Regularity, Interference, and Capacity of Large Ad Hoc Networks

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***Abstract*— In the analysis of large random ad hoc networks, the underlying node distribution is almost ubiquitously assumed to be the homogeneous Poisson point process. Using tools from stochastic geometry, we make an effort to derive interference and capacity results for clustered processes. We also reveal connections between the regularity of a point process and these metrics.**

I. Introduction

A. Motivation and overview

A common and analytically convenient assumption for the node distribution in large wireless networks is the homogeneous (or stationary) Poisson point process (PPP) of intensity λ, where the number of nodes in a certain area A is Poisson with parameter λA, and the numbers of nodes in two disjoint areas are independent random variables. For sensor networks, this assumption is usually justified by claiming that sensor nodes may be dropped from aircraft in large numbers; for mobile ad hoc networks, it may be argued that terminals move independently from each other. While this may be the case for certain networks, it is much more likely that the node distribution is not "completely spatially random" (CSR), *i.e.*, nodes are either clustered or more regularly distributed. Moreover, even if the complete set of nodes constitutes a PPP, the subset of *active* nodes (*e.g.*, transmitters in a given timeslot or sentries in a sensor network), may not be homogeneously Poisson. Certainly, it is preferable that simultaneous transmitters in an ad hoc network or sentries in a sensor network form more regular processes. On the other hand, many protocols have been suggested that are based on clustered processes. This motivates the need to extend the rich set of results available for PPPs to other node distributions.

In this paper we consider the interference and the probability of successful transmission between a transmitter and receiver pair in an interference-limited channel. The location of transmitters in space is modeled as *Poisson clustered process*, and fading is modeled as Rayleigh. We provide a numerically integrable expression for the outage probability and closed-form upper and lower bounds. We show that when the transmitter receiver distance is greater than the typical cluster radius, the success probability is greater than for a PPP of the same density. We obtain the maximum intensity of transmitting nodes for a given outage constraint, *i.e.*, the transmission capacity [1] of this spatial arrangement and show that it is equal to that of a Poisson arrangement of nodes. Analytical tools from stochastic geometry are used, including the probability generating functional of Poisson cluster processes, the Palm characterization of Poisson cluster process, and the Campbell-Mecke and Slivnyak's theorems.

B. Results for Poisson point processes

There exists a significant body of literature for networks with Poisson distributed nodes. Results on the interference are available in [2]–[5], and the throughput in the presence of interference is analyzed in [6]–[10]. Even in the case of the PPP, the interference distribution is not known for all fading distributions and all channel attenuation models. Only the characteristic function or the Laplace transform of the interference can be evaluated in all the cases. The Laplace transform can be used to evaluate the outage probabilities under Rayleigh fading characteristics [7], [9]. In [11] upper and lower bounds are obtained under general fading and PPP arrangement of nodes. [1] introduces the *transmission capacity*, which is a measure of the area spectral efficiency of the successful transmissions resulting from the optimal contention density.

C. Regularity

Ripley's K-function can be used to assess the regularity of a point process [12]. For a stationary process of intensity λ, the K-function or *second reduced moment function* [13, p. 120f.] is defined as

$$\lambda K(r) = \int \phi(B_0(r)) P_0^{!}(d\phi)\,, \tag{1}$$

where $B_0(r)$ is the ball of radius r centered at 0, ϕ here is interpreted as counting measure, and $P_0^{!}(Y)$ denotes the reduced Palm distribution, defined as $P_0^{!}(Y) = \mathbb{P}(\phi\backslash\{0\} \in Y \mid 0)$ for $Y \in \mathcal{N}$ with $\mathcal{N}$ the σ-algebra of the locally finite and simple sequences of points in $\mathbb{R}^2$. So, $\lambda K(r)$ denotes the number of extra points in a ball of radius r centered at a "typical point" that is not itself counted. For the PPP, $K(r) = \pi r^2$. In regular networks, $K(r)$ is smaller than this (at least for small r), whereas for clustered networks, $K(r)$ is larger than πr^2.

Intuitively we expect that regular networks suffer from smaller interference and therefore permit higher capacity, while clustered networks suffer from lower capacity. This paper is a first attempt to explore whether this intuition is correct.

