Does Theory Matter: Assessing the Impact of Monotonicity and Concavity Constraints on Forecasting Accuracy

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Abstract

This paper investigates the impact of imposing monotonicity and concavity restrictions on the precision of elasticity estimates, firm efficiency scores, and forecasting accuracy of cost frontiers. In particular, the paper focuses on the affect of imposing these restrictions in an application forecasting 1999 costs of electricity generating plants using Kleit and Terrell's (2001) sample 1996 plants. Results indicate improvements in the precision of elasticity estimates, efficiency estimates, and forecasting accuracy. The results also suggest that the gains in precision come at no cost in terms of bias.

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1. Introduction

Economic theory provides strong restrictions on the cost frontier that are often ignored in empirical applications. From a theoretical perspective, monotonicity and concavity of the cost function are basic tenets, and little debate exists on those properties. From an empirical perspective, there may be both costs and benefits to imposing these properties. This paper evaluates the benefits of imposing concavity locally based on measures of precision of elasticities and forecasts and measuring the bias of out of sample forecasts.

The debate on whether to impose restrictions revolves around misspecification. Imposing true restrictions on a correctly specified cost frontier unambiguously improves efficiency of estimation with no adverse consequences. When the empirical model is viewed as an approximation of the true technology, and thus potentially misspecified, the question may be more difficult. Wales (1977) argues that parameterizations of the translog violating concavity may approximate more complex technologies better than a restricted version of the translog. Stated differently, monotonicity and concavity restrictions may increase the bias of estimators. In the case of a misspecified cost frontier, the decision on imposing restrictions may require weighing gains in terms of efficiency versus costs due to potentially larger misspecification bias.

Some earlier attempts to impose concavity support Wales' conjecture. Many of these efforts to impose concavity relied on using parameter restrictions to force the cost frontier to satisfy monotonicity and/or concavity restrictions globally (at all positive prices). For example, Jorgensen and Fraumeni (1981) provide the restrictions needed to limit the flexibility of the functional form. For the translog, Diewert and Wales (1987) showed that this restriction also causes the translog to overestimate the size of own-price elasticities and, for inputs with small own-price elasticities, results in estimates of cross price elasticities near shares. Global restrictions also proved restrictive for other functional forms. For example, Wales (1977), Terrell (1994), and Gagne and Ouellette (1998) find that global monotonicity and concavity restrictions greatly restrict the ability of many common functional forms to approximate an unknown cost frontier.

Terrell (1996) noted that much of the loss in flexibility stems from imposing restrictions at extreme prices where no inferences will be drawn from the study. As a consequence, he suggests imposing monotonicity locally, over the range of prices where inferences will be drawn. Terrell's (1996) results using Bernt-Wood data suggest that imposing local restrictions can ensure a theoretically consistent function without sacrificing much in the way of flexibility. Dorfman and McIntosh (2001) use a Monte Carlo experiment that imposes curvature conditions and find that mean square errors of estimated elasticities are greatly improved in both small and somewhat large samples. Likewise, O'Donnell and Coelli (2004) find significant changes in elasticities and improvements in precision when constraints are imposed on distance functions.

Overall, the literature provides evidence that imposing monotonicity and concavity locally leads to substantial gains in precision for real data and evidence that precision gains offset any increases in bias in simulated data. However, the literature currently contains no efforts to measure bias in a real application. This paper fills that void. We measure efficiency directly through the size of posterior standard deviations by using the frontier estimated using 1996 data to forecast cost for firms in 1999. If the constraints lead to substantial bias, the forecasts of the constrained model should suffer.

Section 2 of this paper provides the translog functional form with monotonicity and concavity restrictions. Section 3 details the data used for the analysis. Section 4 provides the statistical model and Section 5 contains the empirical findings of the forecasting results. Concluding remarks are provided in Section 6.

2. The Cost Frontier

This application consists of estimating the cost frontier for electric power generation. In particular, we will estimate elasticities of substitution using 1996 data for electric power generation facilities to summarize the technology and forecast costs of firms in 1999. Though the cost frontier is unknown, theory does tell us a great deal about the frontier we seek to approximate. Let $c(q, p)$ denote the cost frontier, expressed as a frontier of output vector (q) and input price vector (p). Microeconomic theory requires that cost must satisfy monotonicity in both prices and output, or $\frac{\partial c}{\partial p} > 0$ and $\frac{\partial c}{\partial q} > 0$.

The second fundamental property of the cost frontier is concavity in input prices. Mathematically, concavity requires that $\frac{\partial^2 c}{\partial p \partial p'}$ be negative semidefinite and rules out an upward sloping input demand. Concavity is the property that is most often violated in the empirical literature.[1] Theory also requires that the cost frontier satisfy homogeneity of degree one in the input prices, which requires that $c(tp) = tc(p)$, where $t > 0$.

