On the Long-Run Variance Ratio Test for a Unit Root

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Abstract

This paper investigates the effects of consistent and inconsistent long-run variance estimation on a unit root test based on the generalization of the von Neumann ratio. The results from the Monte Carlo experiments suggest that the tests based on an inconsistent estimator have less size distortion and more stability of size across different autocorrelation specifications as compared to the tests based on a consistent estimator. This improvement in size property, however, comes at the cost of a loss in power. The finite sample power, as well as the local asymptotic power, of the tests with an inconsistent estimator is shown to be much lower than that of conventional tests. This finding resembles the case of the autocorrelation robust test in the standard regression context. The paper also points out that combining consistent and inconsistent estimators in the long-run variance ratio test for a unit root is one possibility of balancing the size and power.

Keywords: Bandwidth; Local Asymptotic Power; von Neumann Ratio.

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1 Introduction

Conventionally, the autocorrelation robust inference relies on the consistent estimation of the long-run variance of the data. In the regression context, such an estimator based on the nonparametric kernel method is often referred to as the heteroskedasiticity autocorrelation consistent (HAC) estimator and is frequently employed to construct standard errors or the Wald type test statistics in the presence of serial correlation of unknown form (see Newey and West, 1987, and Andrews, 1991, for example). HAC estimation, however, is known to suffer from the small sample bias that results in size distortion of the test statistics. Kiefer, Vogelsang, and Bunzel (2000) have recently proposed the autocorrelation robust test statistics standardized by an inconsistent long-run variance estimator instead of a consistent estimator. Their alternative asymptotic approximation to the distribution of the test statistic incorporates the randomness of the (inconsistent) long-run variance estimator and is considered to have some advantages in improving the size properties compared to the conventional approach.

Since an influential paper by Phillips (1987), nonparametric long-run variance estimation has also played an important role in the unit root/nonstationary literature. The nonparametric or semiparametric unit root test designed to incorporate general serial correlation, however, is known to suffer from some size distortion for the same reason as the test with HAC estimation in the standard regression model. Therefore, it seems reasonable to investigate whether the inconsistent estimation of the long-run variance provides a useful alternative approach in the unit root test as well as in the tests in the regression model. In this paper, we conduct theoretical and simulation analyses on the effect of consistent and inconsistent long-run variance estimation in testing for a unit root. In particular, we focus on a class of nonparametric tests based on the generalization of the von Neumann (VN) ratio. This class of the unit root test includes the test considered by Sargan and Bhargava (1983), Bhargava (1986), the class of locally best invariant

(LBI) test considered by Nabeya and Tanaka (1990), the Lagrange multiplier (LM) test of Schmidt and Phillips (1992), the modified Sargan-Bhargava (MSB) test considered by Stock (1994, 1999) and Perron and Ng (1996), and a nonparametric unit root test of Breitung (2002). Its multivariate extension includes the cointegration tests considered by Phillips and Ouliaris (1990), Shintani (2001), and Harris and Poskitt (2004).

The main reason for the choice of the VN ratio test in our analysis, rather than more commonly used nonparametric variation of the Dickey-Fuller type test proposed by Phillips (1987) and Phillips and Perron (1988), is its convenience in considering the properties of the long-run variance estimation under the null and alternative hypotheses. In a typical regression framework, the true long-run variance used to standardize the Wald test statistic is common under both null and alternative hypotheses. In the test for a unit root, this is not the case. To be more specific, estimation of a positive long-run variance of the first differenced observation (or the error term) is often required for the test statistic to have a nuisance parameter free limiting distribution under the null hypothesis of a unit root. Under the alternative hypothesis of a stationary root, however, the long-run variance of the same variable becomes zero because of over-differencing. In contrast, the long-run variance of the variable in level is positive and finite under the alternative, while the corresponding long-run variance cannot be defined under the null hypothesis. The unit root test statistic we consider is constructed using the ratio of the long-run variance estimator of the first differenced series to that of the series in levels. Since the growth rate of the bandwidth in the kernel estimator is the key to distinguishing the consistent estimator from the inconsistent estimator, the various combinations of the bandwidths in the numerator and denominator in the long-run variance ratio offer a systematic way to investigate the effect of new approach under both the null and alternative hypotheses.

The remainder of the paper is organized as follows: Section 2 introduces the long-run variance ratio test for a unit root and derives its limiting distribution under different assumptions on the growth rate of bandwidths. The finite sample size properties of each test is investigated by a Monte Carlo simulation in Section 3. The power of the test is considered in Section 4. Finally, some concluding remarks are made in Section 5. Throughout this paper, we use the symbols "$\Rightarrow$" and "$\xrightarrow{p}$" to signify weak convergence and convergence in probability, respectively. All the limits are taken as the sample size $T \to \infty$.

2 The test statistics

Let $\{y_t\}_{t=1}^{T}$ be a univariate time series generated by

$$y_t = \alpha y_{t-1} + u_t \tag{1}$$

where u_t is a weakly stationary zero-mean error with a strictly positive long-run variance defined by $\omega^2 \equiv \sum_{j=-\infty}^{\infty} \gamma_j$ where $\gamma_j = E(u_t u_{t-j})$. For simplicity, the initial condition is set to $y_0 = 0$. We consider a test for the null hypothesis of $\alpha = 1$ against the alternative hypothesis of $|\alpha| < 1$. Therefore, under the alternative hypothesis, y_t is the zero-mean stationary process with the long-run variance $\omega_y^2 = (1-\alpha)^{-2}\omega^2$.

Throughout this paper, the long-run variance of the zero-mean series x_t is estimated by a nonparametric kernel estimator with the Bartlett kernel,

$$\widehat{\omega}^2(x_t, K) = \sum_{j=-(K-1)}^{K-1} (1 - |j/K|)\, T^{-1} \sum_{t=|j|+1}^{T} x_t x_{t-|j|} \tag{2}$$

where K is the bandwidth/lag truncation parameter. As emphasized in Newey and West (1987), this choice of the kernel function ensures nonnegative estimates, and thus the long-run variance ratio test statistic defined below will always be nonnegative. In addition, this long-run variance estimator is known to be consistent when bandwidth K grows at a rate slower than $T^{1/2}$, with an optimal growth rate being $T^{1/3}$ under some moment conditions (Andrews, 1991). When x_t has a non-zero mean, $\widehat{\omega}^2(x_t - \overline{x}, K)$ where $\overline{x} = T^{-1}\sum_{t=1}^{T} x_t$ will provide the consistent estimator. This estimator, however, becomes inconsistent if K grows too fast, for example, at the rate T.

The VN ratio is usually defined as the ratio of the sample variances of the first differences and the levels of a time series. The ratio is often applied to regression residuals to conduct the Durbin-Watson test for serial correlation. As a test statistic for a unit root hypothesis, however, we utilize the following generalization of VN ratio,

$$R = MT\frac{\widehat{\omega}^2(\Delta y_t, K)}{\widehat{\omega}^2(y_t, M)} \tag{3}$$

where $\Delta y_t = y_t - y_{t-1}$ for $t = 1, ..., T$. This ratio replaces the sample variances of original VN ratio with the sample long-run variances. If u_t is iid, the ratio with the choice of $K = M = 1$ can be used to test the null hypothesis of a unit root. But for the serially correlated u_t, it does not provide the nuisance parameter free distribution under the null. We consider the following combinations of growth rates of K and M that provides asymptotically pivotal test statistics in the presence of serially correlated error, u_t.

C0: $K = kT^{1/3}$ for some $k > 0$ and $M = 1$.