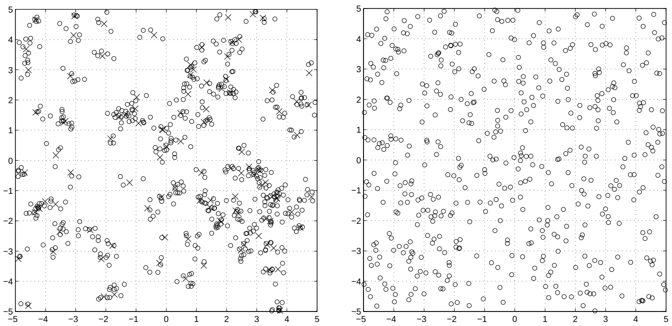

Fig. 1. (Left) Thomas cluster process with parameters $\lambda_p = 1, \bar{c} = 5$, and $\sigma = 0.2$. The crosses indicate the parent points. (Right) PPP with the same intensity $\lambda = 5$ for comparison.

D. Neyman-Scott cluster processes

Neyman-Scott cluster processes [13, Sect. 5.3] are Poisson cluster processes that result from homogeneous independent clustering applied to a stationary Poisson process. The parent points form a stationary Poisson process $\phi_p = \{x_1, x_2, \dots\}$ of intensity λ_p. The clusters N^x are of the form $N^{x_i} = N_i + x_i$ for each $x_i \in \phi_p$. The N_i are a family of identical and independently distributed finite point sets with distribution C independent of the parent process. The complete process ϕ is given by

$$\phi \quad = \quad \bigcup_{x \in \phi_p} N^x \tag{2}$$

The daughter points of the representative cluster N_0 are scattered independently and with identical distribution $F(x) = \int f(x) dx$ around the origin. The intensity of the cluster process is $\lambda = \lambda_p \bar{c}$, where $\bar{c}$ *is the average number of points in the representative cluster*.

If the number of nodes per cluster is Poisson (with intensity $\bar{c}$), the resulting process is a *Poisson cluster process*. We will further specialize to *Matern cluster processes* and *Thomas cluster processes*. For Matern cluster process each point is uniformly distributed in a ball of radius a around the origin. In Thomas cluster process each point is scattered using a symmetric normal distribution with variance σ^2 around the origin, *i.e.*, each child cluster forms an inhomogeneous Poisson process with intensity

$$\lambda(x) = \frac{\bar{c}}{2\pi\sigma^2} \exp(-\|x\|^2/2\sigma^2)$$

so that the mean number of children per parent is $\bar{c}$. A Thomas cluster process is illustrated in Fig. 1 (left).

II. Regularity and Interference

Let ϕ be a stationary, isotropic, and simple point process on $\mathbb{R}^2$. Each transmitter at location x_i is assumed to transmit unit power. The power received by a receiver located at z due to a transmitter at x_i is modeled as $h_i g(x_i - z)$, where h_i denotes the fading (assumed iid), and $g(x)$ generally represents the power law and is usually taken to be $\|x\|^{-\alpha}$, $(1 + \|x\|)^{-\alpha}$ or $\min\{1, \|x\|^{-\alpha}\}$.

$$I_\phi(z) = \sum_{x_i \in \phi} h_i g(x_i - z) = \sum_{x_i \in \phi_{-z}} h_i g(x_i)\,. \tag{3}$$

The conditions required for existence of the quantity $I_\phi()$ are discussed in [14]. Let $P^{!0}$ be the reduced Palm distribution at the origin and $\mathcal{K}_n(\mathbf{B})$ denote the reduced n-th factorial moment measure of ϕ. $\mathbf{B} = \mathbf{B}_1 \times \dots \times \mathbf{B}_{n-1}$, $\mathbf{B}_i \in \mathbb{R}^2$.

$$\mathcal{K}_n(\mathbf{B}) = \int \sum_{x_1,\dots,x_{n-1} \in \phi}^{\neq} \mathbf{1_B}(x_1, \dots, x_{n-1}) P^{!0}(\mathrm{d}\phi)\,. \tag{4}$$

Note that the interference distribution need not be the same for all points (Palm distributions are not stationary in general). First and second moments can be determined using the second and third order reduced factorial moments. The average interference is given by

$$E^{!0} I_\phi(z) = E^{!0}\Big[\sum_{x_i \in \phi} h_i g(x_i - z)\Big] \tag{5}$$

$$= E[h]\lambda \int g(x - z)\mathcal{K}_2(\mathrm{d}x) \tag{6}$$

$$= \frac{E[h]\lambda}{2\pi} \int g(x - z) K'(\|x\|) dx\,, \tag{7}$$

where $K'(r) = dK(r)/dr$. Since the process is stationary, $\mathcal{K}_2(B)$ can be expressed as [13]

$$\mathcal{K}_2(B) = \frac{1}{\lambda^2} \int_B \varrho^{(2)}(x) dx \tag{8}$$

where $\varrho^{(2)}(x)$ is the second order product density[1].