[1] See Diewert and Wales (1987) or Terrell (1996) for further discussion of these violations.

application. With three inputs and two outputs as in this application, the translog cost frontier is:

$$\text{(1)} \quad \ln c(p,q) = a_0 + \sum_{i=1}^{3} a_i \ln p_i + \sum_{i=1}^{2} b_i \ln q_i + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij} \ln p_i \ln p_j + \frac{1}{2}\sum_{i=1}^{2} g_i (\ln q_i)^2 ,$$

where: $a_{ij} = a_{ji}$ for all $i, j = 1,2,3$, and

$$\sum_{i=1}^{3} a_i = 1, \quad \sum_{j=1}^{3} a_{ij} = 0 \quad (i, j = 1,2,3).$$

The translog cost frontier imposes homogeneity of degree one with respect to input prices under these conditions. As a second order approximation to an arbitrary cost frontier, the translog also fulfills Diewert's minimum flexibility requirement for flexible forms. By Shepphard's lemma, the first derivative of the log cost frontier with respect to the log input price produces the share equations associated with a respective input:

$$\text{(2)} \quad s_i(p,q) = a_i + \sum_{j=1}^{3} a_{ij} \ln p_j .$$

For any given price, the regularity conditions can be verified from restrictions derived from the translog cost frontier. Monotonicity in the input prices is ensured by nonnegative values of (2). Let $\mathbf{s}$ represent the vector of *n* shares, $\hat{s}$ denote an $n \times n$ diagonal matrix with the shares making up the main diagonal, and A denotes the $n \times n$ symmetric matrix of the parameters a_{ij}. Diewert and Wales (1987) show that the translog cost frontier satisfies concavity if and only if $A - \hat{s} + ss^T$ is a negative semidefinite matrix.

(3) $b_i + g_i \ln q_i > 0$.

3. Data:

This study uses 1996 data on electric power generation first used by Kleit and Terrell (2001) for estimation. An updated 1999 version of that data is created to assess out of sample forecasts. Both data sets use data from the Utility Data Institute (UDI) that includes plant level information concerning total costs, fuel prices, and two measures of output for electricity generating plants for the years 1996 and 1999. Two measures of output are required to take into account the fact that some power plants exist primarily to provide output during periods of peak demand. This data set also provides information on plant location and the average price of natural gas burned at each plant.

The second data source is the Bureau of Labor Statistics (BLS), which provides county level data on manufacturing wages for 1999. The wage rate in this data set is the average annual manufacturing wage for workers in the county where the power generating plant is located.

For the third data source, Hilt (1996) supplies plant level measures of the capital stock, taxes, overhead, depreciation, and operating and management expenses. Allocating firm level data to each plant derives all these variables. Hall and Jorgenson's (1971) method is used to calculate the price of capital using this data set.

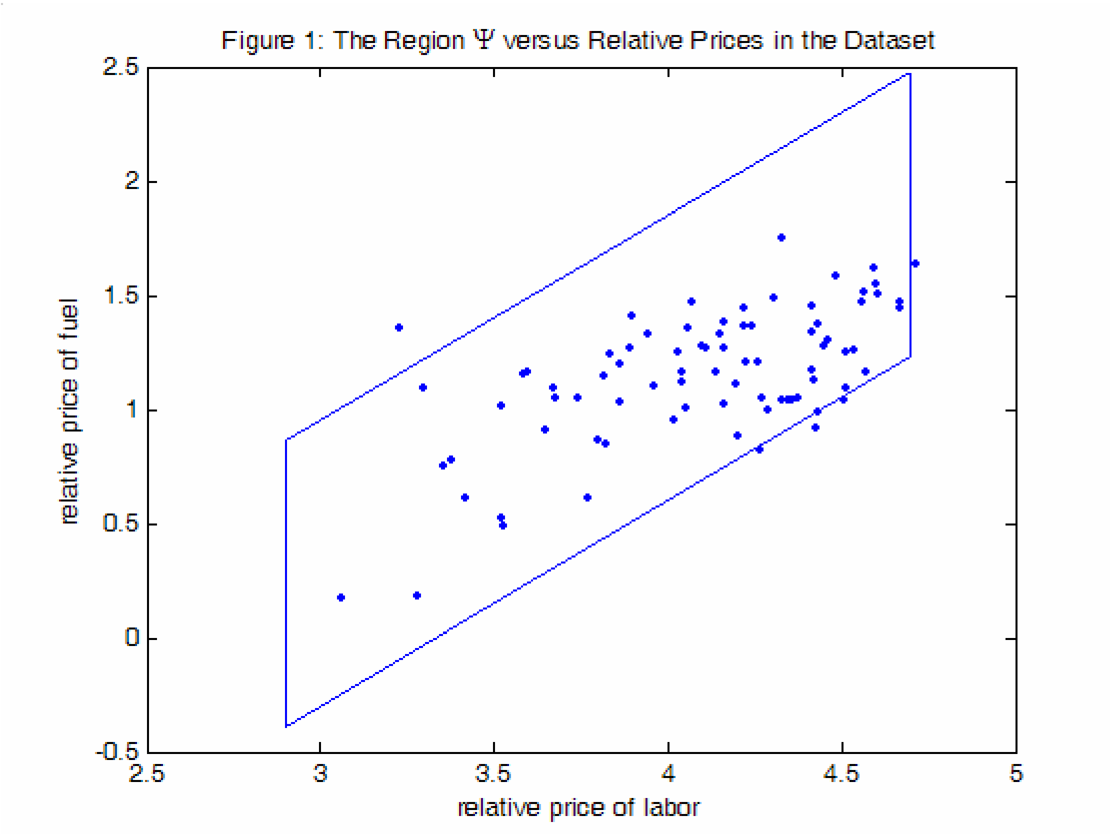
Table 1
Summary Statistics for the 1996 Data

