Local Delay in Poisson Networks with and without Interference

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***Abstract*—Communication between two neighboring nodes is the most basic operation in wireless networks. Yet very little research has focused on the *local delay*, defined as the mean time it takes a node to connect to a nearby neighbor. This problem is non-trivial when link distances are random but static, as is the case when the node distribution of a static network is modeled as a stochastic point process. We first consider the interference-free case, where links are independent, to study how fading and power control affect the delay in links of random distance. We find that power control is essential to keep the delay finite, and that randomized power control can drastically reduce the required (mean) power for finite local delay. Secondly, we study the local delay in a Poisson network with ALOHA, including both interference and noise. In this case we present an analytical bound on the local delay.**

I. Introduction

A fundamental source of delay in wireless networks is the time it takes for a node to successfully connect to a nearby neighbor. Consequently, the *local delay*, defined as the mean time, in numbers of time slots, until a packet is successfully received over a link between nearest neighbors, is an important quantity to study.

We tackle the problem in two steps. First, we ignore interference but consider random link distances. In this case, the "network" is just a collection of independent links, as shown in Fig. 1. The link distance distributions in our analysis will be drawn from the nearest-neighbor distances in Poisson networks, where the nodes' locations form a (homogeneous) Poisson point process (PPP), but other distributions could be considered as well. We take the distances to be static and derive the mean delay (ensemble average) over the links for different types of fading and power control strategies.

Secondly, we analyze the local delay in an actual Poisson network with interference. The channel access scheme is (slotted) ALOHA, and the links considered are the ones from each transmitter to its nearest receiver. The channel fading and the transmit/receive states are assumed iid over time, while the nodes are static[1]. Again power control turns out to be critical. We derive the first analytical bounds on the local delay with noise, interference, and power control.

A mathematical framework for the analysis of the local delay in Poisson ALOHA networks is provided in [1]. We build on this framework to obtain concrete results for different types of fading and power control. In our previous work in [2], we neglected noise and studied the local delay in the interference-limited (noiseless) setting without power control for Rayleigh fading. Here, we focus on the case with noise and provide results for more general fading and link distance distributions, including power control.

We shall see that in some cases, the local delay is infinite. This phenomenon is called *wireless contention phase transition* in [1]. An infinite local delay does not imply that certain nodes cannot reach their nearest neighbors in finite time; in fact, conditioned on the link distance, the local delay is always finite. It implies, however, that the local delay has a heavy tail, or, in other words, that the fraction of links with large local delays is significant.

II. A Collection of Links

Consider a collection of wireless links, as shown in Fig. 1. Assume the links do not interfere, but they have iid random distances and are subject to fading. Focusing on a single link, the received power is

$$P_r = PhR^{-\alpha}\,,$$

where P is the transmit power, h is the (power) fading coefficient, R is the link distance, and α is the path loss exponent. The transmission is assumed successful if $P_r > \theta$ for some threshold θ. Given R, we have

$$p_{s|R} = \mathbb{P}(h > \theta R^\alpha/P) = 1 - F_h(\theta R^\alpha/P)\,,$$

where $F_h(x) = \mathbb{P}(h \leq x)$ is the cumulative distribution function (cdf) of the fading random variable. Fading is assumed iid over time, which implies that the mean delay of successful transmission, conditioned on R, is $p_{s|R}^{-1}$. If R was also is temporally iid, the (unconditioned) delay would simply be

$$D = 1/\mathbb{E}_R(p_{s|R})\,.$$

In this case, we could define $\tilde{h} \triangleq hR^{-\alpha}$ and consider the fading coefficient to be $\tilde{h}$, ignoring the distance uncertainty. We focus on the case of fixed R, in which case the mean delay is the *ensemble average*

$$D = \mathbb{E}_R(1/p_{s|R})\,.$$

The static case is more interesting and practical. The distributions of the link distance R that we consider are drawn from

[1]The case where a new realization of the point process is drawn in each time slot would be much easier to analyze.

