Superposition Coding Strategies: Design and Performance Evaluation

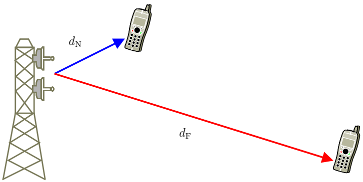
Summary

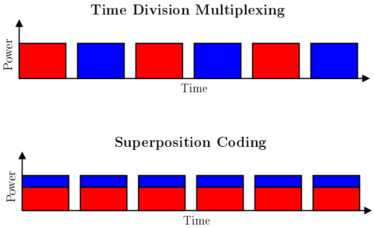
These slides summarize two papers:

- Part 1 (A two-user superposition-coded system prototype): (Vanka12a) S. Vanka, S. Srinivasa, Z. Gong, P. Vizi, K. Statmatiou, and M. Haenggi,"Superposition Coding Strategies: Design and Experimental Evaluation", IEEE Trans. Wireless. Comm., 2012. Accepted.
- Part 2 (Coding gain from practical superposition codes): (Vanka12b) S. Vanka, S. Srinivasa, and M. Haenggi, "A Practical Approach to Strengthen Vulnerable Downlinks using Superposition Coding", in ICC 2012.

A Two-User SC System Prototype [Vanka12a]

What is Superposition Coding?





- BS sends information to *two* users N (near) and F (far) $\leftrightarrow$ Communicating over a **Broadcast Channel** (BC)
- BS has full CSI: *Gaussian* BC [Cover06][1]
- BS has no CSI: *Fading* BC [Zhang09][2]
- Capacity achieved by **Superposition Coding** (SC) and **Successive Decoding** (SD)

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[1] T. Cover, and J. A. Thomas, Elements of Information Theory, 2nd ed., John Wiley & Sons, Inc., 2006.

[2] W. Zhang, S. Kotagiri, J. N. Laneman, On Downlink Transmission Without Transmit Channel State Information and With Outage Constraints," IEEE Trans. IT, Sept. 2009.

The Team Effort

The Team: Sundaram Vanka, Sunil Srinivasa, Peter Vizi, Zhenhua Gong, Kostas Stamatiou

Contributions

1. Superposition coding techniques that work for small to medium-sized packets ($100 - 500$ bytes).
2. Designed the complete SC physical layer
3. Developed the reference Matlab model and provided extensive assistance in C code integration, testing and debugging
4. Proposed practical approaches to leverage the coding gain from superposition-based multiuser channel codes
5. Designed efficient experiments that measure performance gains from SC

SC with Finite Blocklength Channel Codes

- IT result **existential**, not constructive
- Need to understand how SC works with well-known codes
- Identify key practical issues that arise in its implementation

Definition (Code Library)

A collection of $M < \infty$ encoder-decoder function pairs with spectral efficiencies (aka "rates") $r_1 < r_2 \cdots < r_M$

Definition: Packet Error Rate (PER)

The probability of codeword decoding error

Definition ($\epsilon-$Feasible on a Link)

A code with rate r is $\epsilon-$feasible on a link if the PER of a codeword encoded at r is no greater than ϵ

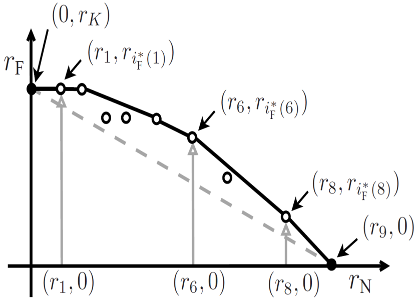
Achievable Rates with a Code Library

- Need (γ_n, γ_f) to specify BC
- Set γ_n s.t. $r_n = r_M$ is feasible
- Set γ_f s.t. $r_f = r_K < r_M$ is feasible
- For $m = 1, 2, \ldots, M$

$$\max_{\{r_1,\ldots,r_M\}} r_f$$

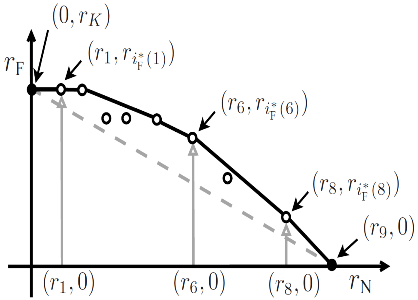
s.t. (r_f, r_k) is jointly ϵ-feasible

- Transmission scheme decides joint feasibility!



