SHOT NOISE MODELS FOR THE DUAL PROBLEMS OF COOPERATIVE COVERAGE AND OUTAGE IN RANDOM NETWORKS

Jagadish Venkataraman, Martin Haenggi and Oliver Collins
Department of Electrical Engineering, University of Notre Dame
Notre Dame, IN
email: {jvenkata, mhaenggi, ocollins}@nd.edu

ABSTRACT

The performance analysis of ad hoc networks requires the characterization of the network's self-interference. However, thus far, the exact distribution of the interference has been analytically tractable only for a path loss exponent of 4. Further, conventional power law decay models for signal propagation have a singularity at the origin that causes all the moments of the network self-interference to diverge, resulting in no useful insight into the statistics of the interference. In this paper, we employ a 2D shot noise process with a stochastic power law impulse response function together with a bounded path loss to model the interference in large wireless networks, where the nodes are distributed according to a Poisson point process. We further show that this interference problem and the problem of coverage in cooperative sensor networks are duals of each other — they can be solved using the same shot noise-based approach.

I. INTRODUCTION

Large, self-organizing wireless networks, variously referred to as ad hoc or sensor networks, have recently attracted a lot of attention [1][2]. Such networks are typically modeled as a randomly deployed set of nodes that compete for common network resources, potentially interfering with every other transmitter in the network. An accurate statistical characterization of the interference is a prerequisite for the performance analysis of such networks. The most commonly adopted path loss model for signal propagation in literature is one where the signal strength falls off as a decaying power law of the distance of transmission. For this model, however, when the nodes are distributed in $\mathbb{R}^2$ according to a Poisson point process (PPP), the exact distribution of the interference is analytically tractable only for a path loss exponent of 4 for the additive white Gaussian noise (AWGN) [4] and Rayleigh fading channels [5]. Path loss exponents, however, can assume values in a continuous range. In such cases, even if the exact distribution of the interference is unavailable, the first few moments can prove useful in obtaining performance bounds. Unfortunately, the singularity at the origin for the power law decay model causes all the moments of the interference to diverge for all decay exponents [6], thereby not providing any meaningful insight.

The objective of our paper is to propose the use of a 2D shot noise process with a stochastic impulse response function to model the network self-interference over a wide range of path loss exponents. The stochastic nature of the process can be used to model random channel flucutations, variable transmission powers etc. Further, we apply a bounded modification prevalent in literature to the decaying power law in order to eliminate the singularity at the origin, so that the signal propagation model becomes physically meaningful for arbitrarily small distances as well. This bounded shot noise model can be successfully applied to both finite and infinite networks to derive performance measures. To illustrate this concept, we use this shot noise model to solve the dual problems of cooperative coverage and outage in large wireless networks.

The first part of this paper models coverage in cooperative sensor networks. Coverage is a fundamental issue in wireless sensor networks [7][8] and is a measure of how well a target point or region is covered by a given network of sensors. References [9][10] introduce the notion of *information coverage* which is based on the assumption that distributed sensing among nodes is possible. In this paper, we use the shot noise interference model to derive bounds on the sensor node density required to cover a target region when the sensing relies on wave propagation laws equal to those which guide signal propagation in wireless ad hoc networks. We, therefore, derive these bounds for 2 cases - one where sensing occurs over a medium that introduces additive noise and large scale path loss and the other where the medium additionally introduces small scale fading as well. This analysis also presents a limiting lower bound by using a Gaussian approximation in the limit when infinite processing power is available for node cooperation.

In the latter part of this paper, we consider the dual to the coverage problem - the outage problem. Given an outage constraint, we use the shot noise model to derive the transmission capacity [11], i.e., the maximum allowed intensity of simultaneously transmitting nodes in an ad hoc network setting with peer-to-peer transmission. We present this metric for a multihop cooperative transmission scheme operating under a *threshold link model* [12]. The conclusion follows.

II. RELATED WORK

Baccelli *et al.* [13][14] study the effect of interference on connectivity in random ad hoc networks by constructing a signal to interference ratio graph. Reference [13] also considers a bounded modification to the path loss attenuation model but does not incorporate the modified function in the shot noise model to study outage. References [15][16] present cooperative communications models in a random ad hoc network. However, [15] only investigates the diversity gain for an additive noise channel and does not include the effects of self-interference. Similarly, [16] focuses on a power allocation strategy in a multihop network that does not include the effects of self-interference. Reference [17] talks about duality of connectivity and coverage, but again does not incorporate interference. The information coverage models used in this paper are derived from [9][10], which present lower bounds on the node density for the AWGN channel. Finally, the transmission capacity ideas used in this paper are derived from [11].

III. SYSTEM MODEL

The system model is as follows:

- Nodes are distributed in $\mathbb{R}^2$ according to a homogeneous Poisson point process (PPP) Π of density λ.
- Nodes make independent decisions on whether to transmit or listen, and each node is equipped with an omnidirectional antenna. In any given time slot, a node transmits with probability α so that the set of all transmitting nodes forms a PPP Π_α of intensity $\lambda\alpha$.
- All transmissions are nearest neighbor transmissions. Transmitters can all use either a constant transmission power or choose to draw their powers independently from the same known distribution.
- We present the performance results for an "average" network realization, i.e., by averaging over all possible network realizations.
- Each node generates information packets of fixed length, and all transmissions are assumed to be synchronized slot-wise (slotted ALOHA).
- The interference model assumes that each transmitting node contributes to the interference seen at any receiving node. For transmission over a distance r, the power law decay is given by $r^{-\eta}$, where η is the decay exponent.
- The total interference seen at a typical receiver node is $\mathcal{I} = \sum_{i=1}^{n} \mathcal{I}_i$, where the summation is over all transmitting nodes and $n \to \infty$ for infinite networks. In order to keep $\mathbb{E}[\mathcal{I}]$ finite, it is necessary (but not sufficient) that the path loss exponent $\eta > 2$ (Maclaurin and Cauchy criterion) [18].
- An Outage occurs when the signal-to-interference ratio (SIR) γ is less than a certain threshold Θ, i.e., $\mathcal{O} = \mathbb{P}(\gamma < \Theta)$. The background noise power, σ_n^2, is assumed to be much smaller than the network self-interference and is ignored in the outage analysis. In the Rayleigh fading case, noise and interference can be treated independently [19], so the noise simply yields an additional factor in the reception probability.