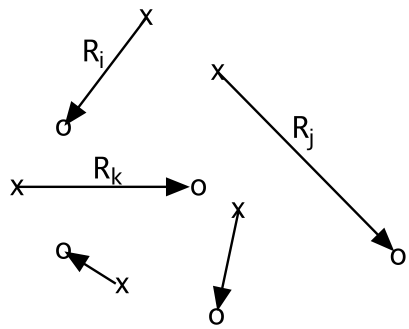

Fig. 1. A collection of links with random distances. Transmitters are denoted by x and receivers by o.

the nearest-neighbor distributions in Poisson networks, where the nodes are distributed according to a PPP. For the fading, we start with Rayleigh and then extend to the Nakagami-m model. As we shall see, power control will be necessary to avoid infinite mean delays.

A. The Rayleigh/Rayleigh case

Here we consider the case where the link distance R is Rayleigh distributed with mean $(2\sqrt{\lambda})^{-1}$, corresponding to the nearest-neighbor distance in a Poisson network of intensity λ [4]:

$$f_R(r) = 2\lambda\pi r \exp(-\lambda\pi r^2)\,, \quad r \geq 0. \tag{1}$$

The fading random variables are also Rayleigh, with mean 1. In this case, the success probability of a transmission over distance R is $p_{s|R} = \exp(-\theta R^\alpha/P)$. For constant P, the mean delay

$$D = 2\pi\lambda \int_0^\infty \exp(\theta r^\alpha/P) r \exp(-\lambda\pi r^2) \mathrm{d}r$$

diverges to infinity as soon as $\alpha > 2$. Hence:

Fact 1 *With Rayleigh fading and Rayleigh link distances, power control is needed as soon as the path loss exponent is larger than* 2. *In other words, any constant power is not sufficient to keep the mean delay finite if* $\alpha > 2$.

For $\alpha = 2$ and all constant P, we have

$$D = \frac{\lambda\pi}{\lambda\pi - \theta/P}\,. \tag{2}$$

For $\alpha > 2$, the transmit power needs to be chosen as a function of the link distance R. So we will henceforth assume that the transmitter knows R, which is a reasonable assumption given that the distance remains constant forever, and that it chooses its transmit power as

$$P \triangleq aR^{\alpha-2+b}\,, \tag{3}$$

where a the *power control factor*, and b the *power control exponent*. In this case, the success probability becomes $p_{s|R} = \exp(-\frac{\theta}{a}R^{2-b})$, and the mean delay, now a function of a and b, is

$$D(a,b) = 2\lambda\pi \int_0^\infty \exp\left(\frac{\theta}{a} r^{2-b}\right) r \exp(-\lambda\pi r^2) \mathrm{d}r\,. \tag{4}$$

For $b < 0$, the integral diverges for all values of the remaining parameters (not enough power if the nearest neighbor is far). For $b > 2$, the integral diverges since there is not enough power for receivers that are very close ($R \ll 1$). This second type of divergence is due to the singularity of the path loss law at the origin. If a bounded path loss law is used, say $(1+R)^{-\alpha}$ (and the corresponding transmit power), the first exponential in (4) is to be replaced by $\exp(\frac{\theta}{a}(1+r)^{2-b})$, which results in a finite delay for all $b \geq 0$. This integral does not admit a closed-form expression, though. We will therefore continue with the unbounded path-loss law and restrict ourselves to the regime $b \in [0, 2]$, knowing that for bounded path loss the delay could be reduced by choosing $b > 2$, *i.e.*, by *over-compensating* for the large-scale path loss.

1) $b = 0$: We obtain

$$D(a,0) = \frac{\lambda\pi}{\lambda\pi - \theta/a}\,, \qquad \theta < a\lambda\pi\,, \tag{5}$$

which shows that the mean delay exhibits a *phase transition*. There is tension between the delay given the distance r, call it $D(r)$, for which $\log D(r) = c_2 r^2$, and the density $f_R(r)$, for which $\log f_R(r) \sim -c_1 r^2$ as $r \to \infty$. Hence the local delay is finite if $c_1 > c_2$, which is exactly the condition in (5).