Achievable Rates with a Code Library

- Convexify solution set $\{(r_k, r_i^*(k)) : k \in [M]\}$ to get the rate-region boundary
- Solution requires finding desired Tx power split for SC
- α_k: N's share for rate r_k
- $\bar{\alpha}_k \triangleq 1 - \alpha_k$: F's share for rate $r_i^*(k)$



The BICM Code Library

- Pairs powerful binary codes with well-known modulation techniques [Caire98][3]
- Combines the advantages of signal space coding with well-known binary codes
- Flexible and easy to implement
- Coding technique in DSL, Wi-Fi, WiMAX...

In our library:

- Modulations: BPSK, QPSK, 16-QAM
- Channel codes:
  - Standard const. length 7 rate-1/2 convolutional code with generator matrix [133,171]
  - Rates 2/3, 3/4, 5/6 punctured versions of mother code

[3] G.Caire, G. Taricco and E.Biglieri, "Bit-Interleaved Coded Modulation", IEEE Trans. IT, May 1998.

SC-BICM Rate Region in the High Reliability Regime

Can approximate PER as a function of SNR: PER at N:

- Pick $\gamma_n \gg \gamma_{\mathrm{f}}$ so N can almost certainly decode F's packet
- If F's signal is *perfectly cancelled* at N, N decodes its packet from the matched filter outputs

$$Y_{\mathrm{n}}(m) = \alpha X_{\mathrm{n}}(m) + W_{\mathrm{n}}(m),\ m \in [L]$$

- For this p2p case [Caire98][4]

$$\mathrm{PER}_{\mathrm{n}} \lessapprox NWQ\left(D_{\mathrm{n}}\sqrt{\frac{C_{\mathrm{n}}\gamma_{\mathrm{n}}}{2}}\right), \quad \gamma_{\mathrm{n}} \to \infty.$$

- N: payload size, W: # free distance error events D_{n} : constellation min. distance, C_{n} : free distance of N's conv. code

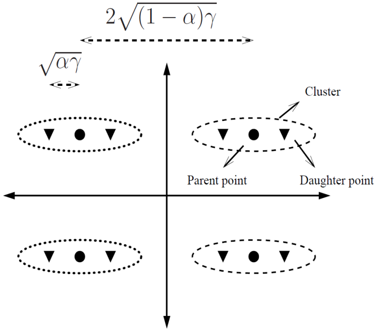
[4] G. Caire and E Viterbo, "Upper Bound on the frame error probability of terminated trellis codes", IEEE Comm. Letters, Vol. 2, pp. 2-4, 1998

SC-BICM Rate Region in the High-Reliability Regime

F decodes its packet from

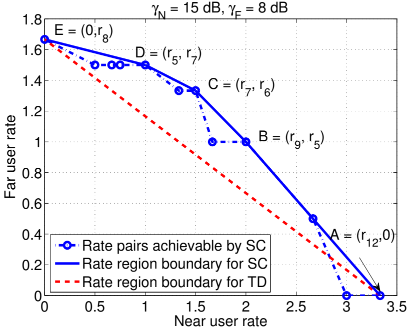
$$Y_{\mathrm{f}}(m) = \bar{\alpha} X_{\mathrm{f}}(m) + \alpha X_{\mathrm{n}}(m) + W_{\mathrm{n}}(m), \; m \in [L]$$

- Discrete interference $\Longrightarrow$ **Symbol Clusters**
- ML demodulation for F's symbols: Find the nearest **cluster**
- Expression similar to N, but "Constellation Min. Distance" = Closest cluster separation



Rate Region for "Practical" PERs

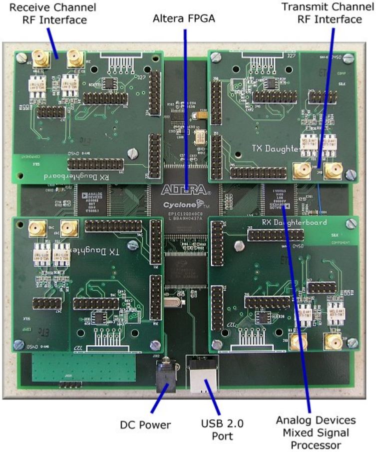
- Problem: Bound on PER_f too loose at practical PERs ($\lesssim 0.1$), esp. for small intercluster separations
- Numerically find the rate region