IV. SHOT NOISE BACKGROUND

Shot noise results when a memoryless linear filter is excited by a train of impulses derived from a homogeneous PPP with arrival rate μ [20]. The impulse response of the filter, $f(t)$, can assume different shapes like a triangle, rectangle, decaying exponential, decaying power law etc. More generally, the impulse shapes can be stochastic and may be randomly chosen from a family of shapes, $f(\mathsf{k}, t)$, with a random variable k. In this paper, we consider the stochastic impulse response model. Specialization to the deterministic case is trivial. The shot noise amplitude is given by

$$\mathcal{I}(t) = \sum_j f\left(\mathsf{k}_j, t - \mathsf{t}_j\right). \tag{1}$$

The arrival times[1] $\{\mathsf{t}_j\}$ are Poisson with rate μ and $\{\mathsf{k}_j\}$ are iid random variables drawn from a common distribution and independent of $\{\mathsf{t}_j\}$. All impulse functions $f(\mathsf{k}, t)$ are assumed to be causal and integrable over $-\infty < t < \infty$ so that the series in (1) converges in distribution. As the driving rate μ increases, under some weak conditions on the characteristic time duration of the impulse response function, the amplitude distribution of shot noise approaches a Gaussian distribution [20][21]. This is true for many impulse response functions. However, for a decaying power law, the amplitude distribution does not tend to a Gaussian for any value of μ [6]. In this paper, we are interested only in the decaying power law shot noise process to model the large scale path loss in wireless networks.

The Laplace transform of $\mathcal{I}(t)$, $\Phi(s) = \mathbb{E}\left[e^{-s\mathcal{I}(t)}\right]$, is obtained as follows. Let the k_j's be drawn from a discrete set $\{K_1, K_2, \cdots\}$ with probabilities $p_1, p_2, \cdots$. The shot noise process can then be written as the sum of independent shot noise processes, i.e., $\mathcal{I}(t) = \mathcal{I}_1(t) + \mathcal{I}_2(t) + \cdots$, where $\mathcal{I}_i(t)$ is the sum of deterministic impulse responses with a Poisson arrival and a constant parameter K_i, i.e.,

$$\mathcal{I}_i(t) = \sum_j f(K_i, t - \mathsf{t}_j)\delta\left(\mathsf{k}_j - K_i\right) \tag{2}$$

where $\delta(\cdot)$ is the discrete unit impulse. Since the I_i are independent,

$$\begin{aligned} \Phi(s) &= \mathbb{E}\left[e^{-s(\mathcal{I}_1(t)+\mathcal{I}_2(t)+\cdots)}\right] \\ &= \Phi_1(s)\Phi_2(s)\cdots. \end{aligned} \tag{3}$$

For a deterministic impulse response, it is a well-known result [22] that

$$\Phi_i(s) = \exp\left\{-\mu p_i \int_{-\infty}^{\infty} \left(1 - \exp\left[-sf(K_i, t)\right]\right) dt\right\}, \tag{4}$$

where μp_i is the arrival rate of impulse responses with parameter K_i. After evaluating every $\Phi_i(s)$ using (4), $\Phi(s)$

[1] Time is just a hypothetical variable motivated from the study of actual noise phenomena. This is replaced with distance in the following section.

is given by

$$\Phi(s) = \exp\left\{-\mu \sum_i p_i \int_{-\infty}^{\infty} \left(1 - \exp\left[-sf(K_i, t)\right]\right) dt\right\}$$
$$= \exp\left\{-\mu \int_{-\infty}^{\infty} \mathbb{E}_{\mathsf{k}} \left(1 - \exp\left[-sf(\mathsf{k}, t)\right]\right) dt\right\}, \quad (5)$$

where $\mathbb{E}_{\mathsf{k}}[\cdot]$ is expectation w.r.t k. Though k is assumed to be drawn from a discrete distribution, the above expression can be extended to continuous distributions using limiting arguments so that (5) is true in general.

A decaying power law impulse response function is given by $f(\mathsf{k}, t) = \mathsf{k}t^{-\eta}$, whose Laplace transform, after simplification using integration by parts, is given by [6]

$$\begin{aligned}\Phi(s) &= \exp\left\{-\mu \int_A^B \mathbb{E}_{\mathsf{k}} \left[1 - \exp\left(-s\mathsf{k}t^{-\eta}\right)\right] dt\right\} \\ &= \exp\Big\{\mu A \mathbb{E}_{\mathsf{k}} \left[1 - \exp\left(-s\mathsf{k}A^{-\eta}\right)\right] \\ &\quad - \mu B \mathbb{E}_{\mathsf{k}} \left[1 - \exp\left(-s\mathsf{k}B^{-\eta}\right)\right] \\ &\quad + \mu \mathbb{E}_{\mathsf{k}} \left[(s\mathsf{k})^{1/\eta} \Gamma\left(1 - 1/\eta, s\mathsf{k}A^{-\eta}\right)\right] \\ &\quad - \mu \mathbb{E}_{\mathsf{k}} \left[(s\mathsf{k})^{1/\eta} \Gamma\left(1 - 1/\eta, s\mathsf{k}B^{-\eta}\right)\right]\Big\}, \quad (6)\end{aligned}$$

where $\Gamma(a, x) = \int_x^{\infty} t^{a-1} e^{-t} dt$ is the incomplete Gamma function. For the special case when $A = 0$ and $B = \infty$, (6) reduces to

$$\Phi(s) = \exp\left(-\mu \mathbb{E}_{\mathsf{k}} \left[\mathsf{k}^{1/\eta}\right] \Gamma\left(1 - 1/\eta\right) s^{1/\eta}\right). \quad (7)$$

This completes the description of the 1D shot noise process which can equivalently be used to model interference powers decaying with distance according to a power law where the arrival times are replaced with the node locations. Extending the Laplace transform in (6) to a 2D PPP is straightforward and the derivation is given in Appendix A. Intuitively, this derivation implies that if the ordered node distances of the interferers from the origin are originally $r_1, r_2, ...$ in a plane, then r_i^2 represent Poisson arrival times on a line with constant arrival rate $\pi\lambda$. Equivalently, the PPP can be projected on to r_i resulting in a non-homogeneous process in which the intensity of the transmitting nodes increases linearly as $\lambda\alpha r$ (follows from the Mapping Theorem [23]). Upon using this projection in (6) and assuming a uniform angle distribution for the node locations, we get back the expression derived in the appendix to within a constant.

The following section adapts this 2D shot noise process to model the interference in a random ad hoc network, which is then used to derive coverage and outage bounds.

V. INTERFERENCE MODELING

The distribution of the point process in $\mathbb{R}^2$ is unaffected by the addition of a transmitter node at the origin (by Slivnyak's Theorem [23]). Given this transmitter node, we consider a receiver at unit distance from this transmitter[2], shift the origin to this receiver node, and develop the interference model around this "typical" receiver node. This conditional distribution is sometimes referred to as the Palm distribution and since the network is homogeneous, the interference measure at the origin is representative of the interference seen by all other receiver nodes in the network.