CC: $K = kT^{1/3}$ and $M = mT^{1/3}$ for some $k, m > 0$.

CI: $K = kT^{1/3}$ for some $k > 0$ and $M = T$.

II: $K = T$ and $M = T$.

The choice of C0 is a combination of the bandwidth growth rates that ensures the numerator providing a consistent estimator of ω^2 under the null, and the denominator providing a consistent estimator of variance of y_t (or the autocovariance of order zero) under the alternative. With the choice of CC, the numerator provides a consistent estimator of ω^2 under the null, and the denominator provides a consistent estimator of ω_y^2 under the alternative. CI is the case of the denominator being an inconsistent estimator of ω_y^2 under the alternative, while the numerator is still the consistent estimator of ω^2 under the null. Finally, II is the combination in which both the numerator and denominator are inconsistent estimators under the null and alternative, respectively. Note that employing a rate other than $T^{1/3}$ is also possible in C0, CC, and CI, and theoretical results will not be affected as long as it provides a consistent estimator. The $T^{1/3}$ rate is employed here simply because it is the optimal rate and this particular rate will be used in the simulation in the next section.

When non-zero mean stationarity or trend stationarity is allowed as an alternative hypothesis, a demeaned and detrended version of the unit root test is often employed in practice. The long-run variance ratio test can be also extended to these more empirically relevant cases. Suppose $\overline{y} = T^{-1}\sum_{t=1}^{T} y_t$, $\overline{\Delta y} = T^{-1}\sum_{t=1}^{T}\Delta y_t$, $\widetilde{y}_t = \sum_{j=1}^{t}\left(\Delta y_j - \overline{\Delta y}\right)$ and $\overline{\widetilde{y}} = T^{-1}\sum_{t=1}^{T}\widetilde{y}_t$. The demeaned and detrended test statistics are given by

$$R_\mu = MT\frac{\widehat{\omega}^2(\Delta y_t, K)}{\widehat{\omega}^2(y_t - \overline{y}, M)} \tag{4}$$

and

$$R_\tau = MT\frac{\widehat{\omega}^2(\Delta y_t - \overline{\Delta y}, K)}{\widehat{\omega}^2(\widetilde{y}_t - \overline{\widetilde{y}}, M)}. \tag{5}$$

When $K = M = 1$, R_μ corresponds to the test of Sargan and Bhargava (1983) and R_τ corresponds to the R_2 test proposed by Bhargava (1986). Note that R_τ is based on a detrending procedure that is efficient under the null. Schmidt and Phillips (1992) also showed that, for a Gaussian likelihood, the LM principle leads to these tests. With the choice of C0, the test is equivalent to the nonparametric extension of the VN ratio test considered by Nabeya and Tanaka (1990) and Schmidt and Phillips (1992). It is also equivalent to the MSB test considered by Stock (1994, 1999) and Perron and Ng (1996). The one-dimensional case of the cointegration tests considered by Phillips and Ouliaris (1990), Shintani (2001), and Harris and Poskitt (2004) reduces to the same unit root test under C0. R and R_μ under CC are equivalent to $P^*(1,0)$ and $P_\mu^*(1,0)$ of Shintani (2001), respectively. For II, the long-run variance ratio test is closely related to the test considered by Breitung (2002). For example, for the demeaned case, Breitung's test is (the inverse of) the variance ratio $BR = T^2 \sum_{t=1}^T (y_t - \overline{y})^2 / \sum_{t=1}^T S_t^2$ where $S_t = \sum_{j=1}^t (y_j - \overline{y})$. Simple computation reveals that the relation between the two tests are given by

$$R_\mu = BR + \left\{ T^{-1} y_T^2 - 2T^{-1} y_T \sum_{j=1}^T y_j \right\} \Big/ 2T^{-4} \sum_{j=1}^T S_j^2. \tag{6}$$

We now introduce the following assumption on the error term.

Assumption 1. *(a)* $u_t = C(L)\varepsilon_t = \sum_{j=0}^\infty c_j \varepsilon_{t-j}$, $c_0 = 1$, $|C(1)| > \delta > 0$ *and* $\sum_{j=0}^\infty j|c_j| < B < \infty$ *where* δ *and* B *are some positive constant.*

(b) ε_t *is iid with zero mean, variance* σ^2*, and finite fourth cumulants, and* $\varepsilon_s = 0$ *for* $s \leq 0$.

Under Assumption 1, we have $\omega^2 = C(1)^2\sigma^2$ and $T^{-1/2} \sum_{t=1}^{[Ts]} u_t \Rightarrow \omega W(s)$ where $[Ts]$ signifies the integer part of Ts and $W(s)$ denotes a standard Brownian motion defined on $C[0,1]$. The limiting distribution of the long-run variance ratio test is given in the following theorem.

Theorem 1. *Suppose that* $\{y_t\}_{t=1}^T$ *is generated by* (1) *with* $\alpha = 1$ *and assumption 1 is satisfied. Then,*

(a) (Standard test)

$$
R \Rightarrow \begin{cases} \left\{\int_0^1 W(r)^2 dr\right\}^{-1} & \text{for C0 and CC,} \\ \left\{2\int_0^1 \overline{W}(r)^2 dr + \left(\int_0^1 W(r)dr\right)^2 - 2\left(\int_0^1 W(r)dr\right)\left(\int_0^1 \overline{W}(r)dr\right)\right\}^{-1} & \text{for CI,} \\ \left\{2\int_0^1 W(r)^2 dr + W(1)^2 - 2W(1)\int_0^1 W(r)dr\right\} & \\ \quad \times \left\{2\int_0^1 \overline{W}(r)^2 dr + \left(\int_0^1 W(r)dr\right)^2 - 2\left(\int_0^1 W(r)dr\right)\left(\int_0^1 \overline{W}(r)dr\right)\right\}^{-1} & \text{for II,} \end{cases}
$$

where $\overline{W}(r) = \int_0^r W(s)ds$*.*

(b) (Demeaned test)

$$
R_\mu \Rightarrow \begin{cases} \left\{\int_0^1 W_\mu(r)^2 dr\right\}^{-1} & \text{for C0 and CC,} \\ \left\{2\int_0^1 \overline{W}_\mu(r)^2 dr\right\}^{-1} & \text{for CI,} \\ \left\{2\int_0^1 W(r)^2 dr + W(1)^2 - 2W(1)\int_0^1 W(r)dr\right\} \Big/ 2\int_0^1 \overline{W}_\mu(r)^2 dr & \text{for II,} \end{cases}
$$

where $W_\mu(r) = W(r) - \int_0^1 W(s)ds$ *and* $\overline{W}_\mu(r) = \int_0^r W_\mu(s)ds$*.*

(c) (Detrended test)

$$
R_\tau \Rightarrow \begin{cases} \left\{\int_0^1 V_\mu(r)^2 dr\right\}^{-1} & \text{for C0 and CC,} \\ \left\{2\int_0^1 \overline{V}_\mu(r)^2 dr\right\}^{-1} & \text{for CI,} \\ \int_0^1 V_\mu(r)^2 dr \Big/ \int_0^1 \overline{V}_\mu(r)^2 dr & \text{for II,} \end{cases}
$$

where $V_\mu(r) = V(r) - \int_0^1 V(s)ds$*,* $V(r) = W(r) - rW(1)$ *and* $\overline{V}_\mu(r) = \int_0^r V_\mu(s)ds$*.*

The limiting distribution of each test statistic is a function of Brownian motion or Brownian bridge. Evidently, this contrasts to the autocorrelation robust test in regression where only the test with an inconsitent long-run variance estimator has a nonstandard limiting distribution. Critical values for the limiting distribution of the long-run variance ratio tests with all the combination of bandwidth growth rates are provided in Table 1. Numbers are obtained by simulation using an approximation of Brownian motion by partial sums of standard normal random variables with 10,000 steps and 10^7 iteration. In the following section, we evaluate the finite sample size property of each test using these asymptotic critical values. Note that the test rejects the null hypothesis for large values of the long-run variance ratio and the critical region is constructed accordingly. The consistency of the tests is also provided in the following theorem.