Variable	Mean	S.D.
Cost (C)	50,653,645.79	51,246,491.93
Annual Output (q_1)	1,537,843.24	1,741,523.54
Peak Output (q_2)	649.32	548.47
Wage (P_L)	45,342.54	7,196.22
Price of Fuel (P_F)	2.71	0.46
Price of Capital (P_K)	1.02	0.39
Log Relative Wage	3.85	0.16
Log Relative Fuel Price	1.03	0.40

Table 2
Summary Statistics for 1999 Data

Variable	Mean	S.D.
Cost (C)	58,475,601.66	88,083,689.22
Annual Output (q_1)	1,666,551.19	2,003,655.62
Peak Output (q_2)	588.10	554.17
Wage (P_L)	53,553.19	15,869.57
Price of Fuel (P_F)	2.70	0.34
Price of Capital (P_K)	0.91	0.28
Log Relative Wage	4.10	0.46
Log Relative Fuel Price	0.99	0.11

using 1996 data and for the 1999 data set constructed for this paper. Note that annual output, cost, and wages rose over the three-year period. The methodology used in this paper requires defining a region Ψ, of relative price combinations where monotonicity and concavity will be imposed.[2] Figure 1 contains a graph of the relative prices in Ψ versus the 1999 data and shows that most price combinations lie inside ψ.



[2] See Terrell (1996) or Kleit and Terrell (2001) for additional discusion on choosing ψ.

This application uses the Bayesian cost frontier model initially introduced by Van den Broeck, Gary Koop, J. Osiewalski, and M. F. Steel (1994).[3] We impose monotonicity and concavity through the prior. The cost of an efficient firm is represented by the cost frontier ($f(p_i, q_i)$), which yields the cost that an efficient plant faces given a vector of prices (p_i) for inputs used to produce a given level of outputs (q_i). If a plant's observed cost exceeds that which would be provided by the frontier, then that deviation is partly attributable to inefficiency. Therefore, any deviations from the frontier can be used to measure plant inefficiency.

As in earlier work, express the log total cost for the plant as:

(4) $$\ln(c_i) = f(p_i, q_i) + u_i + v_i .$$

The deviation of plant $i's$ cost from the frontier is comprised of two stochastic error terms, inefficiency (v_i) and measurement error (u_i). The inefficiency error term follows an exponential distribution with a scale parameter l and $u_i \sim IIDN(0, s^2)$.

Combining the cost frontier above with the translog cost frontier, this yields a linear model which can be expressed as:

(5) $$\begin{aligned} y_i &= X_i b + u_i + v_i \\ u_i &\sim N(0, s^2) \\ v_i &\sim EXP(l). \end{aligned}$$

where y_i is the log cost for plant i, X_i is a row vector of independent variables used in order to create the translog frontier, b is a column vector representing the coefficients of

[3] The Bayesian frontier model has been widely used in the literature. For example, see Koop, G., J. Osiewalski, and M. F. Steel (1994), Lewis and Anderson (1999), or Lewis, Springer and Anderson (2003).

one-sided (non-negative) error term measuring plant inefficiency.

We choose a flat prior for β, and gamma priors for λ^{-1} and σ^2,

$$
\begin{aligned}
& p(b) \propto 1, \\
(6) \qquad & p(l^{-1}) = f_G(l^{-1} \mid 1, -\ln(r^*)), \\
& p(s^{-2}) = f_G\left(s^{-2} \mid \frac{t}{2}, \frac{s_p^2}{2}\right)
\end{aligned}
$$

where $f_G(\cdot \mid u_1, u_2)$ denotes a gamma density with mean υ_1/υ_2 and variance υ_1/υ_2^2. Note that with y_i defined as log cost, $r_i=\exp(-v_i)$ measures the efficiency of the ith firm and r^* is simply the prior median for efficiency. Following van den Broek, Koop, Osiewalski, and Steel (1994), and Koop, Osiewalski, and Steel (1994), we set r^* to 0.875.[4] Fernandez, Osiewalski, and Steel (1997) show that an uninformative prior on σ^2 may generate an improper posterior in a cross-sectional application such as this one. For the prior on σ^2, we choose τ to be one and set s_p^2 to .03, which implies a weak prior on σ^2.