So, if the power control factor a is large enough, the local delay will be finite even if the power is adjusted in proportion to $R^{\alpha-2}$ only — thus the compensation for the large-scale path loss does not have to be complete. In particular, for $\alpha = 2$, the transmit power can be chosen to be the same for all nodes, irrespective of their nearest-neighbor distance (see (2)). In other words, the distances do not need to be known at the transmitter for $\alpha = 2$.

2) $b = 2$: With complete compensation for the large-scale path loss, the integration in (4) becomes obsolete since the success probability does not depend on the distance R, and we obtain immediately

$$D(a,2) = \exp\left(\frac{\theta}{a}\right)\,. \tag{6}$$

In this case, the delay increases exponentially in θ, or $\log D = \Theta(\theta)$ as $\theta \to \infty$.

3) $b = 1$: Let $t \triangleq \frac{\theta}{2a\sqrt{\lambda\pi}}$. Then

$$\begin{aligned} D(a,1) &= 1 + \frac{\theta \exp(t^2)\,(1 + \operatorname{erf} t)}{2a\sqrt{\lambda}} \\ &= 1 + \sqrt{\pi} t \exp(t^2)(1 + \operatorname{erf} t)\,. \end{aligned} \tag{7}$$

So in this case, $\log D = \Theta(\theta^2)$. We observe that there is no phase transition for $b = 1$ or $b = 2$.

4) General $b \in (0,2]$:

Proposition 1 *If $P \propto R^{\alpha-2+b}$, for any $0 < b \leq 2$, the links can support arbitrary rates at finite mean delays.*

Proof: Letting $x \triangleq r^2$, the delay (4) is of the form

$$c_1 \int_0^\infty \exp\left(-x(c_1 - c_2 x^{-b/2})\right)\mathrm{d}x\,, \quad c_1, c_2 > 0\,.$$

For $b \leq 2$, the integral can only diverge due to the upper integration bound. To show that it converges even for $b \ll 1$, we compare the integrand with $\exp(-bx)$. We observe that

$$\exp\left(-x(c_1 - c_2 x^{-b/2})\right) < \exp\left(-\frac{c_1 x}{2}\right) \quad \text{for } x > \left(\frac{2c_2}{c_1}\right)^{2/b},$$

which proves finiteness of the delay for all $0 < b \leq 2$, $c_1 > 0$ and $0 < c_2 < \infty$. For a transmission rate ρ, $\theta = 2^\rho - 1$ (or a multiple of it, depending on the code), so $c_2 = \theta/a$ is finite for all rates. ■

The delays will become extremely large as $b \to 0$, $a \to 0$, and/or $\theta \to \infty$, but there is no phase transition.

B. The gamma/Rayleigh case

Here we consider the scenario where the link distance is gamma distributed, parametrized by an integer n:

$$f_{R_n}(r) = \frac{2}{\Gamma(n)}(\lambda\pi)^n r^{2n-1} \exp(-\lambda\pi r^2)\,.$$

We will refer to this link distance model as the gamma(n) model. The fading is still Rayleigh.

The gamma distribution models the case where a node transmits to its n-th nearest neighbor in a Poisson network [4]. Calculating $\mathbb{E}_{R_n} \exp(\frac{\theta}{a} R_n^{2-b})$, we find:

Proposition 2 *For a transmit power $P = aR_n^{\alpha-2}$, the mean delay for the gamma(n) case, D_n, is the mean delay in the Rayleigh case, D_1, raised to the n-th power:*

$$D_n(a,0) = (D_1(a,0))^n\,, \qquad n \in \mathbb{N}\,. \tag{8}$$

If the path loss is fully compensated for, i.e., $P = aR_n^\alpha$, *$D_n(a,2) = \exp(\theta/a)$, irrespective of n.*

In this result, the transmit powers are adjusted according to n, so the nearest-neighbor and the second-nearest-neighbor delays, related by (8), are achieved using different powers. If the transmit power is chosen according to the distance to the second-nearest neighbor, the time to connect to the nearest neighbor is bounded as

$$D_1(a,0) < \sqrt{D_2(a,0)} \tag{9}$$

and

$$D_1(a,2) < D_2(a,2)\,, \tag{10}$$

since $R_2 > R_1$ a.s.