$\gamma_\mathrm{n} = 18$ dB, $\gamma_\mathrm{f} = 8$ dB, $\epsilon = 0.1$

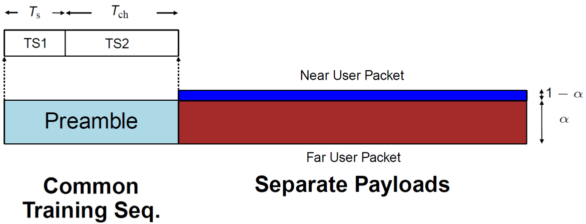
Point	α	$(r_\mathrm{n}, r_\mathrm{f})$
A	1	(3.33,0)
B	0.21	(2,1)
C	0.13	(1.5,1.33)
D	0.06	(1,1.5)
E	0	(1.67,0)

Towards an SC-BICM Prototype on a USRP Platform



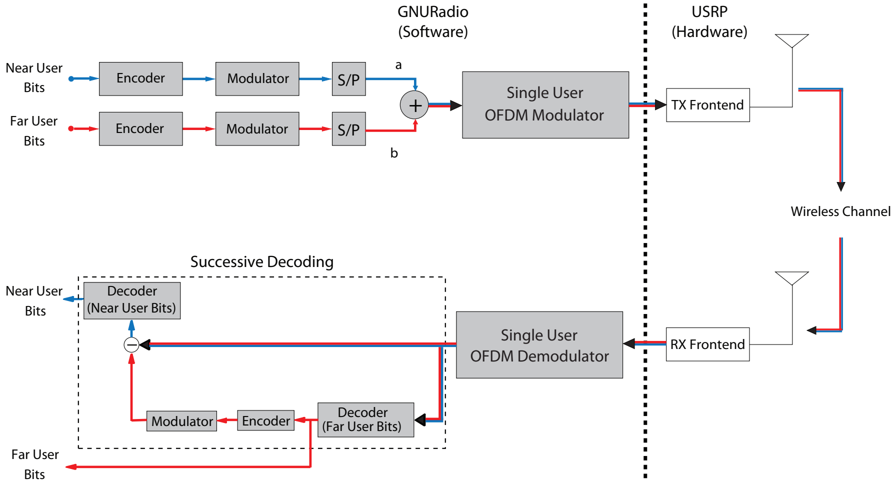
- Flexible
  - Multi-Protocol
  - Multi-Band
- Board has FPGA, DAC/ADC, RF Frontends
- USB 2.0 Interface with Linux PC
- Software-based DSP on GNURadio
  - Open Source
  - In-built USRP drivers

Frame Structure



- TS1: Packet acquisition, timing and frequency sync. Duration $T_s = 48\mu s$
- TS2: Channel estimation. Duration $T_{ch} = 34\mu s$

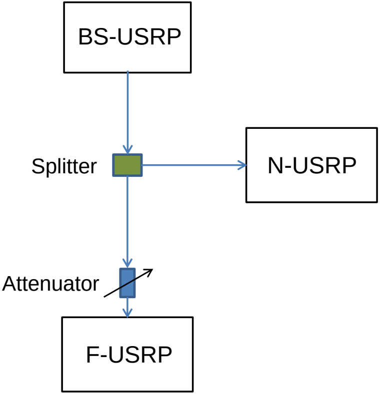
Top-level Block Diagram



Emulating a Gaussian BC

Step 1: Only BS→N active

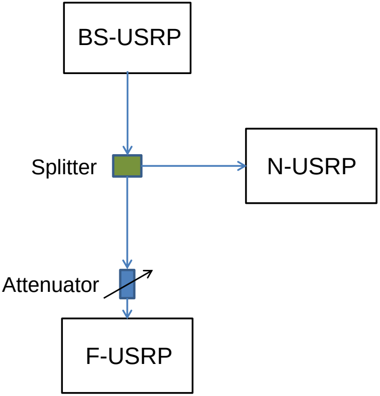
- Increase BS power P, measure PER for highest rate
- Find P_n = Smallest P s.t. highest rate is feasible
- Note down largest backoff β_k from P_n for rate r_k to be feasible $k \in [M]$
- β_k = initial guess for α_k
- Special case: PER curves (PER for all P, r_k)