We expect $E^{!0} I_\phi(y)$ to increase with decreasing regularity[2].

Example: Thomas Cluster Process. In this case, from

$$\frac{\varrho^{(2)}(x)}{\lambda^2} = 1 + \frac{1}{4\pi\lambda_p\sigma^2} \exp\Big(\frac{-\|x\|^2}{4\sigma^2}\Big) \tag{9}$$

where $\lambda = \lambda_p \bar{c}$. we obtain

$$E^{!0} I_\phi(z) = E\, I_{\text{Poi}} + \frac{1}{4\pi\lambda_p\sigma^2} \int g(x - z) \exp\Big(\frac{-\|x\|^2}{4\sigma^2}\Big) dx \tag{10}$$

which shows that the mean interference is indeed larger than for the PPP.

III. Outage and Transmission Capacity of Clustered Processes

A. System model and assumptions

In this section we focus on Poisson cluster processes with transmitter locations $\phi = \{x_i\}$ on $\mathbb{R}^2$. The receiver is assumed to be at a distance R from the transmitter and not to belong to the cluster process.

Let W denote the additive Gaussian noise at the receiver. We say that the communication from a transmitter at location x to a receiver situated at z is successful if and only if

$$\frac{h_x g(x - z)}{W + I_{\phi \setminus \{x\}}(z)} \geq T \quad \text{or} \quad \frac{h_x g(x - z)}{W + I_\phi(z)} \geq \frac{T}{1 + T} \tag{11}$$

Note that there is no point of the point process at z (wp1), and $\|x - z\| = R$ by the assumption of a transmit-receiver distance of R.

[1] Intuitively, this indicates the probability that there are two points are separated by $\|x\|$. For PPP, it is $\varrho^{(2)}(x) = \lambda^2$ independent of x.

[2] Note that for $g(x) = \|x\|^{-\alpha}$, $E I_\phi(y)$ is diverging.

The reduced Palm distribution $P_0^!$ of a Neyman-Scott cluster process ϕ is given by [13], [15]

$$P_0^! = P * \tilde{C}_0^! \quad (12)$$

where P is the distribution of ϕ, and $\tilde{C}_0^!$ is the reduced Palm distribution of the finite representative cluster process N_0. "$*$" denotes the convolution of distributions, which corresponds to the superposition of ϕ and N_0. The Palm distribution $\tilde{C}_0^!$ is given by

$$\tilde{C}_0^!(Y) = \frac{1}{\bar{c}} E\Big(\sum_{x\in N_0} 1_Y(T_x\phi \setminus \{0\})\Big) \quad (13)$$

where $T_x\phi = \phi - x$, i.e, the translated point process. Let $Y_x = Y + x$

$$\tilde{C}_0^!(Y) = \frac{1}{\bar{c}} E\Big(\sum_{x\in N_0} 1_{Y_x}(\phi \setminus \{x\})\Big) \quad (14)$$

Let $C()$ denote the probability distribution of the representative cluster. Using the Campbell-Mecke theorem [13], we get

$$\tilde{C}_0^!(Y) = \frac{1}{\bar{c}} \int_{\mathbb{R}^2}\int_N 1_{Y_x}(\phi) C_x^!(d\phi)\bar{c}F(dx) \quad (15)$$

