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Partial differential equations in several complex variables.

Generations of students have learned the theory of the $\overline{\partial}$-Neumann problem from the monograph of G. B. Folland and J. J. Kohn [The Neumann problem for the Cauchy-Riemann complex, Ann. of Math. Stud., 75, Princeton Univ. Press, Princeton, N.J., 1972; MR 57#1573]. In the three decades since the publication of that little book, the theory of the multidimensional Cauchy-Riemann equations has undergone tremendous development. Now two prominent researchers in the field, both former students of Kohn, have written a “next generation” volume that may serve both as a text for students and as a reference for workers in the area.

The first three of the twelve chapters introduce background material about multidimensional complex analysis: the biholomorphic inequivalence of the ball and the polydisc, the Cauchy and Bochner-Martinelli integral representations, holomorphic extension phenomena, pseudoconvexity, and the Levi problem. The next three chapters are devoted to the Hilbert space approach to solvability and regularity of the $\overline{\partial}$-equation: the $L^2$ existence theory on pseudoconvex domains, the $1/2$-subelliptic estimate for the $\overline{\partial}$-Neumann problem on strongly pseudoconvex domains, Sobolev estimates for the $\overline{\partial}$-Neumann problem on more general pseudoconvex domains, boundary regularity of biholomorphic mappings, irregularity of the Bergman projection on worm domains.

“The second half of the book is intended as a self-contained introduction to the tangential Cauchy-Riemann equations”, according to the authors’ preface. Chapter 7 introduces the tangential Cauchy-Riemann complex and discusses Lewy’s equation. Chapter 8 proves a $1/2$-subelliptic estimate and local regularity for $\Box_b$ under condition $Y(q)$, while Chapter 9 establishes the $L^2$ existence theory of $\Box_b$ on pseudoconvex boundaries. Then the theme changes to integral representations: Chapter 10 constructs a fundamental solution for $\Box_b$ on the Heisenberg group, and Chapter 11 uses integral formulas to study $L^p$ and Hölder estimates for solutions of $\Box$ and $\Box_b$ on strictly convex domains. The concluding chapter addresses the embeddability of abstract CR structures.

Compared to a text on the same subject by S. G. Krantz [Partial differential equations and complex analysis, CRC, Boca Raton, FL,
1992; MR 94a:35002], the book under review has less preparatory background about partial differential equations but a much more extensive account of contemporary researches on $\overline{\partial}$ and $\partial_b$. Anyone planning to do research in this area will want to have a copy of the book.  

_Harold P. Boas (1-TXAM)_