As Terrell (1996) notes, the prior can also incorporate monotonicity and concavity restrictions. Let h(β) be an indicator function, equal to one if the stochastic frontier satisfies monotonicity and concavity for all price combinations in a region of prices and output Ψ, and zero otherwise. The full prior incorporates the restrictions from theory by using this indicator function to slice away the portion of the density violating concavity and monotonicity:

$$(7) \qquad p(b, s^2, l^{-1}) \propto p(b)p(l^{-1})p(s^{-2})h(b).$$

[4] Given the number of observations in this sample, this implies a weak prior on λ.

where inferences will be drawn or, equivalently sets the probability of parameter values which violate microeconomic theory at relevant prices to zero. Combining the prior and the likelihood produces the posterior density, denoted $p(q)$, where $q = (b, s^2, l, v)$. Elasticities, efficiency measures, and returns to scale are all functions of θ. Let g(θ) denote a vector containing these functions of interest. The posterior mean for these functions of interest is:

(8) $$E[g(q)] = \int g(q)\,p(q)\,dq\ .$$

As in most Bayesian applications, these integrals cannot be computed analytically and are instead computed using Monte Carlo integration. Koop, Osiewalski, and Steel (1994) first introduced the Gibbs sampler for the stochastic frontier model. This paper uses the variant of the algorithm employed by Kleit and Terrell (2001), which includes a proper prior for σ^2 and adds an accept-reject element to impose monotonicity and concavity restrictions.

5. Results:

Table 3 presents the posterior moments for the model parameters estimated using the 1996 data set. The results produce very similar estimates of mean efficiency, 14.3% for the unconstrained model and 13.2% for the constrained model which imposes monotonicity and concavity. The posterior standard deviations also show a slight improvement in precision for estimates of efficiency and more dramatic increases in efficiency for other parameters. The parameters themselves are difficult to interpret, and the intuition behind efficiency gains is better understood by looking at shares and elasticities.

Table 3: Posterior Moments for Model Parameters
(A) Unconstrained

Parameter	Mean	Std. Dev.	5th percentile	95th percentile
a_0	8.269	2.808	3.534	12.740
a_1	1.400	1.433	-0.876	3.813
a_2	0.267	0.945	-1.281	1.769
a_{11}	-0.403	0.430	-1.115	0.278
a_{12}	-0.609	0.393	-1.240	0.053
a_{22}	0.280	0.336	-0.265	0.830
b_1	-0.409	0.184	-0.704	-0.102
g_1	0.042	0.007	0.030	0.054
b_2	0.887	0.239	0.494	1.278
g_2	-0.060	0.020	-0.092	-0.027
s^2	0.006	0.002	0.003	0.011
l	0.143	0.025	0.107	0.187

(B) Constrained

Parameter	Mean	Std. Dev.	5th percentile	95th percentile
a_0	9.947	1.100	8.117	11.769
a_1	0.614	0.349	0.091	1.234
a_2	0.405	0.342	-0.216	0.895
a_{11}	-0.140	0.110	-0.327	0.026
a_{12}	-0.147	0.136	-0.386	0.064
a_{22}	0.114	0.116	-0.060	0.320
b_1	-0.442	0.195	-0.759	-0.112
g_1	0.044	0.008	0.031	0.056
b_2	0.845	0.260	0.419	1.278
g_2	-0.057	0.022	-0.092	-0.021
s^2	0.010	0.003	0.006	0.014
l	0.132	0.022	0.099	0.171

Note: All computations are based on 5000 iterations of the Gibbs sampler with the first 500 dropped to avoid sensitivity to starting values.

the means for prices and output in the 1999 data set. Focus first on the posterior moments for the shares. The point estimate implied by the posterior mean[5] for the labor share is implausibly close to zero. The 90% highest density region of [-.28,.28] contains both negative shares and more plausible estimates for the share of expenditure devoted to labor. While the point estimate for the fuel share is more plausible, the 90% highest density region is large and contains values greater than one. Panel (B) contains results with monotonicity and concavity imposed. Note that the highest density regions for shares shrink substantially and do not include values less than zero or greater than one. The point estimates implied by posterior means also now appear more in line with expectations.

For elasticities, the point estimates deviate more and the gains in precision are also more dramatic. The point estimate for the own price elasticity of labor is .371 and predicts that plants will increase their demand for labor in response to an increase in wages. However, a closer look at the posterior standard deviations and highest density regions suggests that it is very difficult to draw any strong conclusions from the unconstrained model.[6] The posterior standard deviation shrinks from 190.9 in the unconstrained model to 1.1 in the constrained model. The constrained model produces point estimates and highest density regions that adhere to economic theory and also appear similar to those produced in previous studies. Simply stated, imposing

[5] The use of the posterier mean as our point estimate assumes a quadratic loss function.

[6] This result is not an artifact of evaluating the elasticities at 1999 means rather 1996 means, though the intervals are slightly larger. For example, the highest density region for ε_{LL} at 1996 means is [-27.38,24.36].

shares and elasticities in this application.

Table 4: Shares and Elasticities

(A) Unconstrained

	Posterior Mean	Posterior Std. Dev.	5^{th} percentile	95^{th} percentile
S_L	0.004	0.174	-0.284	0.282
S_F	0.860	0.171	0.583	1.140
S_K	0.135	0.112	-0.042	0.322
e_{LL}	0.371	190.892	-17.825	0.824
e_{FF}	-0.792	0.415	-1.273	-0.818
e_{KK}	-11.774	584.440	-13.550	-3.737
e_{LF}	3.031	207.908	-15.962	-0.569
e_{LK}	-3.402	167.845	-12.098	-0.435
e_{FL}	0.262	0.379	-0.243	0.313
e_{FK}	0.530	0.232	0.196	0.505
e_{KL}	-0.557	167.513	-6.970	1.179
e_{KF}	12.332	745.610	-14.373	2.611