The mean transmit power is

$$a\mathbb{E}(R_n^{\alpha-2+b}) = a(\lambda\pi)^{1-\alpha/2-b/2} \frac{\Gamma(n+\alpha/2+b/2-1)}{\Gamma(n)}\,. \tag{11}$$

C. The Rayleigh/Nakagami case

We return to the case of Rayleigh link distances (or gamma(1)). The delay results for the Rayleigh fading case can be generalized to Nakagami-m fading:

Proposition 3 (Nakagami fading) *With Nakagami-m fading, $m \geq 1/2$, and $b = 0$, the mean delay is finite if*

$$\theta < \frac{a\lambda\pi}{m} \tag{12}$$

and infinite if

$$\theta > \frac{a\lambda\pi}{m}\,. \tag{13}$$

For $b = 2$, the mean delay is

$$D(a,2) = \frac{\Gamma(m)}{\Gamma(m, m\theta/a)}\,, \tag{14}$$

where $\Gamma(\cdot,\cdot)$ is the upper *incomplete gamma function.*

Proof: Let H be a Nakagami-m (power) fading random variable. From

$$\mathbb{P}(H < x) = \frac{\Gamma(m, mx)}{\Gamma(m)}$$

follows

$$p_{s|R} = \frac{\Gamma(m, m\theta R^{2-b}/a)}{\Gamma(m)}\,,$$

and examining the range where $\mathbb{E}(p_{s|R}^{-1})$ is finite yields the result for $b = 0$. For $b = 2$, the delay is simply p_s^{-1}, which is independent of R. ■

Remarks. For $b = 0$ it is interesting to note that the phase transition occurs at a value of θ that is directly proportional to the amount of fading or the variance of the fading random variable h. The more fading (the smaller m), the higher the threshold can be chosen while still achieving finite delay. If $m > a\lambda\pi/\theta$, then the delay becomes infinite due to a lack of diversity. For $b = 2$, the delay is decreasing (to 1) with increasing m if $\theta/a < 1$ and increasing (diverging to ∞) if $\theta/a > 1$. This is intuitive since without fading, the delay is 1 if $\theta/a < 1$, in which case transmissions always succeed, and infinity otherwise.

D. Induced fading: Random power control

We focus again on the case of Rayleigh distances. Comparing the expression for temporally iid link distances, $1/\mathbb{E}_R(p_{s|R})$, and the expression for the static case, $\mathbb{E}_R(1/p_{s|R})$, it is apparent from Jensen's inequality that much can be gained by temporal fluctuations in the received power. With static link distances, such an effect can be realized by *random power control*. It seems plausible that inducing fading by randomly varying the transmit power helps keep the mean delay finite. Since heavier-tailed distributions can be expected to yield better results, we use the Pareto distribution with complementary distribution

$$\mathbb{P}(H > x) = \left(\frac{k-1}{kx}\right)^k\,, \quad k > 1,\ x \geq 1 - 1/k\,,$$

parametrized with a single parameter k such that $\mathbb{E}(H) = 1$ for all $k > 1$. The transmit power is then chosen to be $P = HR^{\alpha-2+b}$, with H temporally independently Pareto. Assuming no channel fading, it follows that

$$p_s(R) = \begin{cases} \left(\frac{k-1}{k\theta R^{2-b}/a}\right)^k & \text{for } R^{2-b} > \frac{a(k-1)}{\theta k} \\ 1 & \text{otherwise} \end{cases}$$

For $b = 0$ and integer $k \geq 2$, the local delay is of the form

$$D^{\mathrm{NNT}}(a, 0) = 1 + Q(\xi) \exp\left(-\frac{k-1}{k\xi}\right),$$

where $\xi \triangleq \theta/(\lambda\pi a)$ and

$$Q(\xi) = c_1\xi + c_2\xi^2 + \ldots + c_k\xi^k$$

is a polynomial of order k. Straightforward yet tedious calculation yields

$$c_j = \frac{k^{j+1}}{(k-1)^{j-1}} \frac{\Gamma(k-1)}{\Gamma(k-j+1)}, \quad j \in \{1, 2, \ldots, k\}.$$