Theorem 2. *Suppose that $\{y_t\}_{t=1}^{T}$ is generated by (1) with $|\alpha| < 1$ and assumption 1 is satisfied. Then, for any bandwidth growth rate combinations C0, CC, CI, or II,*

$$P\left[R > c^*\right], P\left[R_\mu > c^*\right], P\left[R_\tau > c^*\right] \to 1$$

for any fixed constant c^*.

In practice, the OLS residuals from the regression model (1) are often used to estimate the long-run variance of u_t to ensure the consistency of the unit root test. Theorem 2, however, shows that the long-run variance estimator using the over-differenced series Δy_t under the fixed alternative, still provides the consistency of the long-run variance ratio tests. This result is based on the fact that the long-run variance estimator based on Δy_t converges to zero from a positive value at a sufficiently slow rate under the alternative. In the simulation below, we focus on the case with the long-run variance estimator using Δy_t rather than using the quasi-differenced series from the OLS residuals. Nevertheless, the residual-based long-run variance ratio test seems to be a reasonable alternative to our test.

3 Finite sample size of the tests

In this section, the finite sample size properties of each test introduced in the previous section are investigated by a Monte Carlo simulation. We follow previous experimental studies in the unit root testing literature and consider the autoregressive (AR) and moving-average (MA) models to introduce serial correlation in the error term. In particular, our data generating process is (1) with $\alpha = 1$ using the following three different error structures

$$u_t = \begin{cases} \varepsilon_t & \text{(iid error)} \\ \rho u_{t-1} + \varepsilon_t & \text{(AR(1) error)} \\ \varepsilon_t + \theta\varepsilon_{t-1} & \text{(MA(1) error)} \end{cases}$$

where ε_t is an iid standard normal random variable, $\rho = -0.8, -0.5, 0.5, 0.8$ and $\theta = -0.8, -0.5, 0.5, 0.8$. Initial values y_0, u_0 and ε_0 are set to 0. In all cases we use 10,000 replications. There is fairly general agreement that the data-based bandwidth selection method in the long-run variance estimation have very important effects on improving the finite sample performance of the semiparametric and nonparametric unit root tests (e.g., see Stock, 1994, Phillips and Xiao, 1998). For this reason, we use Andrews'(1991) automatic bandwidth selection procedure (designed for the Bartlett kernel) to select K when tests based on C0, CC and CI are applied to AR(1) and MA(1) errors. For the test with CC, the value of the automatic bandwidth selected for K is also used for M. For iid errors, we simply use $K = 1$.

Table 2 reports the rejection frequency of the standard long-run variance ratio test, R, with an asymptotic level of 5% for the sample of five different sizes, T =25, 50, 100, 200, and 500. For each pair of bandwidth growth rates, the first column shows the empirical size when the unit root process has an iid error. With an exception of a slight under-rejection for C0/CI case when $T = 25$, the empirical size of the long-run variance ratio tests is very close to the asymptotic level for all combinations of bandwidth growth rates. The difference among the various choices of bandwidth, however, becomes more evident when the error terms are serially correlated.

Consistent with the finding by Schwert (1989) for the semiparametric unit root tests, the long-run variance ratio tests suffer from size distortion mostly in the case of the near MA unit root ($\theta = -0.8$). Over-rejection is observed for all tests, which implies that the tests are too liberal. However, when inconsistent asymptotics are used for both the numerator and the denominator (II), the size distortion becomes smaller and the empirical size approaches the asymptotic level as sample size increases. In contrast, the size distortion of other tests for the near MA unit root case do not disappear even for $T = 500$. The size distortion appears to be largest when the test is based on CC. The problem seems to be less severe when the combination of the consistent and inconsistent estimators (CI) is employed in comparison with the conventional case (C0 and CC) when sample size increases. For the positively correlated MA error ($\theta = 0.5, 0.8$), the tests based on CC, CI, and II have their empirical size quite close to the nominal size. In this case, only the test with C0 has a noticeable size distortion that results in the conservative test.

On the whole, the size distortion seems to be somewhat less severe for the AR errors compared to the MA errors. The empirical size of the test with C0 is smaller than the nominal size for the entire range of AR parameters, which suggest that the test is too conservative. The largest deviation from the nominal size is observed in the test with CC when AR errors are positively correlated ($\rho = 0.5, 0.8$). In contrast to the C0 case that under-rejects for all the cases, the test with CC over-rejects when AR parameters are positive ($\rho = 0.5, 0.8$), but under-rejects when AR parameters are negative ($\rho = -0.8, -0.5$). As in the MA error results, the AR error results again favor the tests that involves inconsistent estimators, namely, CI and II cases. When the sample size increases, both tests have a size that is very close to asymptotic level for all different values of AR parameters.

Tables 3 and 4 report the same results for R_μ and R_τ, respectively. For the iid error and positively correlated error, the size performance of the demeaned and detrended tests is very similar to that of the standard case except for a very small sample ($T = 25$). For the negatively correlated case, the problem of size distortion becomes more severe in general. In particular, rejection frequency increases substantially with CC for the negatively correlated MA error. When II is used, however, stability of size remains for $\theta = -0.5$, and increases in the rejection frequency seems to be very modest, even for $\theta = -0.8$, compared to the other choice of bandwidths.

In summary, consistent with the previous findings in the literature, the test based on a consistent nonparametric long-run variance estimator suffers from substantial size distortion when the errors are negatively correlated. In contrast, the empirical size of the test using a pair of inconsistent estimators seems to be very close to nominal size on the whole regardless of the serial correlation structure. Therefore, in terms of the stability and accuracy of size, this choice of bandwidth growth seems to be most effective one, with the combination of consistent and inconsistent estimators the second best.

4 Power of the tests

In the previous section, we found that it was possible to improve the size of the long-run variance ratio test for a unit root by introducing inconsistent long-run variance estimators. This section investigates the power properties of the same tests.

First, we consider the limiting distribution under the local alternative $\alpha = 1 + T^{-1}c$ for a particular value of $c < 0$. As in the case of other unit root tests, the limiting distribution involves the functional of a diffusion $J_c(r) \equiv \int_0^r \exp\{(r-s)c\}dW(s)$. Under this local alternative, all the asymptotic results in Theorem 1 hold by replacing $W(r)$ with $J_c(r)$. This can be shown by using the argument similar to Stock's

(1999) in his analysis of the local asymptotic power of the MSB test. The local asymptotic power functions of R, R_μ, and R_τ for various bandwidth growth rates based on the 5 percent level are plotted in Figures 1, 2 and 3, respectively. They are approximated by discrete Gaussian random walks with 500 steps with 10,000 replication. For the standard test (R) in Figure 1, C0/CC and CI cases have similar power when c is close to zero. The difference between their power and that of the II case is evident even if c is close to zero. While the power of the CI case becomes slightly below the C0/CC power function for the moderate value of c, both power functions become 1.00 relatively fast for distant alternatives. In contrast, the power for the II case is considerably lower for entire alternatives. It becomes only about 0.5 even if the c is as small as -24. The local asymptotic power functions of the demeaned and detrended tests (R_μ and R_τ in Figures 2 and 3) show the reduction of the local power by detrending the data compared to their corresponding power for the standard test. However, in terms of the ranking and pattern among the different choices of bandwidth, they are very similar to those of the standard tests. In summary, the asymptotic power function of the tests based on the pair of inconsistent estimators (II) is well below other power functions for the entire range of c for all cases.