(B) Constrained

	Posterior Mean	Posterior Std. Dev.	5^{th} percentile	95^{th} percentile
S_L	0.144	0.061	0.063	0.144
S_F	0.737	0.071	0.607	0.737
S_K	0.119	0.044	0.051	0.119
e_{LL}	-2.084	1.057	-4.021	-0.646
e_{FF}	-0.457	0.175	-0.763	-0.198
e_{KK}	-1.343	0.305	-1.706	-0.727
e_{LF}	1.743	1.066	0.331	3.749
e_{LK}	0.341	0.526	-0.395	1.313
e_{FL}	0.293	0.153	0.069	0.569
e_{FK}	0.164	0.085	0.039	0.316
e_{KL}	0.268	0.537	-0.708	0.995
e_{KF}	1.074	0.537	0.283	2.028

Note: All computations are based on 5000 iterations of the Gibbs sampler with the first 500 dropped to avoid sensitivity to starting values.

the improvement in precision. Notice that the some posterior mass is associated with negative share for labor and capital in Figure 2. Figure 3 shows that the prior slices away that mass in the constrained model. This slicing away of mass is also clear in a comparison of e_{LL}, where substantial mass associated with positive own price elasticities from Figure 2 is eliminated in Figure 3 by imposing concavity.

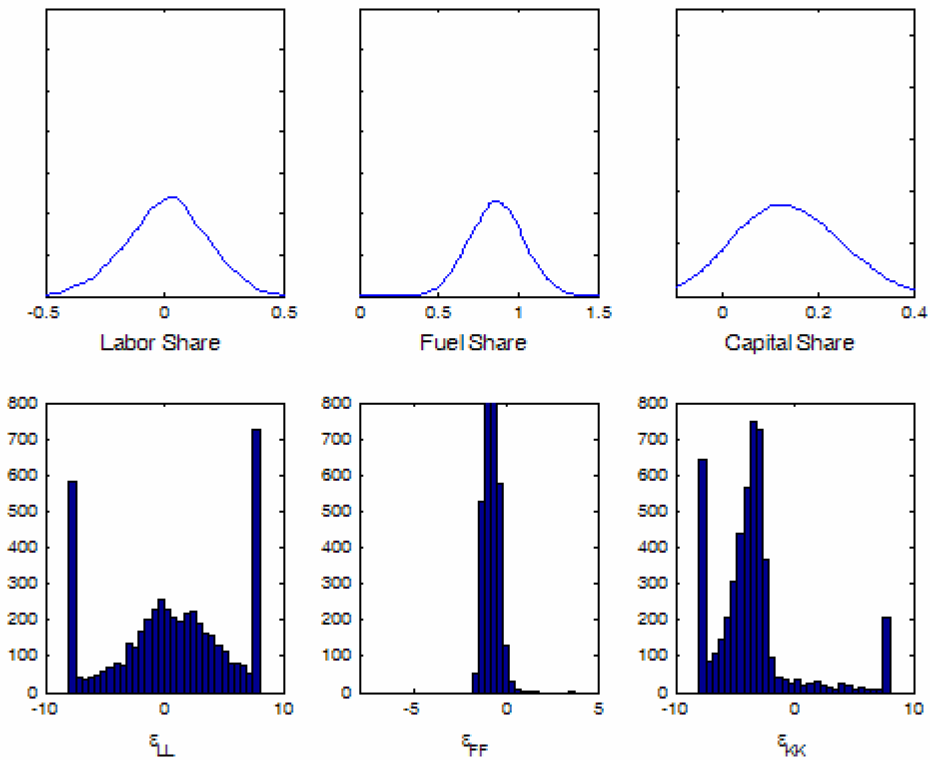


Figure 2—Marginal Density Plots for Shares and Elasticities-Unconstrained Model

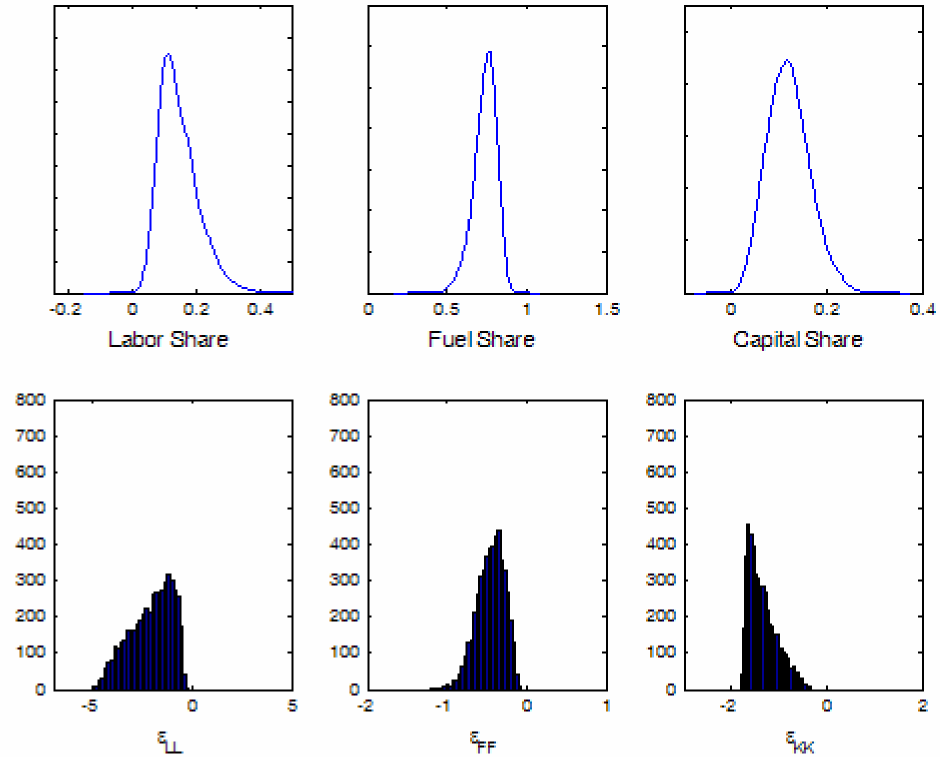


Figure 3—Marginal Density Plots for Shares and Elasticities-Constrained Model

The very large posterior standard deviations for elasticities in the unconstrained model are also easily explained in Figure 2. The marginal density plot for the labor share in Figure 2 shows a significant mass near zero. Because the own price elasticity computations involve division by the shares, the mass near zero translates into very extreme values of elasticities. To permit graphs of marginal densities for elasticities, these extreme values are all grouped into the last single bar. For both e_{LL} and e_{KK}, the histograms show significant posterior mass associated with extreme values, which translates into large posterior standard deviations.

Frequentist studies using unconstrained models often do not exhibit the problems of the unconstrained Bayesian model for two reasons. First, by evaluating elasticities at of the distribution. Second, standard errors and confidence intervals are often computed using the delta method which fails to capture problems associated with shares near zero. Anderson and Thursby (1986) note that the delta method can provide very misleading estimates of confidence intervals for elasticities, even in relatively large samples. A closer look at Gallant and Golub's (1984) large bootstrap standard errors indicate that frequentist studies are not immune to the problem when standard errors are estimated more accurately.

The results above show that incorporating monotonicity and concavity restrictions generates substantial improvements in precision, particularly for elasticities. However, the gain in precision could be offset by increased misspecification bias. To assess the degree of misspecification bias, we examine the out-of-sample forecasts for both models. In particular, we use the posterior density computed for 1996 data to forecast cost in 1999 and then compare our forecasts to the observed values. If the constraints generate substantial biases, the forecast accuracy should deteriorate in the constrained model.