Unless $a \gg \theta$, which is impractical, the minimum local delay is attained at $k = 2$, as expected, since this choice of k produces the heaviest tail. In this case,

$$D(a, 0) = 1 + (4\xi + 8\xi^2) \exp(-1/(2\xi)),$$

which is finite for all choices of θ and a, and $D(a, 0) = \Theta(\theta^2)$, $\theta \to \infty$! So, inducing fading with a polynomial-tail distribution ensures the finiteness of the local delay for all choices of parameters, and it achieves much better asymptotic scaling of the delay with respect to θ than Rayleigh fading, where the delay scales at least exponentially in θ. So we observe that fading with exponential tail appears to result in a delay that increases at least exponentially in θ, whereas fading with a polynomial tail results in a delay that increases only polynomially in θ.

Fig. 2 shows a comparison of the local delay in the case of Rayleigh fading and Pareto induced fading. For small power levels, only the Pareto delay is finite, where for larger power levels, the Rayleigh delay is slightly smaller. In the limit, as the power increases, the local delay approaches 1 in both cases, as expected.

For $b = 2$,

$$D(a, 2) = \begin{cases} \left(\frac{k\theta/a}{k-1}\right)^k & \text{if } \theta k > a(k-1) \\ 1 & \text{otherwise.} \end{cases}$$

which is again minimized for $k = 2$. The asymptotic scaling with respect to θ is not improved by the larger b. The fact that $D(a, 2) = \Theta(\theta^k)$ is interesting; it confirms that the delay scaling is closely tied to how fast the tail of the (complementary) fading distribution decays.

In conclusion:

Fact 2 *Drawing the transmit power from a Pareto distribution in an iid fashion in each time slot drastically reduces the mean power required to keep the delay finite.*

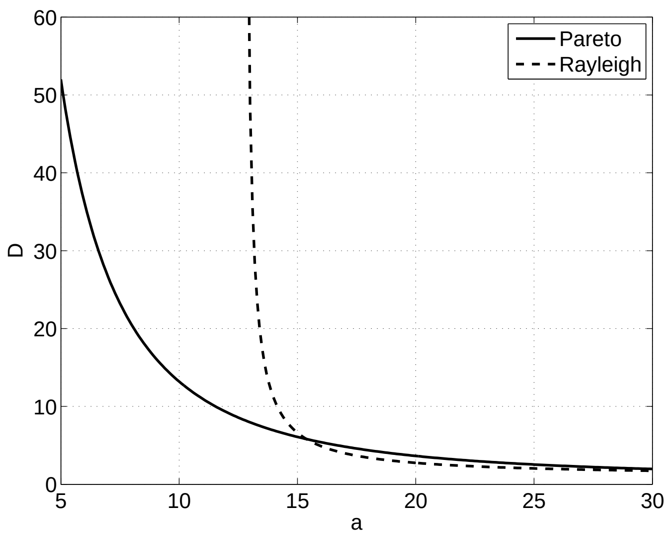

Fig. 2. The local delay for Rayleigh fading case and Pareto random power control ($k = 2$) for $b = 0$, $\theta = 10$, $\lambda = 1/4$. The phase transition in the Rayleigh case occurs at $a = \theta/(\lambda\pi) = 40/\pi \approx 12.7$.

The disadvantage of Pareto power control is the high peak-to-average power ratio.

III. Poisson Networks with Noise and Interference

The collection of links considered in the previous section can be regarded as a "network" without interference. In this section, we include interference, *i.e.*, we are considering an actual wireless network. To reflect the fact that now the transmissions take place between two nearby nodes in a network, we now speak of the *local delay* instead of just the (mean) delay.