Second, we investigate the finite sample power properties using the simulation design similar to the one used in the previous section. We first obtain small sample size-adjusted critical values based on the results in Tables 2 to 4. Note that the size-adjusted critical values are computed for all combinations of data generating process and sample size. We then generate the data from (1) with $\alpha = 0.9$ using the same values of the error term used for $\alpha = 1$ case and apply the unit root test to the data. The frequencies of the rejection of the null hypothesis using the size-adjusted critical value are reported in Tables 5 to 7. The three main findings from the tables are as follows. First, in agreement with the local asymptotic power result, the finite sample size-adjusted power of the test with II is much lower than that of the tests with C0, CC, and CI. For example, when the error is iid, the power of standard, demeaned and detrended tests is only 0.76, 0.53 and 0.25, respectively, even for the large sample with $T = 500$. For the same sample size, the power of other tests are 1.00 or at least close to 1.00. Second, the difference among the size-adjusted power of tests with C0, CC, and CI is modest compared to the much lower power of II case. Among the group of C0, CC, and CI, the test with CC performs somewhat better than the other two when the error is negatively correlated, namely, AR(1) error with $\rho = -0.8, -0.5$ and MA(1) errors with $\theta = -0.8, -0.5$. Third, with the exception of the detrended test with a large sample ($T = 250, 500$), the test based on CI has a good finite sample power property comparable to C0 and CC cases. This fact is interesting given the finding of the previous section that the test with CI has a much better size property than tests based on C0 and CC.

The summary of this section follows. Both local asymptotic power and size-adjusted finite sample power of the long-run variance ratio tests based on the pair of inconsistent estimators (II) are found to be dramatically lower than those of the test based on the pair of consistent estimators (C0 and CC). This suggests that the stability of the size in the II case seems to be too costly to justify the usefulness of the inconsistent long-run variance estimator in the long-run variance ratio test for a unit root. However, at the same time, the tests based on the combination of consistent and inconsistent long-run variance estimators (CI) are found to have reasonable power as well as a good size. Therefore, among the various pairs of bandwidth growth rates in the long-run variance ratio, the choice of CI may have some practical use in testing for a unit root.

5 Conclusion

In this paper, we investigated the properties of the long-run variance ratio tests for a unit root, a generalization of a test based on the von Neumann ratio. Our main interest was in evaluating the effect of introducing the inconsistent long-run variance estimation on the size and power of the unit root tests.

Based on the results of the Monte Carlo simulation designed to evaluate the finite sample property, the unit root tests with an inconsistent long-run variance estimator were found to have much less size distortion compared to the tests with conventional asymptotics that provide a consistent long-run variance estimator. This finite sample size improvement, however, came at the cost of a loss in power. The finite sample power, as well as the local asymptotic power, of the tests with an inconsistent long-run variance estimator was shown to be much lower than that of conventional tests. This finding resembles the case of the autocorrelation robust test in the standard regression context, where the test with better size property proposed by Kiefer, Vogelsang, and Bunzel (2000) has lower power compared to the test based on the conventional HAC asymptotics. In the autocorrelation robust inference literature, some efforts have recently been made to improve the power while maintaining the good size property of the inconsistent asymptotic-based test (e.g., Jansson, 2004). For the long-run variance ratio test for a unit root, we have shown that one possibility to balance between the power and size is to employ on the combination of consistent and inconsistent estimators. Alternatively, while not pursued in this paper, (i) the introduction of GLS detrending (Elliott, Rothenberg, and Stock, 1996), and (ii) the use of an autoregressive spectral density estimator instead of a kernel-based estimator (Berk, 1974), seems to be a promising direction in which to extend the analysis of size and power of the long-run variance ratio test with (or without) an inconsistent long-run variance estimator.

References

Andrews, D.W.K. (1991) Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica* 59, 817-858.

Berk, K.N. (1974) Consistent autoregressive spectral estimates, *Annals of Statistics* 2, 489-502.

Bhargava, A. (1986) On the theory of testing for unit roots in observed time series. *Review of Economic Studies* 53 3, 369-384.

Breitung, J. (2002) Nonparametric tests for unit roots and cointegration. *Journal of Econometrics* 108, 343-363.

Elliott, G., T. Rothenberg, & J.H. Stock (1996) Efficient tests for an autoregressive unit root. *Econometrica* 64, 813-836.

Harris, D. & D.S. Poskitt (2004) Determination of cointegrating rank in partially nonstationary processes via a generalised von-Neumann criterion. mimeo.

Jansson, M. (2004) The error in rejection probability of simple autocorrelation robust tests. *Econometrica* 72, 937-946.

Kiefer, N.M. & T.J. Vogelsang (2002) Heteroskedasticity-autocorrelation robust standard errors using the Bartlett kernel without truncation. *Econometrica* 70, 2093-2095.

Kiefer, N.M., T.J. Vogelsang, & H. Bunzel (2000) Simple robust testing of regression hypotheses. *Econometrica* 68, 695-714.

Nabeya, S. & K. Tanaka (1990) Limiting power of unit-root tests in time-series regression. *Journal of Econometrics* 46, 247-271.

Newey, W.K. & K.D. West (1987) A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55, 703-708.

Park, J.Y. & P.C.B. Phillips (1988) Statistical inference in regressions with integrated processes: Part 1. *Econometric Theory* 4, 468-497.

Park, J.Y. & P.C.B. Phillips (1989) Statistical inference in regressions with integrated processes: Part 2. *Econometric Theory* 5, 95-131.

Perron, P. & S. Ng (1996) Useful modifications to unit root tests with dependent errors and their local asymptotic properties. *Review of Economic Studies* 63, 435-464.

Phillips, P.C.B. (1987) Time series regression with a unit root. *Econometrica* 55, 277-301.

Phillips, P.C.B. & S. Ouliaris (1990) Asymptotic properties of residual based tests for cointegration. *Econometrica* 58, 165-193.

Phillips, P.C.B. & P. Perron (1988) Testing for a unit root in time series regression. *Biometrika* 75, 335-346.

Phillips, P.C.B. & Z. Xiao (1998) A primer on unit root testing. *Journal of Economic Surveys* 12, 423-469.

Sargan, J.D. & A. Bhargava (1983) Testing residuals from least squares regression for being generated by the Gaussian random walk. *Econometrica* 51, 153-174.

Schmidt, P. & P.C.B. Phillips (1992) LM tests for a unit root in the presence of deterministic trends. *Oxford Bulletin of Economics and Statistics* 54, 257-287.

Schwert, G.W. (1989) Tests for unit roots: A Monte Carlo investigation. *Journal of Business and Economic Statistics* 7, 147-159.

Shintani, M. (2001) A simple cointergrating rank test without vector autoregression. *Journal of Econometrics* 105, 337-362.

Stock, J.H. (1994) Unit roots, structural breaks and trends. In R.F. Engle, & D. McFadden (eds.), *Handbook of Econometrics*, Vol. 4., pp.2740-2841. Amsterdam, North-Holland.

