1 Executive Summary
Graduate student Radha Krishna Ganti presented his work “Interference and Outage in Clustered Wireless Ad Hoc Networks” in class.

2 Introduction
1. Consider PPP on $\mathbb{R}^2$ and path loss exponent $\alpha > 2$.
   - Interference can be written in this case as
     \[ I(z) = \sum_{x \in \varphi} h_x g(x - z), \quad g(x) = ||x||^{-\alpha} \] (1)
   - The moment generating function is
     \[ \mathcal{L}_{I_{\varphi}(z)}(s) = \exp \left( -\lambda s^{2/\alpha} \mathbb{E}[h^{2/\alpha}] c(\alpha) \right) \] (2)
     - Does not depend on $z$;
     - Heavytailed;
     - Effect of fading only depend on $\mathbb{E}[h^{2/\alpha}]$.

2. Regular $\leftarrow$ PPP $\rightarrow$ Clustered.

3 Cluster Processes
1. Start with a parent process $\varphi_P$, then for every point, replace with a finite daughter process
   \[ \varphi = \bigcup_{x \in \varphi_P} N_x \] (3)
   where $N_x = N_i + x$ and $N_i$’s are all i.i.d.. $N_0$ is the representative daughter cluster.

2. Homogeneous-independent clustering: $\varphi_P \sim \text{PPP}(\lambda) + \text{homogeneous and independent daughter process} = \text{Neyman-Scott cluster}$.

3. $N_0$: Finite point process $\varphi_0$, $\varphi_0(\mathbb{R}^D) < \infty$ almost sure. Examples:
   - Binomial PP;
   - PPP($\lambda$) is not finite, but the following are finite: 1) $\varphi|_A$ for compact $A$ is finite; 2) can change intensity $\lambda(x) = \text{two dimensional Gaussian (} \int \lambda(x) < \infty)$.
4 Analysis

1. Let $\mathbb{E}[N_0(\mathbb{R}^2)] = \bar{c}$, $N_0(\mathbb{R}^2) \sim \text{Poi}(\bar{c})$ and $f(x) : \mathbb{R}^2 \rightarrow \mathbb{R}$, then $\lambda(x) = \bar{c} f(x)$ and $N_0 \sim \text{PPP}(\bar{c} f(x))$.

2. Let $v(x) : \mathbb{R}^2 \rightarrow \mathbb{R}$ and $0 \leq v(x) \leq 1$:

$$G(v) = \mathbb{E} \left[ \prod_{x \in \varphi} v(x) \right]$$

$$= \mathbb{E} \left[ \prod_{x \in \varphi} \prod_{y \in N_x} v(y) \right]$$

$$= \mathbb{E} \left[ \prod_{x \in \varphi} \mathbb{E} \left[ \prod_{y \in N_x} v(y) \right] \right]$$

$$= M \left( \int_{\mathbb{R}^2} f(y-x)v(y)dy \right)$$

where $M(z) = \exp (-\bar{c}(1-z))$. Hence

$$G(v(x)) = \exp \left( -\lambda \int_{\mathbb{R}^2} \left\{ 1 - M \left( \int_{\mathbb{R}^2} f(y-x)v(y)dy \right) \right\} dx \right)$$

(5)

3. **Theorem 4.1** Let $0 \leq v(x) \leq 1$. The conditional geometry functional for a Poisson cluster process with $N_0(\mathbb{R}^2) \sim \text{Poi}(\bar{c})$ and each point is distributed with density $f(x)$ is

$$\tilde{G}(v) = G(v) \int_{\mathbb{R}^2} G_c(v(\cdot - y))f(y)dy$$

(6)

where $G_c(v(x)) = M(\int_{\mathbb{R}^2} v(x)f(x)dx)$.