A. The network model

We consider a marked Poisson point process (PPP) of potential transmitters $\hat{\Phi}_t = \{(x_i, t_{x_i})\} \subset \mathbb{R}^2 \times \{0, 1\}$, where $\Phi_t = \{x_i\}$ is a homogeneous PPP of intensity λ and the marks $\{t_{x_i}\}$ are iid Bernoulli with $\mathbb{P}(t = 1) = p = 1 - q$. A mark of 1 indicates that the node transmits (in the time slot considered) whereas a 0 indicates it is idle. The receivers form an independent PPP Φ_r of intensity λ_r. We denote the point process of all nodes by $\Phi = \Phi_t \cup \Phi_r$. The large-scale path loss is assumed to be r^α over distance r. A transmission from a node $x \in \Phi_t$ to a node $y \in \Phi_r$ is successful if the signal-to-interference-plus-noise ratio (SINR) exceeds a threshold θ. The SINR is defined as

$$\mathrm{SINR}_{xy} \triangleq \frac{S_{xy}}{W + I_{xy}},$$

where the desired signal is the product of transmit power, transmit mark, fading, and path gain, *i.e.*, $S_{xy} \triangleq P_x t_x h_{xy} \|x - y\|^{-\alpha}$, W is the noise power, and

$$I_{xy} \triangleq \sum_{(z,t_z) \in \hat{\Phi} \setminus \{(x,t_x)\}} P_z t_z h_{zy} \|z - y\|^{-\alpha}.$$

In this section, we assume the fading h_{xy} to be exponential with mean 1 and iid for all transmitter-receiver pairs and over time (block Rayleigh fading). Time is slotted, and transmission attempts are synchronized. We also assume $W \equiv 1$, without loss of generality, which makes the success probabilities $\mathbb{P}(\mathrm{SINR} > \theta)$ compatible with the ones in the previous section, where $I \equiv 0$.

Let $\mathcal{C}_\Phi$ be the event that the typical node situated at the origin $o \triangleq (0,0) \in \mathbb{R}^2$ successfully connects to its nearest receiver $y \in \Phi_r$ in a single transmission (one time slot) *conditioned on* Φ. This is the *nearest-receiver transmission model (NRT)*. Since all events considered are temporally iid, there is no need to add a time index to this event. Conditioning on Φ having a point at the origin o implies that the expectations that involve the point process are taken with respect to the Palm distribution $\mathbb{P}^o$ of Φ and denoted by $\mathbb{E}^o$ [3].

$$\mathbb{P}^o(\mathcal{C}_\Phi) = \mathbb{P}^o(\mathrm{SINR}_{oy} > \theta \mid \Phi)\,.$$

Conditioned on $\mathcal{S}$, the transmission successes are temporally iid, so the *conditional* local delay is geometric with mean $\mathbb{P}^o(\mathcal{C}_\Phi)^{-1}$. The local delay is then obtained by integration with respect to (w.r.t.) Φ:

$$D \triangleq \mathbb{E}^o_\Phi\left(\frac{1}{\mathbb{P}^o(\mathcal{C}_\Phi)}\right)\,.$$

In the next subsections, we evaluate the local delay for different scenarios.

B. Fixed transmit power

Let I be the total received power at the origin,

$$I \triangleq \sum_{(x,t_x)\in\hat{\Phi}_t} P_x t_x h_x \|x\|^{-\alpha}\,, \tag{15}$$

with h_x iid exponential, and

$$\mathcal{L}_I(s \mid \Phi) = \mathbb{E}^o_{t,h}(\exp(-sI \mid \Phi))$$

the conditional Laplace transform given Φ. Instead of conditioning on Φ, we may also condition on Φ_t and R, since the receiver process Φ_r enters the calculation only through the link distance R.