Stock, J.H. (1999) A class of tests for integration and cointegration. In R.F. Engle and H. White (eds.), *Cointegration, Causality and Forecasting: A Festschrift for Clive W.J. Granger*, 135-167. Oxford: Oxford University Press.

Appendix : Proofs

Proof of Theorem 1. In the proof, the limits on integrals over the unit interval is omitted. For example, $\int_0^1 W(r)^2 dr$ is written as $\int W^2$. For part (a), we first investigate the asymptotic properties of the numerator of the long-run variance ratio. Under assumption 1, we have $\widehat{\omega}^2(\triangle y_t, kT^{1/3}) \xrightarrow{p} \omega^2$. To consider the inconsistent case, first note that

$$\widehat{\omega}^2(x_t, T) = 2T^{-2}\sum_{j=1}^{T} S_j^2 + T^{-1}S_T^2 - 2T^{-1}S_T\sum_{j=1}^{T} S_j \tag{A.1}$$

where $S_t = \sum_{j=1}^{t} x_j$. This generalizes equation (1) of Kiefer and Vogelsang (2002) to the case when $S_T \neq 0$. Applying (A.1) to $x_t = \Delta y_t$ and $S_t = \sum_{j=1}^{t} \Delta y_j = y_t$ yields

$$\begin{aligned}
\widehat{\omega}^2(\Delta y_t, T) &= 2T^{-2}\sum_{j=1}^{T} y_j^2 + T^{-1}y_T^2 - 2T^{-1}y_T\sum_{j=1}^{T} y_j \\
&\Rightarrow \omega^2\left\{2\int W^2 + W(1)^2 - 2W(1)\int W\right\}
\end{aligned}$$

where the joint weak convergence results $T^{-2}\sum_{j=1}^{T} y_j^2 \Rightarrow \omega^2 \int W^2$, $y_T/\sqrt{T} \Rightarrow \omega W(1)$ and $T^{-3/2}\sum_{j=1}^{T} y_j \Rightarrow \omega \int W$ follow from Lemma 2.1 of Park and Phillips (1988). For the asymptotic properties of the denominator, first note that $T^{-1}\widehat{\omega}^2(y_t, 1) = T^{-2}\sum_{j=1}^{T} y_j^2 \Rightarrow \omega^2 \int_0^1 W^2$. For $\widehat{\omega}^2(y_t, M)$ with $M/T \to 0$, we have

$$\begin{aligned}
(MT)^{-1}\widehat{\omega}^2(y_t, M) &= M^{-1}\sum_{j=-(M-1)}^{M-1} (1-|j/M|)\,T^{-2}\sum_{t=|j|+1}^{T} y_t y_{t-|j|} \\
&= M^{-1}\sum_{j=-(M-1)}^{M-1} (1-|j/M|)\,T^{-2}\sum_{t=1}^{T} y_t^2 + O_p(M/T) \\
&\Rightarrow \left(\int_{-1}^{1}(1-|x|)dx\right)\omega^2\int_0^1 W^2 = \omega^2\int_0^1 W^2
\end{aligned}$$

where the second equality follows from

$$T^{-2}\sum_{t=j+1}^{T} y_t y_{t-j} = T^{-2}\sum_{t=1}^{T-j} y_t y_{t+j} = T^{-2}\sum_{t=1}^{T-j} y_t^2 + T^{-2}\sum_{t=1}^{T-j} y_t\sum_{s=1}^{j} u_{t+s} = T^{-2}\sum_{t=1}^{T-j} y_t^2 + O_p(M/T)$$

for any $j = 1, ..., M-1$, since $T^{-1}\sum_{t=1}^{T-j} y_t u_{t+j} \Rightarrow (\omega^2/2)\left\{W(1)^2 - 1\right\} - \sum_{s=1}^{j}\gamma_s$ for any $j = 1, ..., M-1$. Finally, for $\widehat{\omega}^2(y_t, T)$, applying (A.1) to $x_t = y_t$ and $S_t = \sum_{j=1}^{t} y_j$ yields

$$\begin{aligned}
T^{-2}\widehat{\omega}^2(y_t, T) &= 2T^{-4}\sum_{j=1}^{T} S_j^2 + T^{-3}S_T^2 - 2T^{-3}S_T\sum_{j=1}^{T} S_j \\
&\Rightarrow \omega^2\left\{2\int \overline{W}^2 + \left(\int W\right)^2 - 2\left(\int W\right)\left(\int \overline{W}\right)\right\}
\end{aligned}$$

where the joint weak convergence results for the I(2) process $T^{-4}\sum_{j=1}^{T} S_j^2 \Rightarrow \omega^2 \int \overline{W}^2$ and $T^{-5/2}\sum_{j=1}^{T} S_j \Rightarrow \omega \int \overline{W}$ are from Lemma 2.1 of Park and Phillips (1989) and

$T^{-3/2}S_T = T^{-3/2}\sum_{j=1}^{T} y_j \Rightarrow \omega \int W$. The required results for part (a) can be obtained by combining the appropriate results under C0, CC, CI, and II, because the convergence of the numerator and the denominator holds jointly and the nuisance parameter ω^2 cancels out for any combination.

For part (b), the numerator is identical as part (a). For the denominator with $M = T$, the formula (A.1) simplifies to $2T^{-2}\sum_{j=1}^{T} S_j^2$ since $S_T = 0$ for $x_t = y_t - \overline{y}$. The rest of the proof is similar to part (a) with the standard Brownian motion replaced by the demeaned Brownian motion. The proof of (c) is entirely analogous to that of parts (a) and (b) except for the use of the demeaned Brownian bridge. ■

Proof of Theorem 2. We show only the consistency of R test since the results for R_μ and R_τ can obtained using a similar argument. Under the fixed alternative, the limiting behavior of the numerator for the over-differenced series is given by

$$\begin{aligned} K\widehat{\omega}^2(\Delta y_t, K) &= \sum_{j=-(K-1)}^{K-1} (K-|j|)\, T^{-1} \sum_{t=|j|+1}^{T} (y_t - y_{t-1})(y_{t-|j|} - y_{t-|j|-1}) \\ &= 2T^{-1}\sum_{j=1}^{T} y_j^2 + (K/T)y_T^2 - 2y_T\overline{y} \\ &\xrightarrow{p} \begin{cases} 2\sigma_y^2 > 0 & \text{for } K/T \to 0 \\ 2\sigma_y^2 + y_\infty^2 > 0 & \text{for } K = T \end{cases} \end{aligned}$$

where $\sigma_y^2 = \sum_{i=0}^{\infty}\left(\sum_{j=0}^{i} \alpha^{i-j} c_j\right)^2 \sigma^2$. For the denominator, since y_t is stationary, we have $\widehat{\omega}^2(y_t, 1) \xrightarrow{p} \sigma_y^2$ and $\widehat{\omega}^2(y_t, mT^{1/3}) \xrightarrow{p} \omega_y^2$. The result in the proof of Theorem 1 can be also used to obtain

$$\widehat{\omega}^2(y_t, T) \Rightarrow \omega_y^2 \left\{ 2\int W^2 + W(1)^2 - 2W(1)\int W \right\}.$$

Combining the all the results yields

$$T^{-2/3}R \xrightarrow{p} 2k^{-1} > 0 \quad \text{for C0,}$$

$$T^{-1}R \xrightarrow{p} 2mk^{-1}\sigma_y^2\omega_y^{-2} > 0 \quad \text{for CC,}$$

$$T^{-5/3}R \Rightarrow 2k^{-1}\sigma_y^2\omega_y^{-2}\left\{ 2\int W^2 + W(1)^2 - 2W(1)\int W \right\}^{-1} > 0 \quad \text{for CI and}$$

$$T^{-1}R \Rightarrow \left(2\sigma_y^2 + y_\infty^2\right)\left\{ 2\int W^2 + W(1)^2 - 2W(1)\int W \right\}^{-1} > 0 \quad \text{for II.}$$

Therefore, all the test statistics diverge in the positive direction as required. ■

Table 1. Critical values

Test	Bandwidth	Level 10%	5%	1%
Standard	C0/CC	13.1	17.8	29.1
	CI	88.1	174	586
	II	31.7	52.7	136
Demeaned	C0/CC	21.8	27.5	40.5
	CI	643	1.10×10^3	2.79×10^3
	II	213	317	657
Detrended	C0/CC	30.3	36.6	51.0
	CI	1.34×10^3	2.10×10^3	4.76×10^3
	II	237	339	680

Note: Results are based on discrete approximation to the Brownian motion by partial sums of standard normal random variable with 10,000 steps and 10^7 replications.