Here N denotes the space of locally finite and simple point sequences [13] on $\mathbb{R}^2$. Since the representative cluster has a Poisson number of points, by Slivnyak's theorem we have $C_x^!(.) = C(.)$. Hence

$$\tilde{C}_0^!(Y) = \int_{\mathbb{R}^2}\int_N 1_{Y_x}(\phi) C(d\phi) f(x)dx \quad (16)$$

$$= \int_{\mathbb{R}^2} C(Y_x) f(x)dx \quad (17)$$

The generating functional $G(v) = E[\prod_{x\in\phi} v(x)]$ of Neyman-Scott cluster processes is given by [13]

$$G(v) = \exp\Big\{-\lambda_p\int_{\mathbb{R}^2}\Big[1 - M\Big(\int_{\mathbb{R}^2} v(x+y)f(y)dy\Big)\Big]dx\Big\} \quad (18)$$

Where $M(z) = \sum_{i=0}^{\infty} p_n z^n$ is the moment generating function of the number of points in the representative cluster. Since this number is Poisson with mean $\bar{c}$,

$$M(z) = \exp(-\bar{c}(1-z))\,. \quad (19)$$

The generating functional for the representative cluster $G_c(v)$ is given by

$$G_c(v) = M\Big(\int_{\mathbb{R}^2} v(x)f(x)dx\Big) \quad (20)$$

Analyzing interference requires evaluation of $E_0^!(\prod_{x\in\phi} v(x))$. From (12), (17), and (18) and basic integral manipulation, it follows that

$$E_0^!(\prod_{x\in\phi} v(x)) = \exp\Big\{-\lambda_p\int_{\mathbb{R}^2}\Big[1 - M\Big(\int_{\mathbb{R}^2} v(x+y)f(y)dy\Big)\Big]dx\Big\} \times \int_{\mathbb{R}^2} M\Big(\int_{\mathbb{R}^2} v(x-y)f(x)dx\Big)f(y)dy\,. \quad (21)$$

The above equation holds when all the integrals are finite. If further $f(x)$ depends only on $\|x\|$ and $v(x) = v(-x)$, then $\int_{\mathbb{R}^2} v(x+y)f(y)dy = \int_{\mathbb{R}^2} v(x-y)f(y)dy = v * f$, then

$$E_0^!(\prod_{x\in\phi} v(x)) = \exp\Big\{-\lambda_p\int_{\mathbb{R}^2}\Big[1 - M\big((v*f)(x)\big)\Big]dx\Big\} \times \int_{\mathbb{R}^2} M\big((v*f)(y)\big)f(y)dy \quad (22)$$

B. *Outage capacity*

In this section we consider the outage probability and transmission capacity for Newman-Scott cluster process, with the number of points in each cluster Poisson with mean $\bar{c}$ and distribution function $f(x)$. Let the transmitter be located at the origin and the receiver at location z at distance $R = \|z\|$ from the transmitter. With a slight abuse of notation we shall be using R to denote the point $(R, 0)$. The probability of success for this pair is given by

$$\mathbb{P}(\text{success}) = \mathbb{P}\Big(\frac{h_x g(z)}{W + I_{\phi\setminus\{0\}}(z)} \geq T\Big) \quad (23)$$

We now assume Rayleigh fading, *i.e.*, the received power is exponentially distributed with the mean μ determined by $g(\cdot)$ and the distance to the transmitter. Adapting the technique given in [7] to cluster processes yields

$$\mathbb{P}(\text{success}) = \int_0^{\infty} e^{-\mu sT/g(z)} d\mathbb{P}(W + I_{\phi\setminus\{0\}}(z) \leq s) \quad (24)$$

$$= \psi_{I_{\phi\setminus\{0\}}(z)}(\mu T/g(z))\psi_W(\mu T/g(z))\,, \quad (25)$$

where ψ_X denotes the Laplace transform of the random variable X. Now the question is about evaluating the Laplace transform of the interference at Y given that there is a point at the origin.

$$\psi_{I_{\phi}(y)}(s) = E_0^!\Big[e^{-s\sum_{x_i\in\phi} h_i g(x_i - z)}\Big] \quad (26)$$

$$= E_0^!\Big[\prod_{x_i\in\phi}\frac{\mu}{\mu + sg(x_i - z)}\Big] \quad (27)$$

At $s = \mu T/g(R)$ we observe that the above expression will be independent of the mean of the exponential distribution μ.