Emulating a Gaussian BC

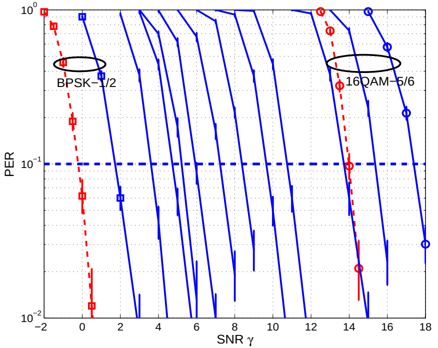
Step 2: Only BS→F active

- Fix target rate r_K for F
- With BS power = P_n, choose largest attenuation a_f s.t. r_K is feasible
- Power control granularity 0.5 dB, attenuator granularity 1 dB



Point-to-Point PER Curves

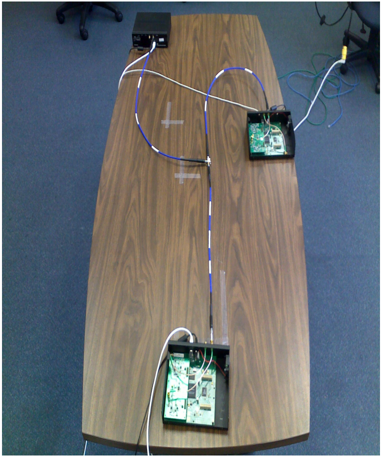
- $\text{SNR} = \frac{\text{Preamble power}}{\text{Noise power}}$
- Digitally measured for fixed amplifier gain setting
- Worst-case implementation loss ≈3.5 dB (16-QAM, rate-5/6 10% PER)



The Rate Region Experiment

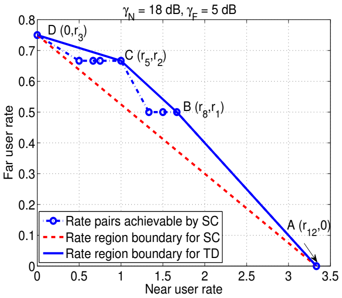
Initialize $r_{\text{prev}} = r_K$
For $k = 1, \ldots, M$:
Step 1: $\alpha_k = \beta_k$; $r_f(k) = r_{\text{prev}}$
Step 2: Measure PER for $r_k, r_f(k)$
Step 3:

- N not feasible: Increase α_k, go to 2)
- N but not F: $r_f(k)$ = Next lowest library rate, go to 2)
- Both N & F: k^{th} solution found. $r_{\text{prev}} = r_f(k)$

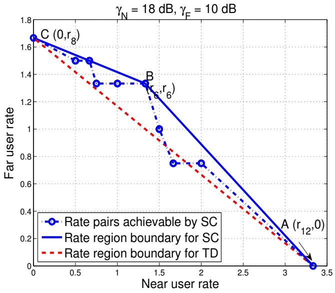


The Choice of F

$P_n = -43$ dBm, $a_f = 9$ dB.
F's single user rate: BPSK-3/4



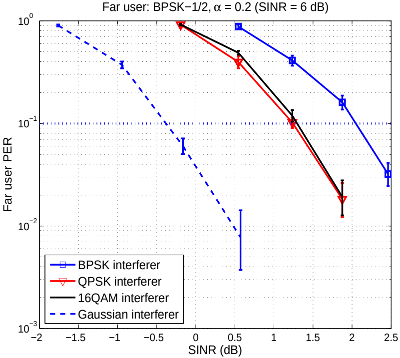
$P_n = -43$ dBm, $a_f = 5$ dB.
F's single user rate: QPSK-5/6



F is "too close" : Not enough disparity. "Too far": no codes to support rate
Sweet spot appears to be between QPSK-5/6 and BPSK-3/4

Interference from N's Symbols at F

High SIR regime: $\alpha \ll 1 - \alpha$

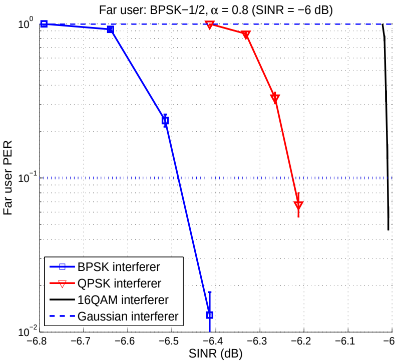


SIR = 6 dB

- Fix F's rate (in this case to BPSK-1/2)
- Compare equal-power Gaussian, BPSK, QPSK and 16-QAM interferers
- BPSK > QPSK/16QAM > Gaussian

Interference from N's Symbols at F

Low SIR regime: $\alpha \gg 1 - \alpha$

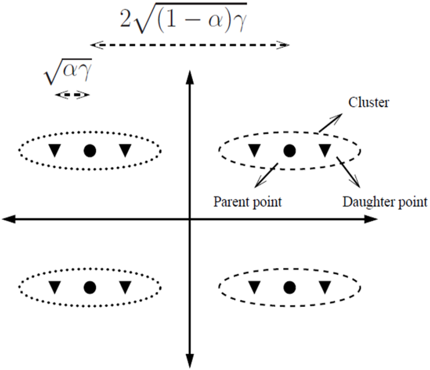


SIR = -6 dB

- Now Gaussian > 16QAM > QPSK > BPSK
- Situation reversed!