The interference power seen by the receiver at the origin can be likened to the amplitude of the shot noise process described in Section IV. Let r_i be the distance of the i^{th} interferer to the origin. The path loss model is the decaying power law impulse response so that $f(\mathsf{k}_i, r_i) = \mathsf{k}_i r_i^{-\eta}$. The driving rate of the arrival process is modeled by the intensity of the transmitting nodes, i.e., $\mu = \lambda\alpha$. The total interference seen at the origin is given by

$$\mathcal{I} = \sum_{i \in \Pi} \mathsf{b}_i f(\mathsf{k}_i, r_i) = \sum_{i \in \Pi_\alpha} \mathsf{k}_i r_i^{-\eta}, \quad (8)$$

where $\{\mathsf{b}_i\}$ is iid Bernoulli with $\mathbb{P}(\mathsf{b}_i = 1) = 1 - \mathbb{P}(\mathsf{b}_i = 0) = \alpha$. For an AWGN channel, k_i is a constant. For a block Rayleigh fading channel, k_i is drawn from an exponential distribution with unit mean and remains constant over one transmission slot. There is also the possibility of the transmitters employing variable transmission powers, in which case k_i is drawn from the distribution for the transmission power.

A. *Decaying power law,* $\mathbf{A = 0, B = \infty}$

The Laplace transform for the decaying power law model $f(\mathsf{k}, r) = \mathsf{k}r^{-\eta},\ 0 \le r < \infty$, is obtained by evaluating (33) in the limit $A = 0$ and $B = \infty$ to be

$$\Phi(s) = \exp\left(-\pi\lambda\alpha \mathbb{E}_{\mathsf{k}} \left[\mathsf{k}^{2/\eta}\right] s^{2/\eta} \Gamma\left(1 - 2/\eta\right)\right). \quad (9)$$

Owing to the singularity at $r = 0$, however, the mean and variance of the interference obtained from this Laplace transform diverge. Nevertheless, this form for $\Phi(s)$ allows for some interesting observations. Notice that the Laplace transform is of the form $\Phi(s) = \exp\left[-(cs)^{2/\eta}\right]$, where c is a constant so that for all positive values of $\lambda\alpha$, the interference is a one-sided Lévy-stable (or α-stable) random variable with asymmetry of dimension $D = 2/\eta$ [24][3]. Similar to a Gaussian distribution, a Lévy-stable distribution has the property that the sum of two Lévy-stable random variables is another Lévy-stable random variable, whose distribution is of the same form as the individual random variables. Therefore, even when the intensity of the interferers is infinite, i.e., $\lambda\alpha \to \infty$, the form of the interference distribution remains the same. The conditions for the central limit theorem are violated as long as $0 < D < 1$, and the interference never converges to a Gaussian distribution. For $\eta \le 2$, $D \ge 1$ and the convergence criterion is satisfied. However, for infinite networks ($B = \infty$), the Maclaurin and Cauchy criterion is violated for these values of η, thus, resulting in a Gaussian

[2]Even if the transmitter-receiver distance is not unity, all distances in the network can be normalized by this distance so that the desired link always has unit distance. This does not affect the homogeneity of the PPP.

[3]This functional form is valid only for $\eta > 1$.

distribution with infinite mean which is of little practical significance.

In addition to the lack of convergence to a Gaussian for $\eta > 2$, when $A = 0$, the singularity at the origin for the decaying power law results in diverging moments for the interference. This motivates the modified path loss model presented in the following subsection.

B. *Bounded power law*

The decaying power law model is accurate if the transmitter or interferer is not too close to the receiver, but the model becomes physically meaningless for distances less than unity. Clearly, the transmit power is a natural bound on the received power since a wireless channel cannot amplify the signal. To avoid this scenario, we use the following bounded power law decay[4]

$$f(\mathsf{k}, r) = \begin{cases} \mathsf{k}, & r < 1 \\ \mathsf{k}r^{-\eta}, & r \geq 1. \end{cases} \tag{10}$$

The validity of this model is verified empirically in [25] for an indoor environment. This modification eliminates the singularity at $r = 0$ present in the original power law and, thus, provides a finite mean and variance for the total interference. We derive these by modeling the total interference caused by all the transmitters at the origin to be the sum of two terms $\mathcal{I}_1$ and $\mathcal{I}_2$, where $\mathcal{I}_1$ is the total interference caused by all transmitters within a distance of 1 from the origin and $\mathcal{I}_2$ is is the total interference power due to all transmitters at distances greater than 1. Since the nodes are distributed according to a PPP, $\mathcal{I}_1$ and $\mathcal{I}_2$ are independent. The Laplace transform for $\mathcal{I}_1$ can easily be obtained as $\Phi_1(s) = \exp\left(-\pi\lambda\alpha\mathbb{E}_{\mathsf{k}}\left[1 - e^{-s\mathsf{k}}\right]\right)$. The corresponding function for $\mathcal{I}_2$ is obtained by substituting $A = 1$ and $B = \infty$ in (33),

$$\begin{aligned} \Phi_2(s) \quad = \quad & \exp\Big\{\pi\lambda\alpha\Big[\mathbb{E}_{\mathsf{k}}\left(1 - e^{-s\mathsf{k}}\right) - \\ & s^{2/\eta}\mathbb{E}_{\mathsf{k}}\left(\mathsf{k}^{2/\eta}\right)\Gamma\left(1 - 2/\eta\right) + \\ & s^{2/\eta}\mathbb{E}_{\mathsf{k}}\left(\mathsf{k}^{2/\eta}\Gamma\left(1 - 2/\eta, s\mathsf{k}\right)\right)\Big]\Big\} \end{aligned} \tag{11}$$

Let k be a unit mean exponential random variable (Rayleigh fading). For a PPP, the mean and variance of $\mathcal{I}_1$ are given by $\mu_1 = \pi\lambda\alpha$ and $\sigma_1^2 = 2\pi\lambda\alpha$. The corresponding values for $\mathcal{I}_2$ are obtained as $\mu_2 = -\frac{d}{ds}\ln\Phi_2(s)|_{s=0} = \frac{2\pi\lambda\alpha}{\eta-2}$ and $\sigma_2^2 = \frac{d^2}{ds^2}\ln\Phi_2(s)|_{s=0} = \frac{2\pi\lambda\alpha}{\eta-1}$. Since $\mathcal{I}_1$ and $\mathcal{I}_2$ are independent,

$$\begin{aligned} \mu_{\mathcal{I}} = \mu_1 + \mu_2 = \frac{\pi\lambda\alpha\eta}{\eta - 2} \\ \sigma_{\mathcal{I}}^2 = \sigma_1^2 + \sigma_2^2 = \frac{2\pi\lambda\alpha\eta}{\eta - 1}. \end{aligned} \tag{12}$$

Thus, the modified path loss model results in finite first and second order moments which, together with other higher order moments, can be used to analyze the convergence of $\mathcal{I}$ to a Gaussian in distribution [23]. Further, as Section VII shows, outage performances for the modified path loss model are given in terms of $\Phi(s) = \Phi_1(s)\Phi_2(s)$.