Lemma 1 (Outage probability) *The probability of successful transmission between the transmitter at the origin and the receiver located at z, when $W \equiv 0$ (no noise), is given by*

$$\mathbb{P}(\textit{success}) = \exp\Big\{-\lambda_p\int_{\mathbb{R}^2}\Big[1 - \exp(-\bar{c}\beta(R,y))\Big]dy\Big\} \times \int_{\mathbb{R}^2}\exp(-\bar{c}\beta(R,y))f(y)dy \quad (28)$$

where

$$\beta(R,y) = \int_{\mathbb{R}^2}\frac{g(x-y-R)}{\frac{g(R)}{T} + g(x-y-R)}f(x)dx \quad (29)$$

Proof: Follows from (22) and (27). □

Remarks. The first term in (28) captures the interference without the cluster at the origin (*i.e.*, without conditioning); it is independent of z since the original cluster process is stationary. The second term is the contribution of the transmitter's cluster; it is identical for all z with $\|z\| = R$, since f and g are isotropic. So the success probability itself is the same for all z at distance R, as expected from stationarity.

Lemma 2 (Lower bound)

$$\mathbb{P}(\textit{success}) \geq P_p(\lambda) P_p(\bar{c} f^*) \tag{30}$$

where $P_p(\lambda)$ *denotes the success probability when* ϕ *is distributed as a PPP,* $f^* = \min_{p=1,2,\dots} \|f\|_p \|f\|_q$*, where* $1/p + 1/q = 1$ *(conj. exponents), and* $\lambda = \lambda_p \bar{c}$*.*

Proof: The first factor in (22) can be lower bounded by the success probability in the standard PPP $P_p(\lambda)$, and the second factor can be lower bounded by $P_p(\bar{c} f^*)$ (details omitted due to space constraints). □

For $a \geq 1/\sqrt{\pi}$ (Matern) and $\sigma \geq 1/\sqrt{2\pi}$ (Thomas), we get $\mathbb{P}(\text{success}) \geq P_p(\lambda) P_p(\bar{c})$. In general, $f^* \leq \|f\|_\infty \|f\|_1$, which is $1/\pi a^2$ for Matern and $1/2\pi\sigma^2$ for Thomas processes. In the latter case, when f is Gaussian, $f * f$ is also Gaussian with variance $2\sigma^2$, hence $f^* \leq 1/4\pi\sigma^2$. Hence for large a or σ, we have $\mathbb{P}(\text{success}) \gtrapprox P_p(\lambda)$. For $g(x) = \|x\|^{-\alpha}$, $P_p(\lambda)$ is given by [7]

$$P_p(\lambda) = \exp(-\lambda R^2 T^{2/\alpha} C(\alpha)) \tag{31}$$

where $C(\alpha) = \big(2\pi\Gamma(2/\alpha)\Gamma(1-2/\alpha)\big)/\alpha = \frac{2\pi^2}{\alpha}\csc(2\pi/\alpha)$ and $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the gamma function.

Lemma 3 (Upper bound) *Let* $\beta^* = \sup_{y \in \mathbb{R}^2} \beta(R, y)$*. Then*

$$\mathbb{P}(\textit{success}) \quad \leq \quad P_p\left(\frac{\lambda}{1 + \bar{c}\beta^*}\right) \tag{32}$$

Proof: Again apply bounding techniques to (28). □

From the above two lemmata, we obtain the bounds

$$P_p(\lambda) P_p(\bar{c} f^*) \leq \mathbb{P}(\text{success}) \leq P_p\left(\frac{\lambda}{1 + \bar{c}\beta^*}\right) \tag{33}$$

We also have $\beta^* \leq \min\{\sup\{f(x)\} R^2 T^{2/\alpha} C(\alpha), 1\}$. From the above inequality $\mathbb{P}(\text{success}) \to P_p(\lambda)$ as $\bar{c}/\sigma \to 0$ or $\bar{c}/a \to 0$, which is intuitive.

The proof for the above lemmata also indicates that it is only by conditioning on the event that there is a point at the origin that the success probability of Neyman-Scott cluster process can be lower than that of the Poisson of the same intensity. This implies that the cluster around transmitter causes the maximum damage. So as the receiver moves away from the transmitter Neyman-Scott cluster process has better success probability than that of Poisson. It is not true in general that Neyman-Scott cluster processes have a lower success probability than PPPs of the same intensity. For example from Figure 3, we see that for $R < 0.8$, the PPP has a better success probability than the Matern process.