Table 2. Empirical size of the standard test with 5% level

	Bandwidth	iid error	AR(1) error, $\rho =$ -0.8	-0.5	0.5	0.8	MA(1) error, $\theta =$ -0.8	-0.5	0.5	0.8
$T = 25$	C0	0.03	0.00	0.01	0.00	0.00	0.09	0.05	0.01	0.01
	CC	0.03	0.19	0.11	0.02	0.04	0.46	0.21	0.04	0.04
	CI	0.04	0.04	0.05	0.02	0.01	0.18	0.08	0.03	0.03
	II	0.04	0.08	0.05	0.03	0.03	0.14	0.07	0.04	0.04
$T = 50$	C0	0.04	0.00	0.02	0.01	0.00	0.17	0.08	0.02	0.02
	CC	0.04	0.14	0.10	0.03	0.02	0.51	0.20	0.05	0.05
	CI	0.05	0.05	0.05	0.03	0.01	0.21	0.09	0.04	0.04
	II	0.05	0.07	0.05	0.04	0.04	0.13	0.06	0.04	0.04
$T = 100$	C0	0.05	0.01	0.04	0.02	0.00	0.32	0.09	0.03	0.02
	CC	0.05	0.12	0.09	0.03	0.02	0.52	0.17	0.05	0.04
	CI	0.05	0.05	0.05	0.03	0.02	0.21	0.08	0.04	0.04
	II	0.05	0.06	0.05	0.05	0.04	0.10	0.06	0.05	0.05
$T = 250$	C0	0.04	0.03	0.05	0.02	0.01	0.39	0.10	0.03	0.03
	CC	0.04	0.09	0.07	0.03	0.02	0.49	0.13	0.04	0.04
	CI	0.05	0.05	0.05	0.04	0.03	0.21	0.08	0.04	0.04
	II	0.05	0.05	0.05	0.05	0.05	0.08	0.05	0.05	0.05
$T = 500$	C0	0.05	0.04	0.05	0.03	0.02	0.40	0.10	0.04	0.04
	CC	0.05	0.08	0.07	0.04	0.03	0.45	0.12	0.05	0.05
	CI	0.05	0.05	0.05	0.04	0.04	0.19	0.07	0.04	0.04
	II	0.05	0.05	0.05	0.05	0.05	0.07	0.05	0.05	0.05

Note: Empirical rejection rate of 5% level tests based on asymptotic critical values when data are generated by (1) with $\alpha = 1$. The data-based bandwidth selection method of Andrews (1991) is applied to the first differenced series for C0, CC and CI. Results are based on 10,000 replications.

Table 3. Empirical size of the demeaned test with 5% level

	Bandwidth	iid error	AR(1) error, $\rho =$				MA(1) error, $\theta =$			
			-0.8	-0.5	0.5	0.8	-0.8	-0.5	0.5	0.8
$T = 25$	C0	0.03	0.00	0.01	0.00	0.00	0.09	0.04	0.01	0.00
	CC	0.03	0.46	0.23	0.05	0.13	0.83	0.37	0.06	0.08
	CI	0.04	0.04	0.05	0.01	0.00	0.34	0.11	0.02	0.02
	II	0.05	0.12	0.07	0.03	0.02	0.30	0.10	0.04	0.04
$T = 50$	C0	0.04	0.00	0.01	0.00	0.00	0.12	0.06	0.01	0.01
	CC	0.04	0.32	0.17	0.04	0.06	0.86	0.33	0.06	0.07
	CI	0.05	0.05	0.05	0.02	0.00	0.39	0.11	0.03	0.03
	II	0.05	0.10	0.07	0.04	0.02	0.29	0.09	0.05	0.05
$T = 100$	C0	0.04	0.00	0.02	0.01	0.00	0.33	0.08	0.02	0.01
	CC	0.04	0.22	0.13	0.03	0.04	0.84	0.27	0.05	0.06
	CI	0.05	0.05	0.06	0.03	0.01	0.37	0.10	0.04	0.03
	II	0.05	0.07	0.06	0.04	0.03	0.22	0.07	0.05	0.05
$T = 250$	C0	0.05	0.01	0.04	0.02	0.00	0.55	0.11	0.03	0.02
	CC	0.05	0.14	0.09	0.03	0.03	0.76	0.19	0.05	0.05
	CI	0.05	0.05	0.05	0.03	0.02	0.31	0.08	0.04	0.04
	II	0.04	0.05	0.05	0.04	0.04	0.13	0.06	0.04	0.04
$T = 500$	C0	0.05	0.03	0.05	0.02	0.01	0.56	0.11	0.03	0.03
	CC	0.05	0.11	0.08	0.04	0.03	0.69	0.16	0.05	0.05
	CI	0.05	0.06	0.06	0.04	0.03	0.28	0.08	0.04	0.04
	II	0.05	0.05	0.05	0.05	0.04	0.09	0.05	0.05	0.05

Note: See the note to Table 2.

Table 4. Empirical size of the detrended test with 5% level

	Bandwidth	iid error	AR(1) error, $\rho =$ -0.8	-0.5	0.5	0.8	MA(1) error, $\theta =$ -0.8	-0.5	0.5	0.8
$T = 25$	C0	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00
	CC	0.02	0.49	0.23	0.02	0.07	0.65	0.34	0.03	0.05
	CI	0.04	0.01	0.02	0.00	0.00	0.14	0.06	0.01	0.01
	II	0.05	0.13	0.07	0.03	0.02	0.18	0.09	0.04	0.04
$T = 50$	C0	0.03	0.00	0.00	0.00	0.00	0.03	0.03	0.01	0.00
	CC	0.03	0.35	0.18	0.02	0.03	0.79	0.36	0.05	0.05
	CI	0.05	0.03	0.04	0.01	0.00	0.24	0.10	0.02	0.02
	II	0.05	0.10	0.06	0.04	0.03	0.21	0.09	0.04	0.04
$T = 100$	C0	0.04	0.00	0.00	0.00	0.00	0.08	0.04	0.01	0.00
	CC	0.04	0.25	0.14	0.02	0.02	0.86	0.30	0.04	0.04
	CI	0.05	0.03	0.05	0.02	0.01	0.30	0.09	0.03	0.03
	II	0.05	0.07	0.06	0.04	0.03	0.20	0.07	0.05	0.05
$T = 250$	C0	0.05	0.00	0.03	0.01	0.00	0.49	0.09	0.02	0.02
	CC	0.05	0.16	0.10	0.03	0.02	0.86	0.24	0.05	0.05
	CI	0.05	0.05	0.05	0.03	0.01	0.32	0.09	0.04	0.04
	II	0.05	0.06	0.05	0.05	0.04	0.14	0.06	0.05	0.05
$T = 500$	C0	0.05	0.01	0.04	0.02	0.00	0.62	0.10	0.03	0.02
	CC	0.05	0.12	0.09	0.03	0.03	0.82	0.19	0.05	0.05
	CI	0.05	0.05	0.05	0.03	0.02	0.30	0.08	0.04	0.04
	II	0.05	0.06	0.05	0.05	0.04	0.10	0.05	0.05	0.05

Note: See the note to Table 2.