[4]This model only accounts for singularities due to large scale path loss and not those due to small scale fading

The remainder of this paper uses this 2D shot noise model based on the modified power law decay to address two seemingly different problems in large wireless networks. The first problem is the coverage analysis in a sensor network, where we estimate the reliability with which an event occurring in any given point in a plane is "covered" via distributed sensing by a group of sensor nodes. The second problem is outage analysis in an ad hoc network setting with peer-to-peer transmission (e.g., Bluetooth). The following sections illustrate how these two analyses present themselves as duals of one another.

VI. COVERAGE ANALYSIS

Traditional coverage analysis assumes a physical coverage model in which a point is said to be covered if it is within the sensing radius of at least one sensor node [8][26]. In such a model, each sensor makes an estimation of a target parameter only by itself and does not cooperate with neighboring sensors to make an improved estimate. References [9][10] introduce the notion of *information coverage*, which is based on the assumption that distributed sensing among nodes is possible. Such a distributed sensing can result in significant reductions in the sensor node density requirements at the cost of increased signal processing power.

Cooperative sensing uses estimation theory to combine measurements from different sensors. So, rather than assuming that a single node can sense with a certain distance-dependent reliability, a point is said to be "information-covered" if the sum of these signals "emitted" from a certain point in the plane and received at a number of sensors exceeds some threshold. Conversely, one could assume that the sensor nodes emit a signal, and if the sum received by a virtual receiver at the point under consideration exceeds a threshold, then that point is covered. This, however, is exactly the interference problem (assuming all nodes transmit). So if the sensing reliability is set equal to the decay of the signal, then a point is information-covered exactly if the interference measured at that point exceeds some threshold when all nodes transmit at a certain power. Having all the nodes transmit is just a (virtual) assumption to help solve the dual problem of information coverage, where the "power decay law" of the interference problem corresponds to the "sensing decay law" of the sensing problem. In particular, we focus on networks where sensing relies on wave propagation laws equal to those which guide signal propagation in wireless ad hoc networks.

We present the coverage analysis over 2 different propagation environments. In the first case the medium introduces large scale path loss and additive noise to the emitted signal. Additionally, in the latter case, the medium also fades the signal amplitude in a random fashion. The choice of these propagation models in the sequel simply correspond to the AWGN and Rayleigh fading channel models in wireless communications. Propagation models for acoustic or pressure signals can be quite different; however, the results presented here can be tailored to represent arbitrary propagation models when the statistics of the propagation medium are available.

A. *Additive noise medium with path loss*

The notation used here is the same as in [9]. Let $r_1, r_2, \cdots, r_M$ denote the distances of M location-aware sensor nodes that cooperate in sensing a given parameter θ (e.g., an acoustic signal). The noise-corrupted measurement of this parameter, x_m, at node m is given by

$$x_m = \theta r_m^{-\eta/2} + n_m, \ m = 1, 2, \cdots, M. \quad (13)$$

The amplitude of the parameter θ decays with distance according to $r^{-\eta/2}$. Equation (13) can be written in matrix form as $\mathbf{X} = \mathbf{D}\theta + \mathbf{N}$, where $\mathbf{X} = [x_1, x_2, \cdots, x_M]^T$, $\mathbf{D} = \left[\min\left(1, r_1^{-\eta/2}\right), \min\left(1, r_2^{-\eta/2}\right), \cdots, \min\left(1, r_M^{-\eta/2}\right)\right]^T$ and $\mathbf{N} = [n_1, n_2, \cdots, n_M]^T$. The components of the noise vector are assumed to be spatially uncorrelated and white with an identical variance of σ_n^2 so that the covariance matrix is given by $\mathbf{R} = \mathbb{E}\left[\mathbf{N}\mathbf{N}^T\right] = \sigma_n^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Since the sensor nodes are location-aware, $\mathbf{D}$ is deterministic and, hence, the estimation algorithm has to only deal with random additive noise.

Let $\hat{\theta}_M$ and $\tilde{\theta}_M = \hat{\theta}_M - \theta$ denote the estimate and the estimation error respectively. When M such measurements are available, [9] proposes the use of the well-known *best unbiased linear estimator* (BLUE) [27] to determine $\hat{\theta}_M$. According to BLUE,

$$\begin{aligned} \hat{\theta}_M &= \left[\mathbf{D}^T\mathbf{R}^{-1}\mathbf{D}\right]^{-1}\mathbf{D}^T\mathbf{R}^{-1}\mathbf{X}, \\ \tilde{\theta}_M &= \left[\mathbf{D}^T\mathbf{R}^{-1}\mathbf{D}\right]^{-1}\mathbf{D}^T\mathbf{R}^{-1}\mathbf{N}. \end{aligned} \quad (14)$$

A point is said to be information-covered with a confidence level of ϵ if $\mathbb{P}\left[\left|\tilde{\theta}_M\right| \le \mathcal{A}\right] \ge \epsilon$, where $\mathcal{A}$ represents the maximum absolute value of the estimation error. Assuming zero-mean, Gaussian noise components, we have

$$\begin{aligned} \mathbb{P}\left[\left|\tilde{\theta}_M\right| \le \mathcal{A}\right] &= 1 - 2Q\left(\frac{\mathcal{A}}{\sigma_n\sqrt{\left(\sum_{m=1}^M r_m^{-\eta}\right)^{-1}}}\right) \\ &= 1 - 2Q\left(\frac{\mathcal{A}}{\sigma_n\sqrt{I^{-1}}}\right) \\ &= 1 - 2Q\left(\sqrt{I}\right), \quad \text{when } \mathcal{A} = \sigma_n. \end{aligned} \quad (15)$$

Here, $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-x^2/2}dx$. I represents the total power received at the M sensor nodes due to the emission at the target point being covered, or equivalently, the total power received by a virtual receiver placed at the target point if all M sensor nodes were to transmit with unit power. The objective is to identify the minimum required node intensity $\lambda_{\min}$ such that $1 - 2Q\left(\sqrt{I}\right) \ge \epsilon$.