A more detailed analysis reveals that a PPP with intensity $\lambda_p \bar{c}$ has a lower success probability than that a clustered process of the same intensity *for large transmit-receiver distances.* On the other hand, for small R, the success probability of the PPP is higher. So there exists an R^* such that $\mathbb{P}(\text{success}) \leq P_p(\lambda_p \bar{c})$ for $R < R^*$ and vice versa.

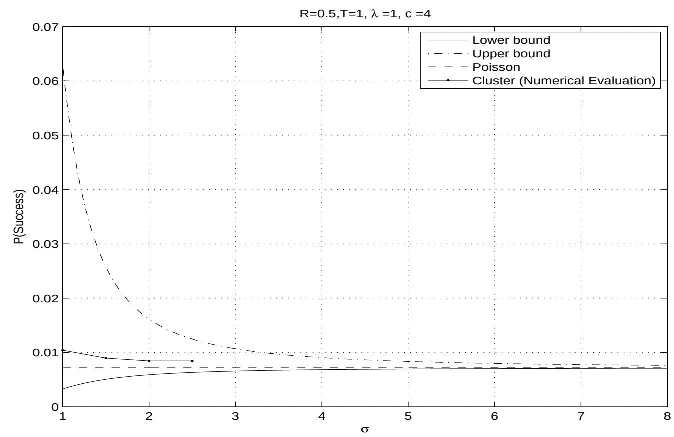

Fig. 2. Comparison of the bounds

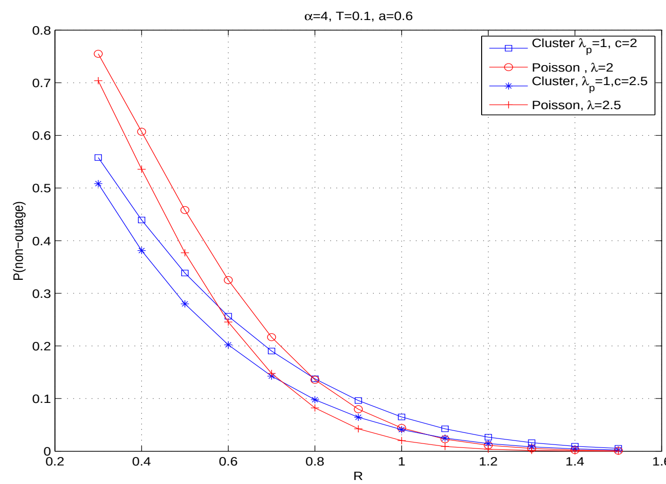

Fig. 3. Comparison of success probability for Poisson and Matern processes.

C. Transmission capacity

Let $P(\lambda, T)$ denote the success probability of the cluster process with intensity $\lambda = \lambda_p \bar{c}$ and threshold T. Transmission capacity is defined [1] as $C(\epsilon, T) = (1-\epsilon)\sup_\lambda\{\lambda : P(\lambda, T) \geq 1 - \epsilon\}$. Let $C_l(\epsilon, T)$ and $C_u(\epsilon, T)$ denote lower and upper bounds to the transmission capacity.

For a PPP we have $P_p(\lambda, T) = \exp(-\lambda R^2 T^{2/\alpha} C(\alpha))$. Hence,

$$\frac{C_p(\epsilon, T)}{1-\epsilon} = \frac{-\ln(1-\epsilon)}{R^2 T^{2/\alpha} C(\alpha)} \approx \frac{\epsilon}{R^2 T^{2/\alpha} C(\alpha)}, \quad \epsilon \ll 1. \tag{34}$$

For Neyman-Scott cluster processes, the intensity $\lambda = \lambda_p \bar{c}$. We first to try to consider both λ_p and $\bar{c}$ as optimization parameters for the transmission capacity, i.e.

$$C(\epsilon, T) := \sup\{\lambda_p \bar{c} : \lambda_p > 0, \bar{c} > 0, \text{outage-constraint}\} \tag{35}$$

without individually constraining the parent node density or the average number of nodes in a cluster.

