Mathematics 30750, Spring 2010

Lecturer: Mei-Chi Shaw, Office Hayes-Healy 244, phone 631-6357

Lectures: 11:45-12:35 MWF, Hayes-Healy 229

TEXT: By Michael C. Reed, Fundamental Ideas of Analysis
ISBN 0-471-15996-4

HOMEWORK: The homework of each week will be due the following Monday. At the end of the semester the homework with the lowest score will be dropped and no late homework will be accepted.

EXAMS and GRADING: Your course grade will be computed as follows:
Test 1 20% (Feb. 17)
Test 2 20% (March 31)
Homework 20%
Final Exam 40%

Office Hours: There will be a regular office hour on every Wednesday, 2:30-3:30 PM at my office or by appointments.

Absence from Exams: A student who is absent from an examination without an official excuse shall receive a grade of zero for that examination. A student who is officially excused will not be penalized. If you miss a test for any reason, call the instructor or the Mathematics Department as soon as possible.

Honor Code: The exams are under the honor code. The honor code does not apply to homework. So you may discuss the homework with other classmates but do not copy each other.
SYLLABUS FOR MATH 30750, REAL ANALYSIS

Prerequisites: Intro. to Math. Reasoning (MATH 20630)

Following is a list of topics that will be covered in Math 30750.

Preliminaries Chapter 1
(1) The real numbers
(2) Sets and functions
(3) Cardinality
(4) Methods of proof

Sequences Chapter 2
(1) Convergence
(2) Limits of sequences,
(3) Cauchy Sequences
(4) Supremum and Infimum
(5) The Bolzano-Weierstrass Theorem

First Exam

The Riemann Integral Chapter 3
(1) Continuous functions, the Intermediate Value Theorem
(2) Definition and properties of the Riemann Integral
(3) Uniform continuity
(4) Integrals of continuous functions
(5) Improper integrals

Differentiation, Chapter 4
(1) Definition and basic properties of differentiable functions
(2) Mean Value Theorem and extrema of differentiable functions
(3) The Fundamental Theorem of Calculus
(4) Chain rule and inverse function theorem
(5) Taylor’s Theorem

Second Exam

Sequences and series of functions, Chapters 5 and 6
(1) Pointwise and uniform convergence
(2) Limit theorems
(3) series, series of functions
(4) power series, Taylor series

Additional topics
(1) Fourier series
(2) Differentiability in \( \mathbb{R}^n \)
(3) inverse and implicit function theorems
(4) Complex Analysis
What is Real Analysis?

Analysis is the mathematics that deals with the ideas of calculus, one of the greatest intellectual achievements of the human mind.

You have already learned the mechanics of calculus of one variable, in Calculus I-II or the equivalent. In Math 30750 you will learn the theory of calculus of one variable. This involves learning definitions, theorems and proofs, proving theorems, writing clear and correct proofs, applying theorems to analyze examples, etc. It does not involve mechanically applying formulas or rules. Of course you will learn some new facts in the course. However, a major aim of the course is to teach you to think in a very logical and rigorous way.

The textbook for this course is much smaller and lighter than your calculus textbook and has far fewer problems. However, some of the problems are quite demanding. You will learn more by spending hours, if needed, on a few problems, or even one problem, than on a dozen exercises which follow the same pattern. I hope you will not be discouraged by difficulty, but rewarded by overcoming it.

The ability to think logically, to analyze problems, and to communicate clearly and precisely will be useful to you in whatever you do after you graduate from Notre Dame, even if the specific mathematical content of this course is not.
SYLLABUS FOR MATH 30750, REAL ANALYSIS

HW#1
1.1 #2, 8, 10, 11
1.2 #2, 6

HW#2
1.3 #1, 4, 7, 8
1.4 #3, 4, 6, 9

HW#3
2.1 #2, 5, 7, 8
2.2 #1, 2, 4, 6, 8

HW#4
2.4 #3, 5, 6, 7, 13
2.5 #1, 2, 3, 4, 5

HW#5
2.6 #1, 2, 3, 4, 7, 8

HW#6
3.1 #3, 4, 5, 7, 10, 11, 12, 13
3.2 #1, 2, 3, 7, 8, 9, 10, 11

HW#7
3.3 #1, 2, 3, 13, 14, 15
3.5 #1, 4, 7, 8

HW#8
3.6 #1, 4, 5, 6, 11, 12
4.1 #1, 2, 3, 6, 8, 9, 10, 11, 12

HW#9
4.2 #1, 4, 5, 6, 10, 12
4.3 #2, 3, 4, 9, 11, 13, 14

HW#10
4.5 #1, 2, 3, 4, 5, 8