However, most sensor networks are deployed in an open field (or a water body) and do not have a static configuration. When a random network has time-varying realizations, it is more meaningful to analyze the average coverage rather than that for a particular realization. I becomes a random variable, denoted by $\mathcal{I}$, whose characteristic function can be determined using the 2D shot noise model presented in the previous section. Using this model, we obtain an upper bound on the probability that a point is not covered on average as follows. Let $\kappa = Q^{-1}\left(\frac{1-\epsilon}{2}\right)$. Then,

$$\begin{aligned} \mathbb{P}\left[1 - 2Q\left(\sqrt{\mathcal{I}}\right) \le \epsilon\right] &= \mathbb{P}\left[\mathcal{I} \le \kappa\right] \\ &= \mathbb{P}\left[e^{-t\mathcal{I}} \ge e^{-t\kappa}\right] \\ &\le \inf_{0 \le t < \infty} \frac{\mathbb{E}\left[e^{-t\mathcal{I}}\right]}{e^{-t\kappa}} \\ &= \inf_{0 \le t < \infty} \frac{\Phi(t)}{e^{-t\kappa}} \\ &= \delta, \end{aligned} \quad (16)$$

where $\Phi(t)$ is the Laplace transform of the interference evaluated at t. The inequality in the above derivation follows from the Markov inequality. δ is referred to as the vacancy probability in [10]. Since the total interference comes from only M (virtual) transmitters, the total received power seen at the target point is only due to nodes spread over a finite disc. Therefore, $\Phi(t)$ is obtained from (33) by letting $\mathsf{k} = 1$ for the AWGN medium and setting $A = 1, B = \sqrt{\frac{M}{\lambda\pi}}$, such that the mean number of nodes present in a disk of radius B is M. We evaluate δ for a given λ by evaluating the Laplace transform over 2 disjoint regions, $[0, 1)$ and $[1, B]$. The following subsection extends this idea to propagation over a medium that has small scale fading as well.

B. *Additive noise medium with fading and path loss*

In this subsection, we extend the coverage analysis to the fading model. For the sake of illustration, we assume a Rayleigh fading medium; however, extending the analysis to other propagation models is straightforward. The measurement model for the Rayleigh block fading medium has the same form, $\mathbf{X} = \mathbf{D}\theta + \mathbf{N}$. The only difference is that $\mathbf{D} = \left[h_1\min\left(1, r_1^{-\eta/2}\right), \cdots, h_M\min\left(1, r_M^{-\eta/2}\right)\right]^T$ is no longer deterministic since $h_i \sim \mathcal{CN}(0, 1), \ i = 1, 2, \cdots, M$. However, the BLUE algorithm requires $\mathbf{D}$ to be deterministic. Hence, we resort to the *weighted least squares estimator* to estimate θ. The estimate and the estimation error are given by

$$\begin{aligned} \hat{\theta}_M &= \left[\mathbf{D}^T\mathbf{W}\mathbf{D}\right]^{-1}\mathbf{D}^T\mathbf{W}\mathbf{X}, \\ \tilde{\theta}_M &= \left[\mathbf{D}^T\mathbf{W}\mathbf{D}\right]^{-1}\mathbf{D}^T\mathbf{W}\mathbf{N}, \end{aligned} \quad (17)$$

where $\mathbf{D}^T$ represents the conjugate transpose of $\mathbf{D}$ and $\mathbf{W}$ is a weight matrix that is used to weigh past and future errors differently [27]. Here, we choose $\mathbf{W} = \mathbf{I}$ and assume $\mathbf{D}$ to be statistically independent of $\mathbf{N}$.

$\mathbf{D}^T\mathbf{D}$ is a scalar and is given by $I = \sum_{m=1}^M |h_i|^2 r_i^{-\eta}$. The estimation error is composed of a weighted sum of the independent noise components given by,

$$\tilde{\theta}_M = I^{-1} \sum_{m=1}^{M} h_i r_i^{-\eta/2} n_i \tag{18}$$

The variance of $\tilde{\theta}_M$ is simply given by $\sigma_M^2 = \sigma_n^2/I$. Once again, for a stochastic network, we are interested in average coverage rather than that for one particular realization and, hence, we model I to be a random variable $\mathcal{I}$. This is the same model that was obtained for the AWGN case with the only difference being that $\mathcal{I}$ includes fading as well as path loss. The remainder of the analysis is identical to the one presented in the previous subsection. In evaluating $\Phi(t)$, however, k represents an exponential random variable with unit mean.

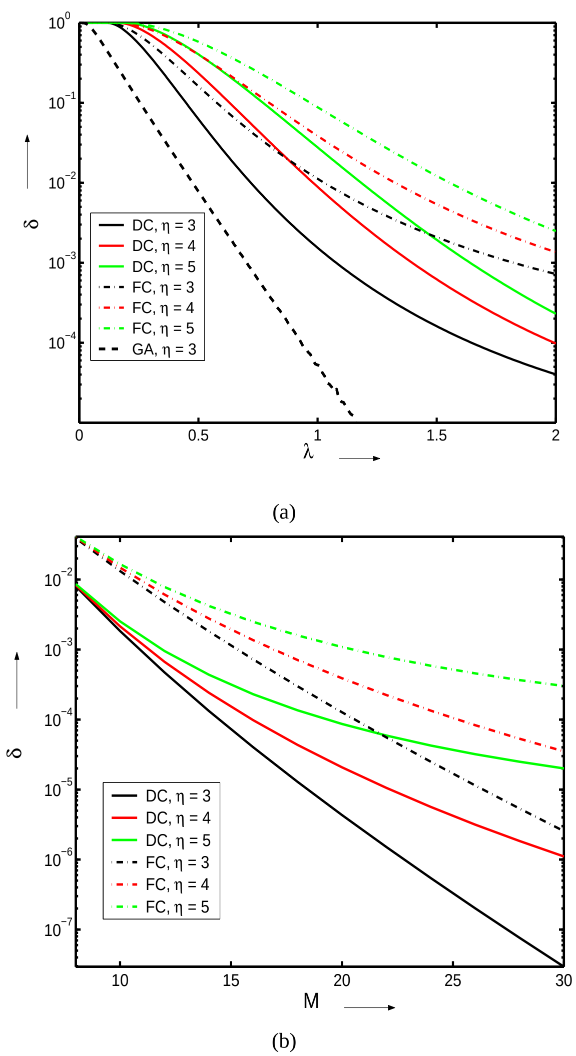

Fig. 1. Variation of the probability of a target point not being covered with respect to the (a) sensor node density (b) number of cooperating sensors for $\epsilon = 0.7$. (DC = AWGN medium, FC = fading channel, GA = Gaussian Approximation)

For both propagation models, we do not concern ourselves with the exact cooperation scheme employed by the sensors. One possible scenario is where all location-aware sensors within a radius of B from the target point communicate with a base station, which then determines an estimate for θ. The processing power available at the base station determines M and, hence, B. Thus, given λ and M, the 2D shot noise model can be used to determine an upper bound on the probability that a given point in a plane is not information-covered and, hence, the fraction of the total target area that is not information-covered. The following subsection presents the variation of this probability as a function of the sensor node density, λ and the number of cooperating nodes, M.

C. *Results*

In this subsection, we present 2 different sets of curves. Figure 1(a) plots the probability of a target point not being covered, δ, as a function of the sensor node density. The solid curves represent δ vs. λ for the AWGN medium for 3 different decay exponents, $\eta = 3, 4$ and 5. These curves illustrate that as the decay exponent increases, the node density that is required to achieve the same δ also increases. This is expected since the strength of θ falls increasingly rapidly with distance as η increases and farther nodes contribute less to the estimation process. The dash-dot curves show the same 3 curves for the Rayleigh fading model. Finally, the black dashed curve depicts the limiting case where all sensors in the network[5] (potentially infinite) cooperate in sensing an event over an AWGN medium for $\eta = 3$. Here, the (virtual) interference is modeled as a Gaussian random variable with the mean and variance obtained by differentiating its Laplace transform.