Why does this happen?

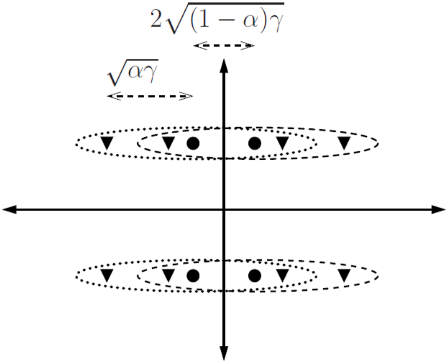
High SIR regime



- Small α: min. distance determined by **cluster separation**
- For a given interference power BPSK perturbs **all** parent points to the max. extent
- Denser interferer constellations place fewer points on the edges

Why does this happen?

Low SIR regime

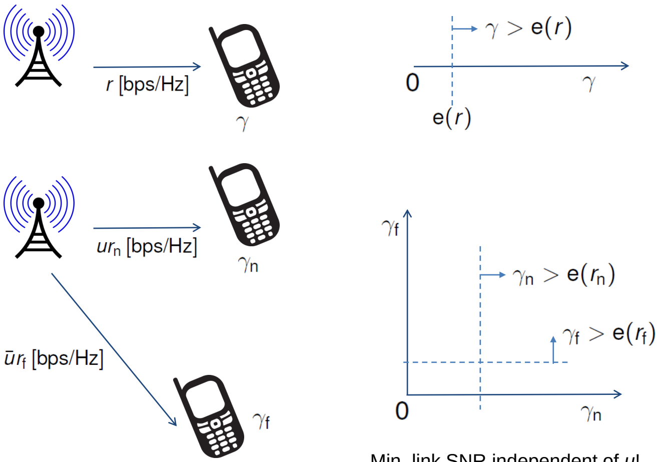


- Large α: min. distance determined by **cluster density**
- For a given interference power BPSK causes the least dense clusters!
- Denser interferer constellations make the problem worse

Conclusion: Must be careful in using the Gaussian approximation in SC systems

The Coding Gain from Practical Superposition Codes [Vanka12b]

Orthogonal Coding on the BC



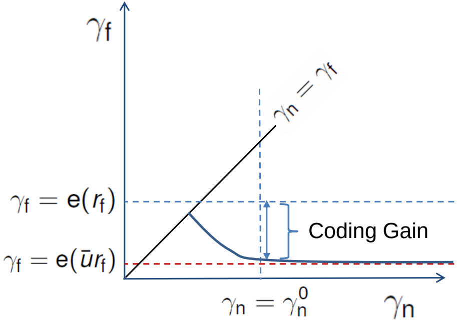
Min. link SNR independent of u!

SC as a Superior Multiuser Channel Code

Constraining $(ur_n, \bar{u}r_f)$ to be feasible with SC

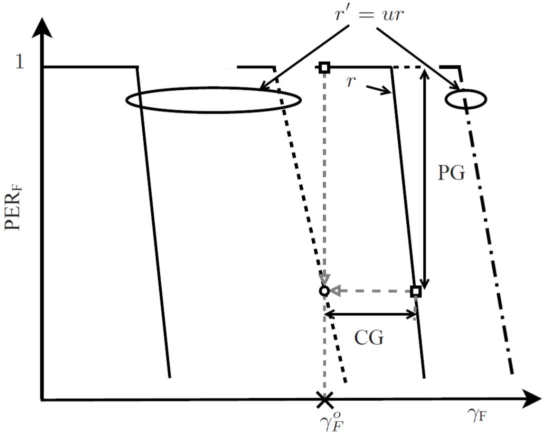
$$\gamma_f^*(\gamma_n; ur_n, ur_f) = \frac{\gamma_n e(\bar{u}r_f)}{\gamma_n - e(ur_n)(1 + e(\bar{u}r_f))}.$$

- Packets encoded **exactly** at $(ur_n, \bar{u}r_f)$
- For each u, require $\alpha > e(ur_n)/\gamma_n$ with SC
- Coding gain increases with γ_n $\Leftrightarrow$ pair F with high-SNR N!