First we analyze the case of fixed (unit) transmit power levels and focus solely on the interference. We have the following relationship between the (conditional) success probability and the Laplace transform:

$$\begin{aligned} p_{s|R,\Phi_t} &= \mathbb{P}(h \geq I\theta R^\alpha \mid R, \Phi) \\ &= \mathbb{E}^o_{t,h}\exp(-I\theta R^\alpha) \\ &= \mathcal{L}_I(\theta R^\alpha \mid \Phi_t) \end{aligned} \tag{16}$$

Given Φ_t and R, the transmission success events are iid; hence, the (conditional) delay is geometric with mean $p^{-1}_{s|R,\Phi}$, and we obtain the local delay by integrating w.r.t. Φ_t and R. We first take the expectation w.r.t. the point process to obtain the local delay conditioned on the link distance R. To this end, we need the following result [2, Lemma 2]:
For $P_x \equiv 1$,

$$\mathbb{E}^o\left(\frac{1}{\mathcal{L}_I(s \mid \Phi_t)}\right) = \frac{1}{p}\exp\left(\frac{p\lambda C(\alpha)s^{2/\alpha}}{q^{1-2/\alpha}}\right)\,, \tag{17}$$

where

$$C(\alpha) \triangleq 2\pi^2/(\alpha\sin(2\pi/\alpha))\,. \tag{18}$$

The factor $1/p$ stems from the ALOHA transmit probability. Replacing s in (17) by θR^α and taking the expectation with respect to the nearest-receiver distance R (per (16)) yields the local delay:

$$D = \frac{1}{p}\mathbb{E}_R\exp\left(\frac{p\lambda C(\alpha)\theta^{2/\alpha}R^2}{q^{1-2/\alpha}}\right) \tag{19}$$

Since the delay given R is proportional to $\exp(cR^2)$, the local delay is finite in the noise-free case provided the transmit probability (and thus interference) p is small enough [2]. However, if noise is included, we have seen in Fact 1 that constant transmit power only result in finite local delay for $\alpha = 2$, even if interference is ignored. But for $\alpha = 2$, $C(\alpha) = \infty$, so the local delay with interference is trivially infinite for $\alpha = 2$. Hence:

Fact 3 *In a static network with noise and interference with the same transmit power at all nodes, the local delay is infinite for all path loss exponents, rates, and transmit probabilities.*

Clearly, power control is needed. With noise and power control,

$$\begin{aligned} p_{s|R,\Phi_t} &= \mathbb{P}(h \geq R^\alpha\theta(I+1)/P \mid R, \Phi_t)) \\ &= \mathbb{E}^o_{t,h}\exp(-R^\alpha\theta/P)\exp(-R^\alpha I/P) \\ &= \mathbb{E}^o_{t,h}\exp(-\theta R^{2-b}/a)\exp(-\theta R^{2-b}I/a)\,, \end{aligned} \tag{20}$$

where the first exponential factor is due to the noise and the second due to the interference. Power control complicates the analysis since it changes the distribution of the interference. In (15), only the fading random variables h_x are temporally iid, whereas the power control random variables P_x stay constant over time.

C. Power control at single transmitter

If only the node under consideration uses the power control scheme $P = aR^{\alpha-2+b}$ while the other nodes transmit at unit constant power, the interference is unchanged, and the local delay (with noise and interference) follows from (19) and (20):

$$\begin{aligned} D(a,b) = 2\pi\lambda_r\int_0^\infty &\exp\left(\frac{\theta}{a}r^{2-b}\right)\cdot \\ &\exp\left(\frac{\lambda p(\theta/a)^{2/\alpha}C(\alpha)r^{4/\alpha-2b/\alpha}}{q^{1-2/\alpha}}\right) r\exp(-\pi\lambda_r r^2)\mathrm{d}r\,, \end{aligned} \tag{21}$$

which is finite whenever $\alpha > 2$ and $b > 0$ or, if $\alpha > 2$ and $b = 0$, for small enough p and large enough a. For $b = 2$, the first two exponentials do not depend on r, and the local delay is given by their product. Of course this is a selfish approach that only works for a single transmitter in the network.