Figure 1(b) plots δ as a function of M, the mean number of sensor nodes whose measurements the base station combines to make the final estimate, for a given λ for both the AWGN and fading models. Once again, these curves illustrate that fading deteriorates the system performance and that for higher η, increasing the number of cooperating nodes provides diminishing returns, similar to the results presented in [10].

The following section solves the dual of the coverage problem using the 2D shot noise and the modified power law decay models.

VII. OUTAGE ANALYSIS

Coverage analysis aims to determine the minimum node density such that the “interference” observed at a target point is above a threshold with a certain reliability. Outage analysis, on the other hand, aims to achieve the opposite. The objective is to determine the maximum allowed intensity of transmitting nodes in the network such that the interference observed by a receiver node is below a threshold with a certain reliability. This, in turn, guarantees a certain minimum throughput when the nodes transmit to each other.

In the following analysis, we consider the threshold link model introduced in [12] for communication over M hops. We do not consider any power allocation schemes similar to those described in [16], but assume the nodes to always

[5] The base station is assumed to have infinite processing power

transmit with the same power P. Transmission from node a to b occurs over M hops and the transmission is successful only if the SINR at every hop exceeds some threshold Θ, i.e., given $\mathcal{O} = \mathbb{P}[\gamma < \Theta]$, the outage probability over M hops is defined as $\mathcal{O}_M = 1 - (1-\mathcal{O})^M$. We present an analysis that aims to satisfy a given outage constraint ϵ_M. Here, we present a general framework to derive the transmission capacity of a network along the lines of the outage analysis in [11] and derive an exact expression for the transmission capacity of a Rayleigh fading channel.

A. *Transmission Capacity*

Recent work by Weber *et al.* [11] introduces the notion of transmission capacity as a performance measure for random ad hoc networks with outage constraints. Transmission capacity is defined as $\mathcal{C}^\epsilon = (\lambda\alpha)_{\max} s(1-\epsilon)$, where s is the constant spectral efficiency with which all transmitting nodes communicate with their receivers and ϵ is the outage constraint such that $\mathcal{O} < \epsilon$. Reference [28] derives upper and lower bounds on the transmission capacity for the AWGN channel for a given ϵ. Here, we evaluate the exact transmission capacity of a multihop network for the fading channel. Given an overall outage constraint ϵ_M over all M hops, the per-link outage constraint is $\epsilon = 1-(1-\epsilon_M)^{1/M}$.

For the Rayleigh block fading channel, k is an exponential random variable with unit mean so that,

$$\begin{aligned}\mathcal{O} &= \mathbb{E}_{\mathcal{I}}\left[\mathbb{P}\left(\mathsf{k} < \Theta\mathcal{I} \mid \mathcal{I}\right)\right] \\ &= 1 - \mathbb{E}_{\mathcal{I}}\left[\exp\left(-\Theta\mathcal{I}\right)\right] \\ &= 1 - \Phi(\Theta). \end{aligned} \tag{19}$$

Let $\Phi(\Theta) = \exp\left[-\lambda\alpha\pi\Psi(\Theta)\right]$. For a given ϵ, the maximum allowed intensity of transmitting nodes is, therefore, derived as

$$\begin{aligned} 1 - \Phi(\Theta) &= \epsilon \\ \Rightarrow (\lambda\alpha)_{\max} &= \frac{\log\left(\frac{1}{1-\epsilon}\right)}{\pi\Psi(\Theta)} \\ &\approx \frac{\epsilon}{\pi\Psi(\Theta)}. \end{aligned} \tag{20}$$

For small ϵ, we use the approximation $-\log(1-\epsilon) \approx \log(1+\epsilon) \approx \epsilon$. For the Rayleigh fading channel, the outage expression presents itself in simple fashion in an analytical closed form. However, this is not so for an AWGN channel or the more practical Ricean fading channel in which nearby interferers, say within a radius x, have a line-of-sight component to the receiver at the origin whereas far away nodes do not. For such a channel model, k would be a squared Ricean random variable for nodes within x and an exponential random variable for interferers beyond x. In such cases, it is useful to decompose the transmission capacity analysis over 2 regions. In the sequel, we present such an analysis and use it to rederive the transmission capacity for the Rayleigh fading channel.

Consider the same setting as before, with the target receiver located at the origin, unit distance away from its transmitter. Let $b(0,x)$ denote a ball of radius x centered at the origin. The outage event is decomposed as follows:

$$\begin{aligned} E_1 &: \text{ Outage only due to interferers in } b(0,x) \\ E_2 &: \text{ Outage only due to interferers in } \bar{b}(0,x). \end{aligned} \tag{21}$$

Events E_1 and E_2 are independent since Π_α is a PPP. For a Rayleigh fading channel, the probabilities of these 2 events are given by

$$\begin{aligned} \mathbb{P}[E_1] &= \mathbb{P}\left[\frac{\mathsf{k}}{\sum_{i\in\Pi_\alpha\cap b(0,x)} f(\mathsf{k}_i, r_i)} < \Theta\right] \\ &= \epsilon_1 \\ \mathbb{P}[E_2] &= \mathbb{P}\left[\frac{\mathsf{k}}{\sum_{i\in\Pi_\alpha\cap \bar{b}(0,x)} f(\mathsf{k}_i, r_i)} < \Theta\right] \\ &= \epsilon_2, \end{aligned} \tag{22}$$

where we define constants ϵ_1 and ϵ_2 to be the outage probabilities due to interferers in regions $b(0,x)$ and $\bar{b}(0,x)$ respectively.