Table 5. Size-adjusted power of the standard test with 5% level

	Bandwidth	iid error	AR(1) error, $\rho =$ -0.8	-0.5	0.5	0.8	MA(1) error, $\theta =$ -0.8	-0.5	0.5	0.8
$T = 25$	C0	0.15	0.11	0.13	0.14	0.10	0.12	0.13	0.14	0.14
	CC	0.15	0.13	0.15	0.12	0.05	0.14	0.14	0.13	0.13
	CI	0.13	0.13	0.13	0.12	0.09	0.14	0.13	0.14	0.13
	II	0.10	0.11	0.11	0.09	0.07	0.15	0.11	0.09	0.10
$T = 50$	C0	0.30	0.21	0.27	0.26	0.19	0.18	0.25	0.28	0.28
	CC	0.30	0.27	0.29	0.25	0.15	0.31	0.29	0.27	0.27
	CI	0.27	0.24	0.25	0.23	0.18	0.30	0.26	0.25	0.25
	II	0.15	0.20	0.17	0.13	0.09	0.27	0.19	0.14	0.14
$T = 100$	C0	0.75	0.45	0.65	0.63	0.47	0.46	0.65	0.68	0.66
	CC	0.75	0.65	0.68	0.61	0.43	0.76	0.70	0.67	0.65
	CI	0.58	0.49	0.52	0.48	0.38	0.65	0.57	0.51	0.51
	II	0.26	0.34	0.28	0.24	0.18	0.55	0.33	0.26	0.25
$T = 250$	C0	1.00	0.97	1.00	1.00	0.97	0.98	1.00	1.00	1.00
	CC	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00
	CI	0.98	0.92	0.95	0.94	0.84	0.99	0.97	0.96	0.95
	II	0.54	0.67	0.59	0.49	0.40	0.91	0.66	0.51	0.52
$T = 500$	C0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	CC	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	CI	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00
	II	0.76	0.86	0.80	0.72	0.63	0.99	0.85	0.75	0.75

Note: Empirical rejection rate of 5% level tests based on size-adjusted critical values when data are generated by (1) with $\alpha = 0.9$. The data-based bandwidth selection method of Andrews (1991) is applied to the first differenced series for C0, CC and CI. Results are based on 10,000 replications.

Table 6. Size-adjusted power of the demeaned test with 5% level

	Bandwidth	iid error	AR(1) error, $\rho =$ -0.8	-0.5	0.5	0.8	MA(1) error, $\theta =$ -0.8	-0.5	0.5	0.8
$T = 25$	C0	0.12	0.05	0.07	0.09	0.09	0.04	0.07	0.10	0.10
	CC	0.12	0.08	0.09	0.06	0.03	0.05	0.08	0.08	0.07
	CI	0.11	0.08	0.09	0.10	0.10	0.05	0.09	0.10	0.10
	II	0.06	0.07	0.07	0.06	0.08	0.02	0.06	0.06	0.06
$T = 50$	C0	0.22	0.08	0.14	0.18	0.15	0.05	0.12	0.20	0.19
	CC	0.22	0.14	0.17	0.15	0.07	0.09	0.15	0.19	0.17
	CI	0.19	0.15	0.17	0.17	0.15	0.11	0.15	0.17	0.17
	II	0.07	0.09	0.08	0.07	0.08	0.05	0.08	0.07	0.07
$T = 100$	C0	0.52	0.17	0.37	0.41	0.31	0.15	0.35	0.46	0.44
	CC	0.52	0.38	0.44	0.38	0.21	0.32	0.45	0.44	0.41
	CI	0.41	0.33	0.36	0.35	0.27	0.30	0.37	0.37	0.37
	II	0.11	0.17	0.12	0.10	0.09	0.17	0.14	0.10	0.10
$T = 250$	C0	1.00	0.72	0.97	0.97	0.87	0.72	0.98	0.98	0.98
	CC	1.00	0.96	0.98	0.97	0.82	0.98	0.99	0.98	0.98
	CI	0.92	0.79	0.85	0.82	0.68	0.88	0.88	0.86	0.84
	II	0.30	0.43	0.34	0.26	0.20	0.62	0.40	0.28	0.28
$T = 500$	C0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	CC	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	CI	1.00	0.98	0.99	0.99	0.95	1.00	1.00	1.00	0.99
	II	0.53	0.68	0.58	0.48	0.39	0.91	0.66	0.51	0.51

Note: See the note to Table 5.

Table 7. Size-adjusted power of the detrended test with 5% level

		iid	AR(1) error, $\rho =$				MA(1) error, $\theta =$			
	Bandwidth	error	-0.8	-0.5	0.5	0.8	-0.8	-0.5	0.5	0.8
$T = 25$	C0	0.07	0.05	0.06	0.06	0.06	0.05	0.06	0.06	0.06
	CC	0.07	0.06	0.06	0.05	0.03	0.06	0.06	0.06	0.05
	CI	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.07
	II	0.05	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.05
$T = 50$	C0	0.11	0.06	0.08	0.10	0.08	0.05	0.07	0.10	0.10
	CC	0.11	0.08	0.10	0.09	0.06	0.08	0.10	0.10	0.09
	CI	0.10	0.08	0.09	0.09	0.08	0.08	0.09	0.10	0.09
	II	0.05	0.07	0.06	0.05	0.05	0.05	0.06	0.05	0.05
$T = 100$	C0	0.27	0.09	0.18	0.23	0.17	0.09	0.17	0.24	0.24
	CC	0.27	0.20	0.23	0.20	0.12	0.18	0.23	0.23	0.22
	CI	0.21	0.16	0.17	0.17	0.14	0.14	0.19	0.18	0.18
	II	0.06	0.09	0.07	0.05	0.05	0.08	0.08	0.06	0.06
$T = 250$	C0	0.91	0.37	0.75	0.79	0.63	0.40	0.76	0.84	0.81
	CC	0.91	0.73	0.82	0.78	0.56	0.71	0.83	0.83	0.80
	CI	0.58	0.41	0.49	0.50	0.39	0.36	0.50	0.52	0.51
	II	0.13	0.19	0.15	0.10	0.08	0.22	0.18	0.12	0.12
$T = 500$	C0	1.00	0.88	1.00	1.00	0.97	0.91	1.00	1.00	1.00
	CC	1.00	0.98	1.00	1.00	0.97	0.96	1.00	1.00	1.00
	CI	0.86	0.62	0.74	0.78	0.67	0.55	0.74	0.79	0.77
	II	0.25	0.30	0.27	0.23	0.18	0.34	0.29	0.24	0.25

Note: See the note to Table 5.