4. **Lemma 4.2** For a Neyman-Scott cluster process

$$P_0^1 = P \ast \tilde{\Omega}_0^1$$

(7)

where $(\varphi = N_0)$

$$\tilde{\Omega}_0^1(y) = \frac{1}{\bar{c}} \mathbb{E} \left[ \sum_{x \in N_0} 1_{y \in (\varphi \setminus \{x\})} \right]$$

$$= \frac{1}{\bar{c}} \mathbb{E} \left[ \sum_{x \in N_0} 1_{y \in (\varphi \setminus \{x\})} \right]$$

**Campbell-Mecke**

$$= \frac{1}{\bar{c}} \int_{\mathbb{R}^2} \int_N 1_{y \in (\varphi \setminus \{x\})} \Omega_x^1(d\varphi) \bar{c} f(x)dx$$

$$= \int_{\mathbb{R}^2} \int_N 1_{y \in (\varphi \setminus \{x\})} \Omega_x^1(d\varphi) f(x)dx$$

$$= \int_{\mathbb{R}^2} \int_N 1_{y \in \varphi} \Omega(d\varphi) f(x)dx$$

$$= \int_{\mathbb{R}^2} \Omega(y_x) f(x)dx$$

(8)
5. From this we have
\[
E^t_0(\prod_{x \in \varphi^T} v(x)) = \int_N \int_N \prod_{x \in \varphi \cup \psi} v(x)P(d\varphi)\tilde{\Omega}_0^t(d\psi)
\]
\[
= \int_N \prod_{x \in \varphi} v(x)P(d\varphi)\int_N \prod_{x \in \psi} v(x)\tilde{\Omega}_0^t(d\psi)
\]
(9)

where
\[
\int_N \prod_{x \in \psi} v(x)\tilde{\Omega}_0^t(d\psi) = \int_{\mathbb{R}^2} \int_N \prod_{x \in \psi} v(x)\Omega(d\psi_y)f(y)dy
\]
\[
= \int_{\mathbb{R}^2} \int_N \prod_{x \in \psi} v(x-y)\Omega(d\psi)f(y)dy
\]
\[
= \int_{\mathbb{R}^2} G_c(\cdot-y)f(y)dy
\]
(10)

6. Thus
\[
G(v) = \exp \left( -\lambda_p \int_{\mathbb{R}^2} \left[ 1 - M((v \ast f)(x)) \right] dx \right) \int_{\mathbb{R}^2} M((v \ast f)(y))f(y)dy
\]
(11)

7. **Theorem 4.3** When the Tx’s form a P.C.P with $\lambda_P$ as parent process intensity, $\bar{c}$ are daughter process intensity and $g(x) = ||x||^{-\alpha}$ ($\alpha > 2$), then the interference $I(z) = \sum_{x \in \varphi} h_x g(x - z)$ is heavy tailed with parameter $2/\alpha$. ($h_x$: fading with $\mathbb{E}[h_x] = 1$.)

8. sketch of proof:
Recall $\lim_{s \to 0} sL_f(s) = \lim_{x \to \infty} f(x)$.

Tauberian Thm: Let $X$ be RV. If $1 - L_x(s) \sim c_1 s^k$, $s \to 0$, $k < 1$, then $P(x \geq y) \sim \frac{c_1 y^{-k}}{1^{-k}}$, $y \to \infty$.

The conditional Laplacian transform of interference is
\[
L_{I_{\varphi}(z)}(s) = E^t_0 \exp(-s \sum_{x \in \varphi} h_x g(x - z))
\]
\[
= E^t_0 \left[ \prod_{x \in \varphi} \exp(-sh_x g(x - z)) \right]
\]
\[
= E^t_0 \left[ \prod_{x \in \varphi} L_h(\rho(x - z)) \right]
\]
(12)
\[
= G(\mathcal{L}_h(\rho(\cdot - z)))
\]
(13)

and then all that remains is to show
\[
G(\mathcal{L}_h(\rho(\cdot - z))) \sim y^{-2/\alpha} \pi \mathbb{E}[h^{2/\alpha}] \rho(2)(z) \lambda^{-1}
\]
(14)

using the previously derived $G(v)$ for Neyman-Scott process in (11).
5 Main Take-Aways

- Palm distribution and Campbell can be used to analyze properties related to complex point process;
- Useful properties of interference can be obtained from transform domain.

6 Sources