Table 5
Forecasting Results

Plant	Actual L(Cost)	$E[v_{it+1}]=\lambda$ Unconstrained Forecast	$E[v_{it+1}]=\lambda$ Constrained Forecast	$v_{it+1}=v_{it}$ Unconstrained Forecast	$v_{it+1}=v_{it}$ Constrained Forecast
Arsenal Hill	16.253	16.238(0.229)	16.340(0.176)		
Atkinson	15.703	15.784(0.192)	15.805(0.170)	15.852(0.154)	15.846(0.138)
Barney Davis	18.535	18.537(0.236)	18.618(0.176)		
Baxter Wilson	18.298	18.394(0.178)	18.415(0.170)	18.369(0.127)	18.395(0.126)
Cedar Bayou	19.227	19.208(0.267)	19.324(0.182)	19.131(0.230)	19.271(0.133)
Collins	18.682	18.332(0.252)	18.441(0.187)		
Cunningham	17.631	17.613(0.208)	17.645(0.175)	17.537(0.163)	17.589(0.127)
Cutler (FL)	16.675	16.635(0.197)	16.650(0.172)	16.527(0.143)	16.565(0.118)
Decordova	18.978	18.903(0.232)	18.981(0.181)	18.836(0.192)	18.918(0.131)
Deepwater(TX)	16.240	16.121(0.258)	16.243(0.176)		
Delta	16.089	16.089(0.183)	16.103(0.171)	16.013(0.125)	16.046(0.120)
Eagle Mountain	17.872	17.675(0.242)	17.809(0.175)	17.671(0.211)	17.795(0.135)
East River	17.621	16.482(0.254)	16.659(0.177)		
Eaton	16.321	15.983(0.217)	16.041(0.181)	15.975(0.162)	16.016(0.132)
Edgewater(OH)	16.225	15.874(0.232)	15.959(0.176)	15.785(0.192)	15.888(0.130)
Fort Phantom	17.623	17.538(0.213)	17.563(0.178)		
Gadsby	16.737	16.621(0.203)	16.657(0.171)	16.632(0.155)	16.675(0.136)
Gordon Evans	17.413	17.462(0.234)	17.547(0.173)	17.348(0.191)	17.455(0.117)
Graham	18.400	18.372(0.217)	18.429(0.176)	18.312(0.175)	18.383(0.130)
Greens Bayou	17.440	17.335(0.240)	17.437(0.173)	17.250(0.189)	17.359(0.120)
Greenwood(MI)	18.128	17.984(0.233)	18.113(0.176)	17.965(0.205)	18.049(0.127)
Handley	18.774	18.559(0.241)	18.686(0.176)	18.545(0.207)	18.666(0.134)
Harvey Couch	16.118	15.976(0.214)	16.009(0.177)		
Hunters Point	17.107	16.677(0.191)	16.715(0.170)		
Hutchinson	16.351	16.199(0.204)	16.238(0.171)	16.229(0.152)	16.260(0.137)
Jones	18.162	18.157(0.210)	18.203(0.173)	18.093(0.165)	18.161(0.126)
Knox Lee	17.849	17.836(0.210)	17.885(0.173)	17.775(0.167)	17.837(0.125)
La Palma	16.998	16.892(0.187)	16.906(0.173)		
Lake Catherine	17.748	17.838(0.189)	17.845(0.170)	17.805(0.144)	17.808(0.127)
Lake Creek	17.596	17.454(0.223)	17.549(0.175)	17.361(0.182)	17.484(0.126)
Lake Hubbard	18.619	18.546(0.258)	18.690(0.176)	18.499(0.232)	18.650(0.131)
Lake Pauline	14.804	14.751(0.269)	14.946(0.201)		
Laredo	17.109	16.902(0.202)	16.920(0.176)		
Lewis Creek	17.474	17.589(0.174)	17.683(0.170)	17.548(0.125)	17.608(0.115)
Lieberman	17.016	16.829(0.226)	16.927(0.173)		
Little Gypsy	18.191	18.256(0.236)	18.338(0.174)		
Lon Hill	17.948	17.916(0.231)	17.992(0.173)	17.818(0.188)	17.919(0.122)
Lone Star	15.201	15.303(0.254)	15.484(0.188)		
Maddox	17.050	16.863(0.221)	16.921(0.181)	16.838(0.176)	16.898(0.137)
Michoud	18.374	18.465(0.203)	18.498(0.171)	18.427(0.161)	18.484(0.130)
Morgan Creek	18.352	18.416(0.205)	18.451(0.171)	18.334(0.157)	18.396(0.121)