Figure 1
Asymptotic power functions: standard test

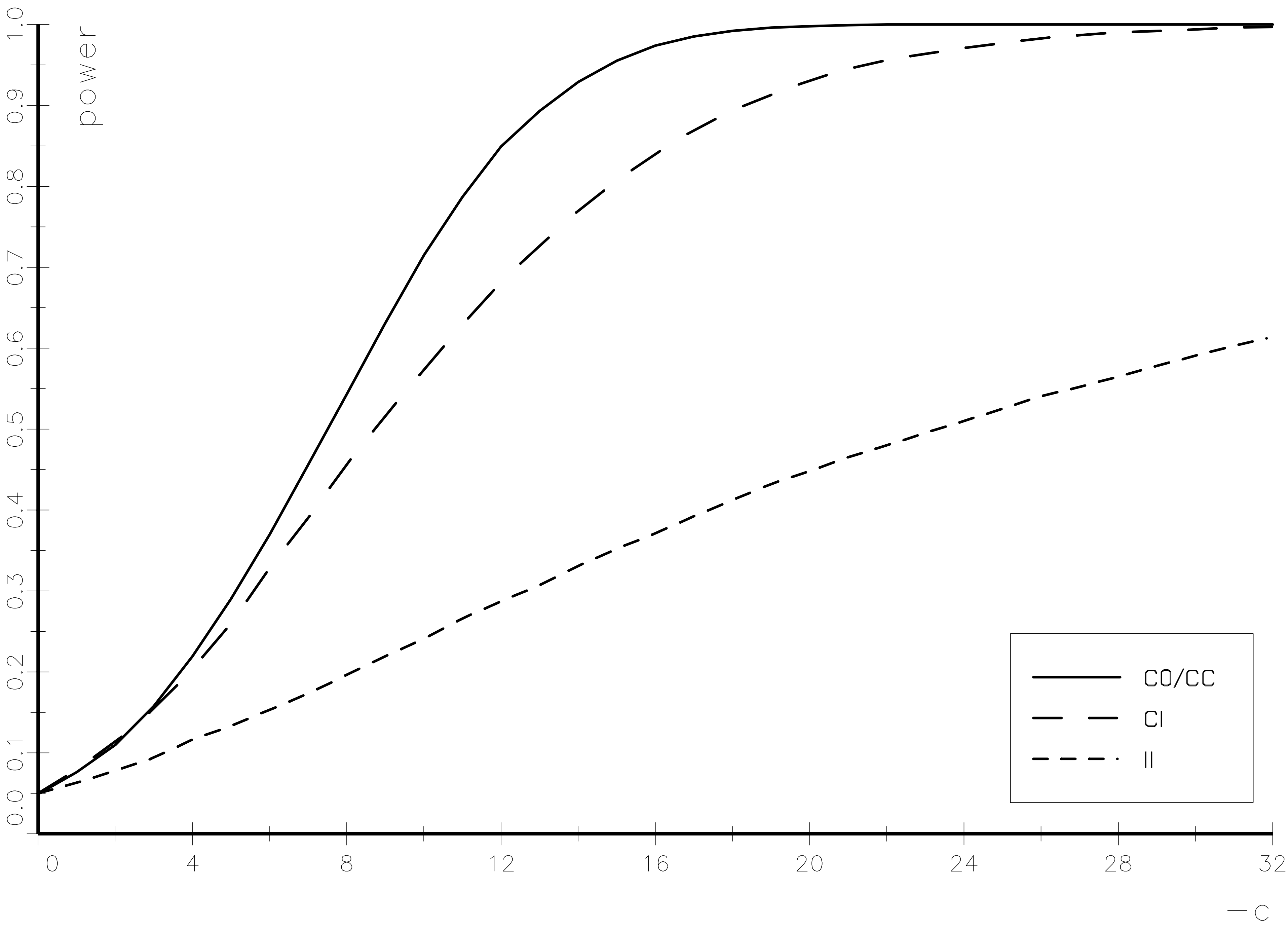

Figure 2

Asymptotic power functions: demeaned test

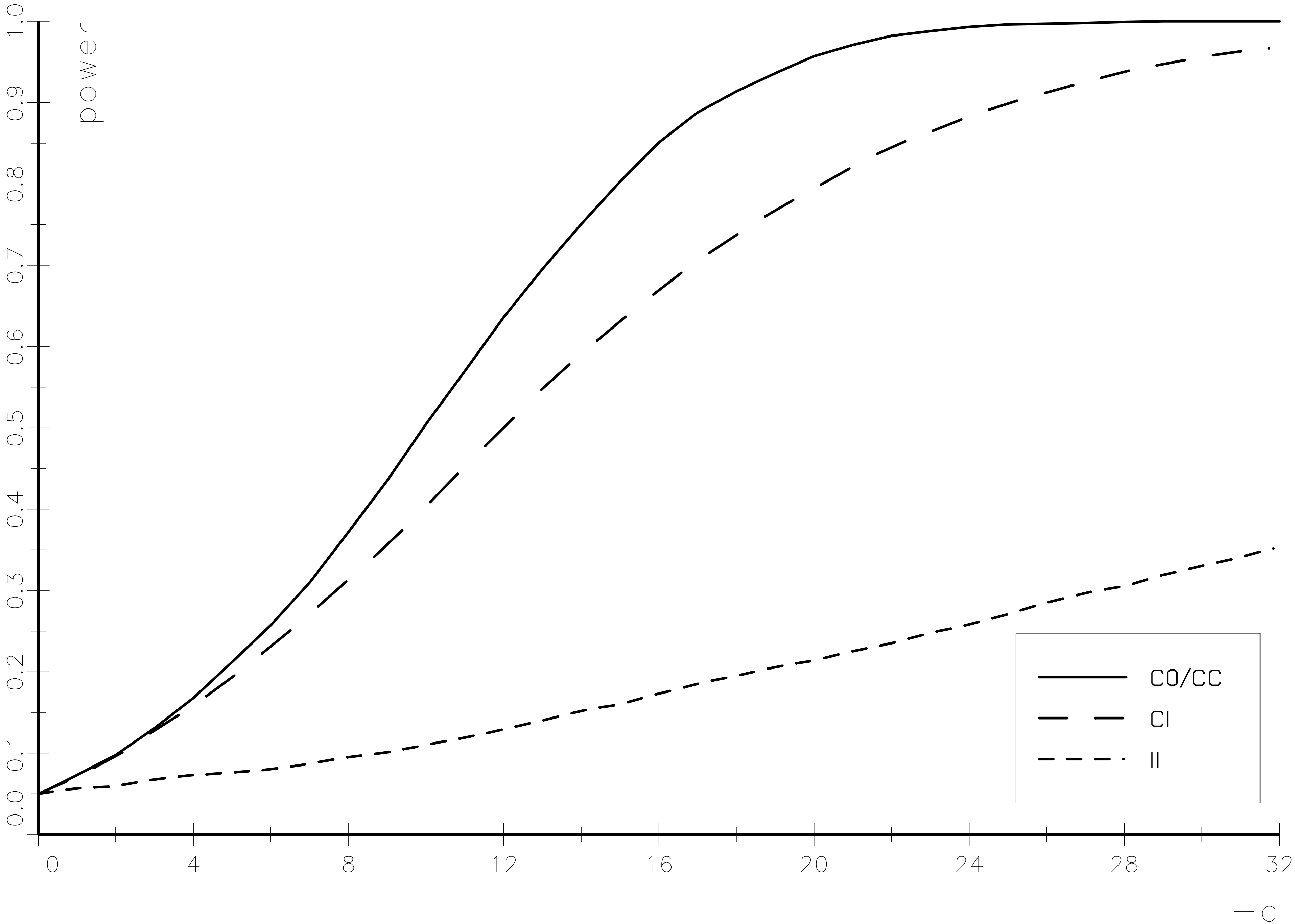

Figure 3
Asymptotic power functions: detrended test