Let $\mathcal{I}_1$ and $\mathcal{I}_2$ be the interferences due to transmitting nodes in $b(0,x)$ and $\bar{b}(0,x)$ with $\Phi_1(s,x)$ and $\Phi_2(s,x)$ as their Laplace transforms respectively. The probabilities of the events E_1 and E_2 are given by $1-\Phi_1(\Theta,x)$ and $1-\Phi_2(\Theta,x)$ respectively. For ease of analysis, we write

$$\begin{aligned} \Phi_1(\Theta,x) &= \exp\left[-\lambda\alpha\pi\Psi_1(\Theta,x)\right], \\ \Phi_2(\Theta,x) &= \exp\left[-\lambda\alpha\pi\Psi_2(\Theta,x)\right]. \end{aligned} \tag{23}$$

The exact expressions for $\Psi_1(\Theta,x)$ and $\Psi_2(\Theta,x)$ are derived in Appendix B. Now, substituting back in the first inequality in (22), we have

$$\begin{aligned} 1 - \exp\left[-\lambda\alpha\pi\Psi_1(\Theta,x)\right] &= \epsilon_1 \\ \Rightarrow (\lambda\alpha)_{\max,1} &\approx \frac{\epsilon_1}{\pi\Psi_1(\Theta,x)}. \end{aligned} \tag{24}$$

Similarly, the outage constraint for the interference from all nodes in $\bar{b}(0,x)$ yields

$$\begin{aligned} 1 - \exp\left[-\lambda\alpha\pi\Psi_2(\Theta,x)\right] &= \epsilon_2 \\ \Rightarrow (\lambda\alpha)_{\max,2} &\approx \frac{\epsilon_2}{\pi\Psi_2(\Theta,x)}. \end{aligned} \tag{25}$$

Appendix B shows that while $1/\Psi_1(\Theta,x)$ is a decreasing function of x, $1/\Psi_2(\Theta,x)$ is an increasing function of x. Therefore, given $(\lambda\alpha)_{\max,1}$ and $(\lambda\alpha)_{\max,2}$ from (24) and (25) respectively, the transmission capacity is given by

$$\mathcal{C}^\epsilon = \min\left\{(\lambda\alpha)_{\max,1}, (\lambda\alpha)_{\max,2}\right\} s(1-\epsilon). \tag{26}$$

The transmission capacity is maximized by letting $(\lambda\alpha)_{\max,1} = (\lambda\alpha)_{\max,2}$, i.e.,

$$\epsilon_2 = \frac{\Psi_2(\Theta,x)}{\Psi_1(\Theta,x)}\epsilon_1. \tag{27}$$

In order to solve for ϵ_1 and ϵ_2 in terms of ϵ, we observe that the total interference $\mathcal{I} = \mathcal{I}_1 + \mathcal{I}_2$, where $\mathcal{I}_1$ and $\mathcal{I}_2$ are independent random variables. Hence, we have

$$\begin{aligned} \Phi(\Theta) &= \Phi_1(\Theta, x)\Phi_2(\Theta, x) \\ &= (1-\epsilon_1)(1-\epsilon_2) \\ \Rightarrow \epsilon &= \epsilon_1 + \epsilon_2 - \epsilon_1\epsilon_2. \end{aligned} \tag{28}$$

This proves that the overall outage event is given by $E = E_1 \cup E_2$ and that the transmission capacity obtained by solving (27) and (28) is identical to the transmission capacity in (20). The final result is independent of the choice of x for the Rayleigh fading channel. Though we do not present them here, these results can easily be extended to direct sequence and frequency hopped CDMA similar to the analysis presented in [28] to analyze the effect of the spreading gain on the system performance.

For other channel models, the transmission capacity is in general a function of x, where x is defined by the channel model itself. We illustrate this dependence in the following subsection. The decomposition approach is similar to the one presented in [11], and since other channel models do not simplify analytically as well as the Rayleigh fading channel, this approach can be used to derive upper and lower bounds on the transmission capacity. However, in doing so, we need to define a third event

E_3 : Outage due to interferers in $b(0, \infty)$ given that $\bar{E}_1 \cap \bar{E}_2$ occurs. (29)

The memoryless property of the exponential random variable obviates the need to consider this event for the Rayleigh fading channel as shown by (28) but for other channel models, the outage event becomes $E = E_1 \cup E_2 \cup E_3$. By definition, the event E_3 is disjoint from $E_1 \cup E_2$. The overall outage probability is, therefore, given by

$$\mathbb{P}[E] = \mathbb{P}[E_1] + \mathbb{P}[E_2] - \mathbb{P}[E_1]\,\mathbb{P}[E_2] + \mathbb{P}[E_3]. \tag{30}$$

We can obtain bounds on the transmission capacity by solving for ϵ_1 and ϵ_2 as before. In the following subsection, we present simulation results for the transmission capacity as a function of ϵ and x.

B. *Results*

In this subsection, we present the transmission capacity as a function of ϵ for the Rayleigh fading channel based on the analytical expression in (20). In order to illustrate the effect of x on the transmission capacity, we also present simulation curves for the transmission capacity when nodes within a radius of x from the intended receiver at the origin have a line-of-sight component in addition to the Rayleigh faded component. The channel model for nodes within x, therefore, corresponds to a Ricean fading channel [29] whereas interferers outside x witness a Rayleigh fading channel.

Fig. 2(a) shows 3 different transmission capacity curves. The $x = 0$ case corresponds to the analytical Rayleigh fading channel curve where no node has a line-of-sight component

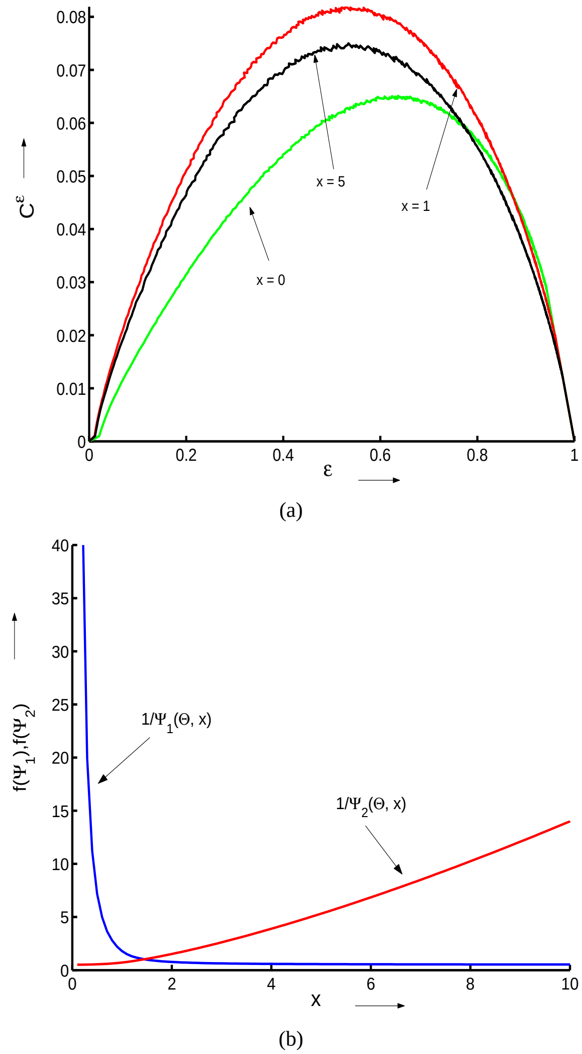

Fig. 2. (a) Variation of the transmission capacity with respect to ϵ for 3 different values of x for the Rician fading channel. (b) Variation of $1/\Psi_1(\Theta, x)$ and $1/\Psi_2(\Theta, x)$ with x.

to the receiver. The remaining two curves correspond to the cases where interferers within $x = 1$ and $x = 5$ have a line-of-sight component to the receiver respectively with a Ricean K-factor of 10. The system parameters used in these simulations are $\eta = 3.4$ and $\Theta = 10$ dB. For both these cases, the intended transmitter has a line-of-sight component at the receiver which explains the surge in the transmission capacity compared to the Rayleigh fading channel model.

