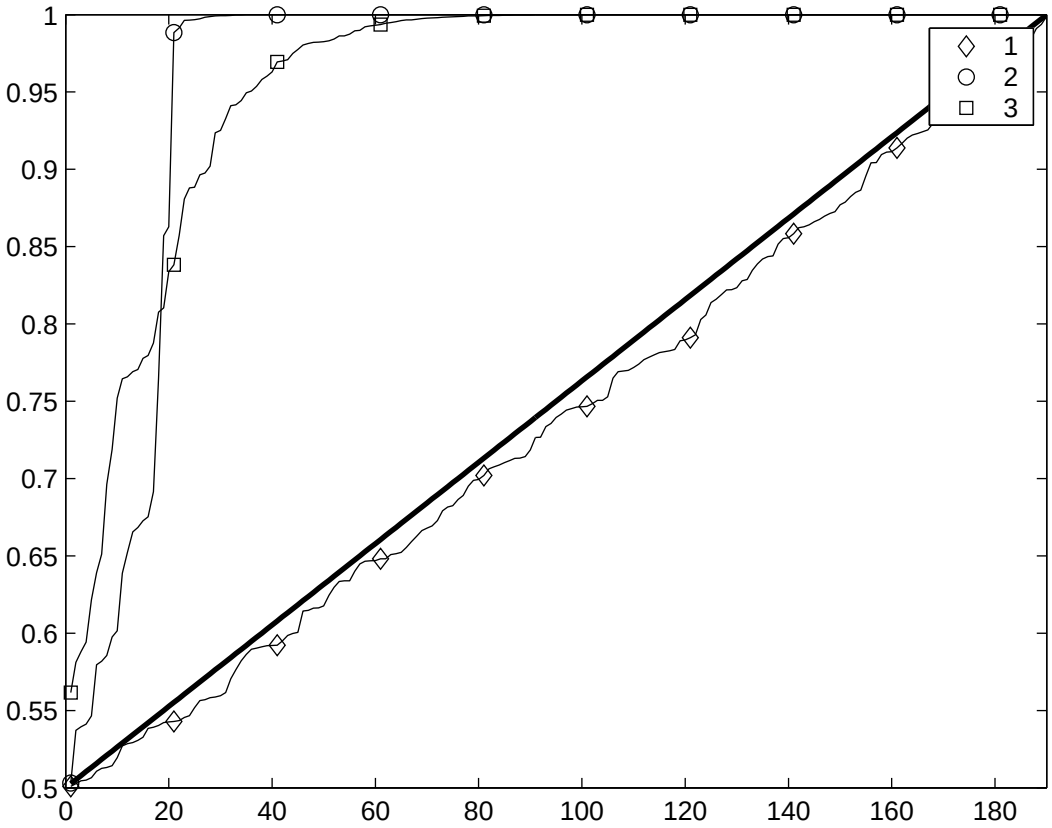


Figure 2: $\bar{\phi}(j)$: DGP 1, 9 and 10



$x_{it} = \lambda_i G_t + e_{it}$, $\lambda_i \sim N(0,1)$, $i = 1, \ldots .8N = 152$, and $\lambda_i = 0, i > .8N$. For DGP 9, $e_{it} \sim N(0,1)$. For DGP 10, $e_{it} \sim N(0,.2)$.

Figure 3: $\bar{\phi}(j)$: DGP 1, 2 and 3



$\bar{\phi}_j$ can be less than one even though DGP 2 (small idiosyncratic errors) and 3 (large idiosyncratic errors) have a factor structure.

Figure 4: $\bar{\phi}(j)$: Growth Rate of Industrial Production, 12 OECD Countries

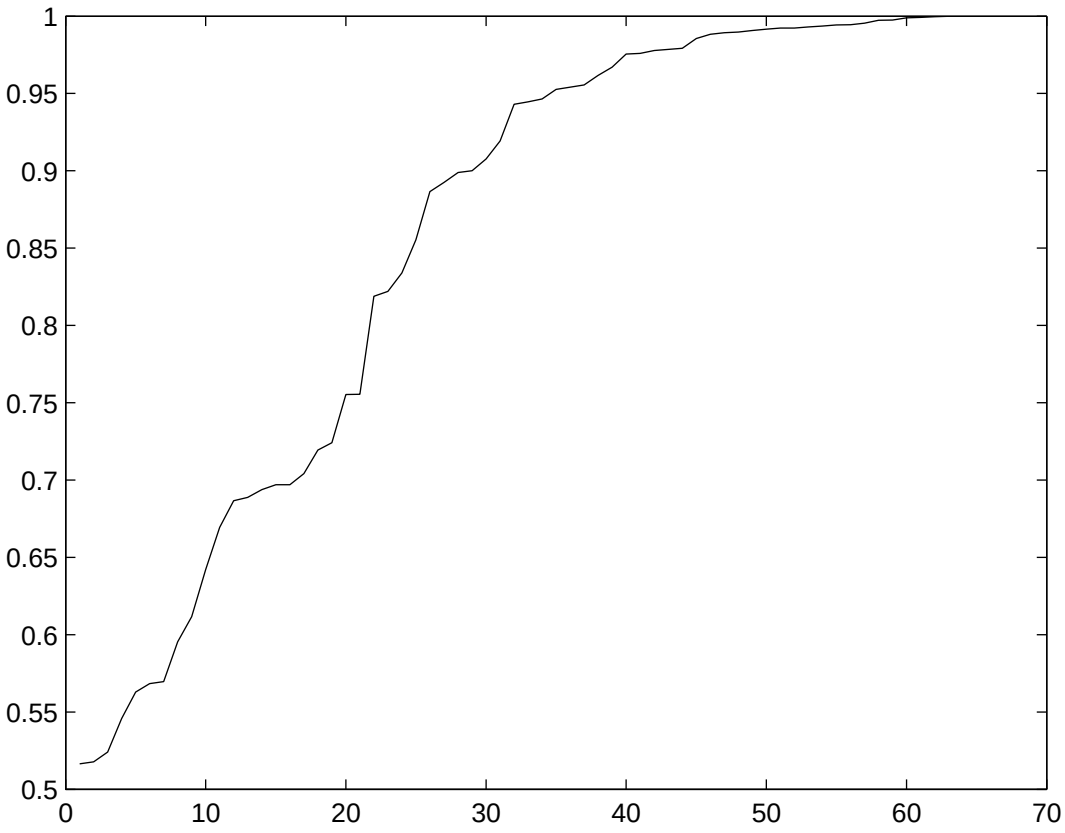


Figure 5: $\bar{\phi}(j)$: Real Exchange Rates, 21 Countries