Note: All computations are based on 5000 iterations of the Gibbs sampler with the first 500 dropped to avoid sensitivity to starting values. Posterior standard deviations are in parenthesis.

Table 5
Forecasting Results (continued)

Plant	Actual L(Cost)	$E[v_{it+1}]=\lambda$ Unconstrained	$E[v_{it+1}]=\lambda$ Constrained	$v_{it+1}=v_{it}$ Unconstrained	$v_{it+1}=v_{it}$ Constrained
Murray Gill	17.101	17.038(0.252)	17.184(0.175)	16.986(0.214)	17.141(0.130)
Mustang	17.600	17.656(0.215)	17.725(0.173)	17.557(0.160)	17.642(0.117)
Nichols	17.790	17.776(0.213)	17.835(0.173)	17.702(0.170)	17.774(0.123)
Ninemile Point	19.324	19.263(0.208)	19.297(0.175)	19.228(0.161)	19.279(0.131)
North Lake	17.921	17.902(0.219)	17.963(0.171)	17.817(0.174)	17.902(0.122)
Northeastern	18.016	18.084(0.212)	18.134(0.171)		
Nueces Bay	18.257	18.295(0.237)	18.377(0.175)	18.200(0.195)	18.306(0.124)
Oak Creek (TX)	16.420	16.203(0.240)	16.253(0.191)		
Ocotillo	17.457	16.890(0.231)	16.989(0.173)	16.828(0.196)	16.930(0.124)
Paint Creek	16.976	16.656(0.315)	16.759(0.210)	16.574(0.289)	16.700(0.172)
Permian Basin	18.591	18.544(0.214)	18.578(0.177)	18.496(0.172)	18.551(0.133)
PH Robinson	19.484	19.433(0.267)	19.550(0.182)	19.378(0.233)	19.517(0.139)
RE Ritchie	17.263	16.818(0.172)	16.905(0.168)	16.788(0.105)	16.849(0.116)
Reeves	15.816	15.830(0.198)	15.852(0.171)	15.798(0.151)	15.829(0.129)
Rex Brown	16.615	16.384(0.178)	16.422(0.169)	16.325(0.125)	16.380(0.121)
Riverside (GA)	15.644	15.158(0.226)	15.244(0.180)	15.196(0.174)	15.259(0.142)
Riverside (MD)	15.982	15.354(0.251)	15.521(0.182)	15.495(0.215)	15.572(0.147)
Riverside (OK)	15.947	15.336(0.248)	15.492(0.182)	15.462(0.211)	15.537(0.147)
Sabine	19.296	19.374(0.232)	19.446(0.179)	19.303(0.188)	19.397(0.130)
Saguaro	17.007	16.673(0.223)	16.762(0.178)	16.600(0.185)	16.691(0.129)
Sam Bertron	17.917	17.820(0.231)	17.903(0.172)	17.760(0.176)	17.846(0.123)
Seminole (OK)	18.974	18.855(0.220)	18.923(0.175)		
Sewaren	16.931	16.487(0.230)	16.577(0.174)		
Southwestern	17.352	17.369(0.196)	17.385(0.170)	17.392(0.146)	17.325(0.120)
Starlington	17.356	17.460(0.224)	17.527(0.172)		
Stryker Creek	18.376	18.423(0.212)	18.473(0.173)	18.377(0.166)	18.443(0.129)
Sweatt	16.166	16.003(0.208)	16.039(0.177)	15.948(0.150)	15.996(0.127)
TH Wharton	17.051	16.670(0.227)	16.732(0.179)	16.739(0.196)	16.791(0.151)
Tradinghouse	19.293	19.245(0.223)	19.319(0.177)	19.216(0.180)	19.302(0.133)
Tulsa	17.280	17.147(0.224)	17.248(0.174)	17.079(0.184)	17.181(0.124)
Turkey Point	18.588	18.641(0.215)	18.680(0.177)	18.558(0.170)	18.617(0.128)
Valley (CA)	16.636	15.354(0.226)	15.415(0.186)	16.938(0.172)	16.987(0.150)
Victoria (TX)	17.414	17.515(0.210)	17.562(0.171)		
Waterford 1 &2	18.126	18.129(0.249)	18.225(0.175)		
Waterside (NY)	17.374	16.849(0.214)	16.912(0.173)	17.334(0.180)	17.388(0.148)
West Springfield	16.458	15.358(0.180)	15.395(0.170)	15.390(0.134)	15.382(0.129)
Wilkes	18.097	18.153(0.202)	18.168(0.173)		
Willow Glen	18.780	18.646(0.259)	18.752(0.179)	18.614(0.228)	18.743(0.139)
Zuni	14.601	14.585(0.206)	14.624(0.181)	14.586(0.138)	14.617(0.129)