Lemma 4 (Lower bound) *With the above definitions,* $C(\epsilon, T) \geq C_l(\epsilon, T) = C_p(\epsilon, T)$*.*

Proof: From the lower bound on $\mathbb{P}(\text{success})$ (see Lemma 2), we have to solve for $\sup\{\lambda_p \bar{c}\}$ under the constraint

$$\lambda_p \bar{c} + \bar{c} f^* \leq \frac{1}{R^2 T^{2/\alpha} C(\alpha)} \ln\left(\frac{1}{1-\epsilon}\right) = \frac{C_p(\epsilon, T)}{1-\epsilon} \quad (36)$$

So we have $C_l(\epsilon, T) = C_p(\epsilon, T)$. This solution requires $\lambda_p \to \infty$, while $\bar{c} \to 0$, such that $\bar{c}\lambda_p = C_p(\epsilon, T)$, which is a degenerate Poisson case. □

With some more work an upper bound can be established:

Lemma 5 (Upper bound) *Let* $\rho(T) = k/\beta$ *with* $k = \int \beta(R, y) f(y) dy$, $\beta = \min\{\sup\{f(x)\} R^2 T^{2/\alpha} C(\alpha), 1\}$. *For* $\epsilon < 1 - e^{-\rho(T)}$, *we have*

$$C(\epsilon, T) \leq C_u(\epsilon, T) = C_p(\epsilon, T) \quad (37)$$

From the two lemmata follows:

Theorem 6 (Transmission capacity I) *For* $\epsilon \leq 1 - e^{-\rho(T)}$ *we have* $C(\epsilon, T) = C_p(\epsilon, T)$.

So for small ϵ, the transmission capacity is equal to the PPP of the same intensity. This capacity is achieved when $\lambda_p \to \infty$ and $\bar{c} \to 0$. This is the scenario in which the cluster process becomes a degenerate PPP. This is due to the definition of the transmission capacity as $C(\epsilon, T) := \sup\{\lambda_p \bar{c} : \lambda_p > 0, \bar{c} > 0, \text{outage-constraint}\}$. Here we have two variables to optimize over for a simple and single function optimization. So we try to fix λ_P as constant and find the transmission capacity with respect to $\bar{c}$. So the new transmission capacity is

$$C^*(\epsilon, T) := \lambda_p (1-\epsilon) \sup\{\bar{c} : \bar{c} > 0, \text{outage-constraint}\} \quad (38)$$

We can establish the following bounds for $C^*(\epsilon, T)$:

Theorem 7 (Transmission capacity II)

$$\frac{\lambda_p C_p(\epsilon, T)}{\lambda_p + f^*} \leq C^*(\epsilon, T) \leq \frac{\lambda_p C_p(\epsilon, T)}{\lambda_p - \beta^* \frac{C_p(\epsilon, T)}{(1-\epsilon)}} \quad (39)$$

IV. Concluding Remarks

We have derived interference and capacity results for Neyman-Scott processes. The main conclusions are that, compared to the PPP, the outage is larger for small transmission distances but smaller for large distances, and that clustering reduces the achievable transmission capacity.

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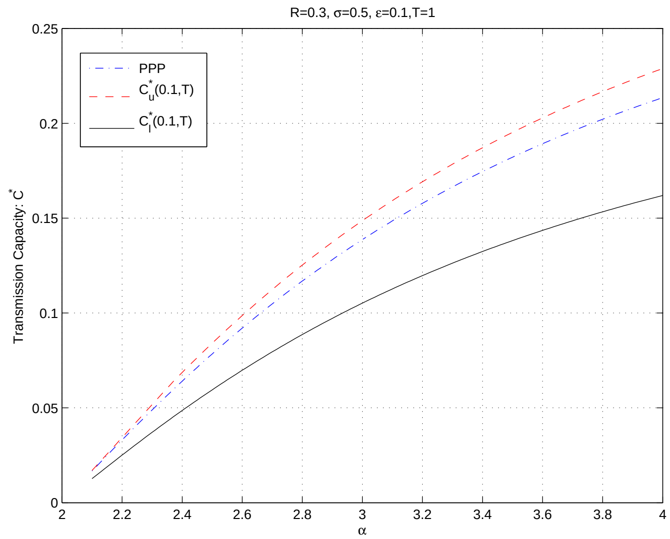

Fig. 4. Transmission capacity bounds for clustered process, $R = 0.3$, $\sigma = 0.5$, $\epsilon = 0.1$, $T = 1$, $\lambda_p = 1$. Note that $\bar{c}$ and the intensity of the PPP are functions of α.

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