Performance Gain in the Finite Blocklength Regime

- Non-zero decoding error probability or Packet Error Rate (PER) ϵ
- At PER $= \epsilon$, typical packet req. $\frac{1}{1-\epsilon}$ to reach F
- Easy to measure the Reliability Gain RG $= \frac{1-\epsilon_{SC}}{1-\epsilon_{TD}}$



SC with Finite Blocklength Channel Codes

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- Need to understand how SC works with well-known codes
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Definition: Packet Error Rate (PER)

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Definition ($\epsilon-$feasible on a link)

A code with rate r is $\epsilon-$feasible on a link if the PER of a codeword encoded at r is no greater than ϵ

SC with a Finite Channel Code Library

Important special case: N close to BS, F at cell-edge.

- $r_n = r_M$, $\bar{u}r_{\mathrm{f}}$ is small (can set to r_1)
- Set $ur_M = r_k$, so that $u_k = r_k/r_M, r_f = r_1/\bar{u}_k,\ k \in \{1, \dots, M\}$
- If library has codes $r_a < r_{\mathrm{f}} < r_b$, time-share between r_a and r_b

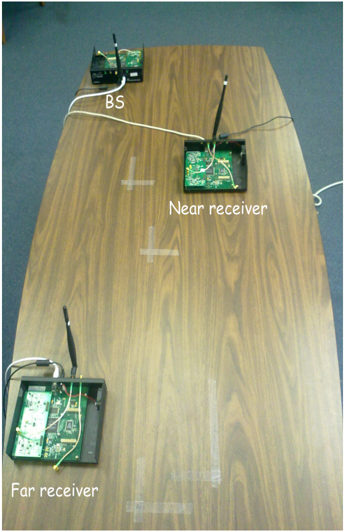
Compare SC using (r_k, r_1) with TD using $(r_M, r_1/u_k)$, for $k = 1, \dots, M$.

Setting up the BC

P : BS power, α: N's share

$$\begin{aligned} \gamma_{\mathrm{n}} &\propto \alpha P &\triangleq P_{\mathrm{n}} \\ \gamma_{\mathrm{f}} &\propto \bar{\alpha} P &\triangleq P_{\mathrm{f}} \end{aligned}$$

Rate r is reliable $\leftrightarrow$ PER $\lessapprox 0.1$
For $k = 1, \ldots, M$:
SC Step:
Step 1: Set $P_{\mathrm{n}} = 0$ & $\uparrow P_{\mathrm{f}}$ s.t. r_1 is reliable
Step 2: $\uparrow P_{\mathrm{n}}$ s.t. r_k is reliable
Step 3: Keeping $P_{\mathrm{n}}/P_{\mathrm{f}}$ constant $\uparrow P_{\mathrm{f}}$ s.t. r_1 is reliable
TD Step: Find $\mathrm{PER}_{\mathrm{f}}$ at BS power $P_{\mathrm{n}} + P_{\mathrm{f}}$ and rate r_1/u_k



Experimental Results

- $\bar{u}r_{\mathrm{f}} = 0.5$ [bps/Hz], SC always uses BPSK-1/2
- 16QAM-5/6 always feasible at N with full power
- SC adjusts N's power and code to provide the same rate as TD

SC			**TD**		
γ_{f} (dB)	SIR (dB)	PER	$\bar{u}$	TD peak rate	PER
8.8	1	7%	0.1	Infeasible	N/A
7.4	1.95	6%	0.2	2.5	100%
5.5	5	3%	0.4	1.25	75%
4.3	5	5%	0.45	1.11	38%
2.7	6	6%	0.8	0.63	37%
2.6	7.5	5%	0.85	0.59	29%

Conclusions

- Experimentally demonstrated a practical approach to exploit superposition codes
- Specific decoding strategies such as demod-and-decode can render the Gaussian approximation for inter-user interference inaccurate
- Signal superposition opens up new possibilities for link-layer scheduling policies [Vizi11][5]

[5] P. Vizi *et al.*, "Scheduling using Superposition Coding: Design and Software Radio Implementation", IEEE Radio and Wireless Week, Jan. 2011.