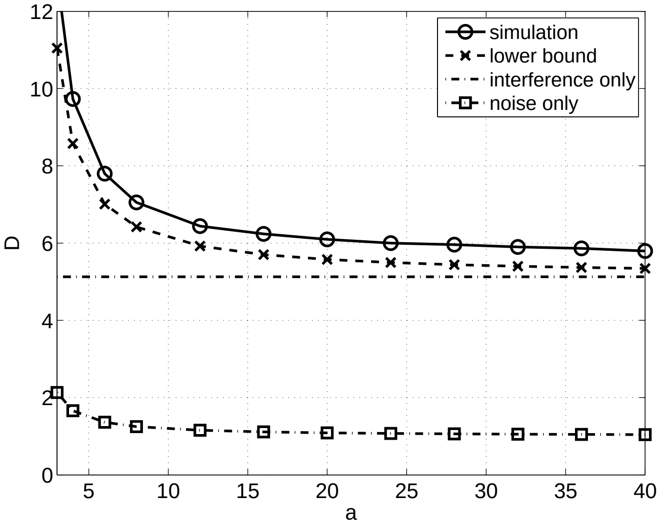

Fig. 3. The local delay in the NRT case with noise and interference as a function of the power control factor a. The solid curve (top) is a simulation, the dashed curve below is obtained by numerical integration of (22). The dash-dotted curve is the local delay if noise is ignored, which is independent of a. The bottom curve is the delay if interference is ignored. Other parameters are $b = 0$, $\lambda = 1/4$, $\lambda_r = 2$, $p = 1/5$, $\alpha = 4$, and $\theta = 10$. The local delay is lower bounded by $1/p = 5$ due to ALOHA.

D. Power control at all nodes

Since it appears impossible to get an exact closed-form solution for the case with full power control, we replace the interferer's transmit powers by their averages (11) (set $n = 1$), which, due to the convexity of the exponential and by Jensen's inequality, yields a lower bound on the local delay:

$$\underline{D}^{\mathrm{NRT}}(a, b) = \frac{1}{p}\mathbb{E}_R\left(\exp\left(\frac{\theta}{a}r^{2-b}\right)\exp(c_3 r^{(4-2b)/\alpha})\right) \tag{22}$$

where

$$c_3 = \frac{\lambda p\Big(\theta(\lambda_r \pi)^{1-\alpha/2-b/2}\Gamma(\alpha/2 + b/2)\Big)^{2/\alpha} C(\alpha)}{q^{1-2/\alpha}}$$

and R is distributed as in (1) but with intensity λ_r.

Fig. 3 shows a simulation result and the result of the numerical integration. As expected, the analytical result is a lower bound on the delay. For comparison, also shown are the curves for the cases where noise and interference only are considered.

IV. Conclusions

We have analyzed the local delay in static Poisson ALOHA networks with noise. First, we considered the interference-free case, *i.e.*, a collection of independent links of fixed random distances. If power control of the form $P = aR^{\alpha-2+b}$ is used, the the delay for Rayleigh fading is finite for $b = 0$ and some conditions on θ, a, and λ, and it is always finite for $b > 0$. For $b = 0$, a similar condition holds for Nakagami fading. If power control is randomized with a distribution with polynomial tail, the local delay is finite even for $b = 0$. So, induced fading can greatly increase the stability region.

Extensions from nearest-neighbor communication to n-th nearest neighbor communication are possible in a fairly straightforward manner, as are extensions to d dimensional Poisson networks.

The analysis of networks with both noise and interference is tricky due to the effect of power control: On the one hand, power control is needed to overcome the noise, on the other, it complicates the interference distribution. In a network where a new realization of the point process is drawn in each time slot, the problem would not exist, for in this case the variations of the transmit power could be combined with the fading. In static networks, however, only the fading states vary in an iid fashion, whereas power control is static over time, as the distance to the nearest neighbor stays constant. We resort to deriving a lower bound by replacing the interferers' actual powers by their averages and invoking Jensen's inequality. This is, to the best of our knowledge, the first analytical bound on the local delay with noise, interference, and power control.

Acknowledgments

The author would like to thank Steven Weber for helpful comments on an earlier version of this paper. The partial support of NSF (grants CNS 04-47869, CCF 728763) and the DARPA/IPTO IT-MANET program (grant W911NF-07-1-0028) is gratefully acknowledged.

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