Note: All computations are based on 5000 iterations of the Gibbs sampler with the first 500 dropped to avoid sensitivity to starting values. Posterior standard deviations are in parenthesis.

An additional forecasting assumption is required because 58 of the 80 firms in our sample also appeared in the 1996 sample. Adding time subscripts to equation (5) helps explain this issue:

(9) $$y_{i,1999} = x_{i,1999}b + u_{i,1999} + v_{i,1999}$$

For firms not included in the 1996 sample, no information exists and based on our initial model $v_{i,1999}$ is distributed exponential with shape parameter λ.[7] The same distribution is appropriate for firms in both samples if we assume efficiency is time specific, or $v_{i,1999}$ and $v_{i,1996}$ are independent. The first two forecast columns of Table 5 provide forecasts under this assumption. The last two columns present forecasts for the 58 firms in both samples assuming $v_{i,1999} = v_{i,1996}$ or that inefficiency is firm, not time, specific. Notice that the forecasts are missing from these columns for firms that do not appear in both samples.

Table 6 contains common forecast evaluation statistics and the average of the posterior standard deviations across all forecasts for three sets of forecasts. The first two rows contain statistics for the unconstrained and constrained models assuming errors are independent over time. The third and fourth rows contain forecast evaluation statistics across 58 firms under the assumption that inefficiency errors are firm specific. Finally, the last two rows contain results assuming independence of errors across time, but using only forecasts for the 58 firms in both samples.

[7] Note that the actual cost is as unknown in the forecast. Koop, Osiewalski, and Steel (1994) provide the conditional distribution for the case where cost is known.

Within each group, constrained models appear slightly better than unconstrained models by all measures.[8] At the very least, the forecast results provide no evidence that monotonicity and concavity restrictions increase bias. The forecasts for firms in the sample for both 1996 and 1999 appear no better than those for firms first appearing in 1999 if efficiency is assumed independent over time. However, forecasting generally improves for the model assuming firm specific inefficiency. With the exception of the mean error criteria, the constrained model with firm specific inefficiency errors outperforms the other models.

Table 6
Measures of Forecasting Accuracy

Model	R^2	Mean Error	MSE	MAE	Mean(SD)
$E[v_{it+1}]=\lambda$,n=80:					
Unconstrained	.917	.153	.097	.188	.222
Constrained	.935	.080	.075	.168	.176
$v_{it+1}=v_{it}$:					
Unconstrained	.948	.159	.069	.185	.176
Constrained	.962	.088	.051	.138	.130
$E[v_{it+1}]=\lambda$,n=58:					
Unconstrained	.916	.157	.098	.188	.220
Constrained	.934	.087	.077	.162	.177

6. Conclusion

The results in this paper find that imposing monotonicity and concavity improves precision and leads to more accurate forecasts in an application to electricity generating plants. Overall the gain in precision is quite large for elasticities, still substantial for shares, and more modest for forecasts of firm level log cost. The intuition behind this

[8] Matched pair t-tests indicate that the mean error, mean square error and mean absolute error of the constrained and unconstrained models are statistically significant for all three pairs.

result lies with an assessment of the information present in the data about each of these variables. One would expect the data provides more information about the cost frontier itself than about its first derivative (shares) or second derivative (elasticities). Thus, our results suggest that monotonicity and concavity restrictions offer the most potential for gains where the information from the data is weakest.

These results clearly suggest that similar empirical studies could benefit from imposing conditions implied by economic theory, particularly if the goal is to estimate elasticities. The next step in this research agenda lies in measuring the impact of constraints in applications using other data sets with varying sample sizes. It would also be useful to measure precision gains and bias for alternative models (SUR models of input demands, distance functions, etc.) and in applications focusing on profit functions and indirect utility functions. In the application the conclusion is clear -- economic theory matters.

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