VIII. Conclusion

In this paper, we have demonstrated the utility of the 2D shot noise process to model the network self-interference in finite as well as infinite networks and used it to perform coverage and outage analyses in large, cooperative networks, where the nodes are distributed in $\mathbb{R}^2$ according to a PPP. Using a modified power law, we have succeeded in deriving an interference model that has finite moments. We have then used this model to derive a lower bound on sensor node density requirement for coverage in a target region. Finally, we use this model to solve the dual problem by deriving an upper bound on the density of transmitting nodes in order to satisfy an outage constraint, thus, illustrating the tradeoff between network coverage and throughput using a single framework.

APPENDIX A

A 2D decaying power law shot noise process has the following Laplace transform:

$$\Phi(s) = \exp\left[-\mu\mathbb{E}_{\mathsf{k}}\left(\psi(s)\right)\right], \tag{31}$$

where

$$\psi(s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 1 - \exp\left[-s\mathsf{k}\left(x^2+y^2\right)^{-\eta/2}\right] dxdy \tag{32}$$

$\psi(s)$ is derived as follows,

$$\begin{aligned}
\psi(s) &\overset{(a)}{=} \int_A^B 1 - \exp\left[-s\mathsf{k}r^{-\eta}\right] 2\pi r dr \\
&\overset{(b)}{=} \pi\left(s\mathsf{k}\right)^{2/\eta}\int_{s\mathsf{k}A^{-\eta}}^{s\mathsf{k}B^{-\eta}} \left(1-e^{-t}\right) d\left(t^{-2/\eta}\right) \\
&= \pi B^2\left[1-e^{-s\mathsf{k}B^{-\eta}}\right] - \pi A^2\left[1-e^{-s\mathsf{k}A^{-\eta}}\right] \\
&\quad + \pi(s\mathsf{k})^{2/\eta}\int_{s\mathsf{k}B^{-\eta}}^{s\mathsf{k}A^{-\eta}} t^{-2/\eta}e^{-t}dt \\
&= \pi B^2\left[1-e^{-s\mathsf{k}B^{-\eta}}\right] - \pi A^2\left[1-e^{-s\mathsf{k}A^{-\eta}}\right] \\
&\quad + \pi\left(s\mathsf{k}\right)^{2/\eta}\Gamma(1-2/\eta, s\mathsf{k}B^{-\eta}) \\
&\quad - \pi\left(s\mathsf{k}\right)^{2/\eta}\Gamma(1-2/\eta, s\mathsf{k}A^{-\eta}).
\end{aligned} \tag{33}$$

(a) is obtained by switching to polar coordinates and generalizing the limits of integration while (b) is obtained through a change of variables and integration by parts.

APPENDIX B

We use the modified path loss model presented in Section V-B to determine $\Phi_1(s,x)$ and $\Phi_2(s,x)$.

Case 1: $x \leq 1$

$$\begin{aligned}
\Phi_1(s,x) &= \exp\left[-2\pi\lambda\alpha\int_0^x \mathbb{E}_{\mathsf{k}}\left(1-e^{-s\mathsf{k}}\right) rdr\right] \\
\Phi_2(s,x) &= \exp\left[-2\pi\lambda\alpha\mathbb{E}_{\mathsf{k}}\left\{\int_x^1 \left(1-e^{-s\mathsf{k}}\right) rdr + \right.\right. \\
&\qquad \left.\left.\int_1^\infty \left(1-e^{-s\mathsf{k}r^{-\eta}}\right) rdr\right\}\right]
\end{aligned} \tag{34}$$

We now evaluate these functions at $s = \Theta$ when k is an exponential random variable with unit mean to obtain

$$\begin{aligned}
\Psi_1(\Theta,x) &= \frac{x^2\Theta}{1+\Theta} \\
\Psi_2(\Theta,x) &= \frac{-x^2\Theta}{1+\Theta} + \Theta^{2/\eta}\Gamma\left(1-2/\eta\right)\Gamma\left(1+2/\eta\right) \\
&\quad - \Theta^{2/\eta}\mathbb{E}_{\mathsf{k}}\left[\mathsf{k}^{2/\eta}\Gamma\left(1-2/\eta,\Theta\mathsf{k}\right)\right]
\end{aligned} \tag{35}$$

Case 2: $x > 1$

$$\begin{aligned}
\Phi_1(s,x) &= \exp\left[-2\pi\lambda\alpha\mathbb{E}_{\mathsf{k}}\left\{\int_0^1 \left(1-e^{-s\mathsf{k}}\right) rdr + \right.\right. \\
&\qquad \left.\left.\int_1^x \left(1-e^{-s\mathsf{k}r^{-\eta}}\right) rdr\right\}\right] \\
\Phi_2(s,x) &= \exp\left[-2\pi\lambda\alpha\int_x^\infty \mathbb{E}_{\mathsf{k}}\left(1-e^{-s\mathsf{k}r^{-\eta}}\right) rdr\right]
\end{aligned} \tag{36}$$

Upon evaluating these functions at Θ, we get

$$\begin{aligned}
\Psi_1(\Theta,x) &= \frac{\Theta x^{2-\eta}}{1+\Theta x^{-\eta}} + \Theta^{2/\eta}\mathbb{E}_{\mathsf{k}}\left[\mathsf{k}^{2/\eta}\Gamma\left(1-2/\eta,\Theta\mathsf{k}x^{-\eta}\right)\right] \\
&\quad - \Theta^{2/\eta}\mathbb{E}_{\mathsf{k}}\left[\mathsf{k}^{2/\eta}\Gamma\left(1-2/\eta,\Theta\mathsf{k}\right)\right]. \\
\Psi_2(\Theta,x) &= \frac{-\Theta x^{2-\eta}}{1+\Theta x^{-\eta}} + \Theta^{2/\eta}\Gamma\left(1-2/\eta\right)\Gamma\left(1+2/\eta\right) \\
&\quad - \Theta^{2/\eta}\mathbb{E}_{\mathsf{k}}\left[\mathsf{k}^{2/\eta}\Gamma\left(1-2/\eta,\Theta\mathsf{k}x^{-\eta}\right)\right].
\end{aligned} \tag{37}$$

Though it is not immediately clear by looking at (35) and (37) how they vary with x, we use numerical integration methods and plot their variations with respect to x. In particular, we are interested in how they affect the transmission capacity. Fig. 2(b) shows that $1/\Psi_1(\Theta,x)$ is a monotonically decreasing function of x while $1/\Psi_2(\Theta,x)$ is monotonically increasing.

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