On Decoding the kth Strongest User in Poisson Networks with Arbitrary Fading Distribution

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***Abstract*—Consider a d-dimensional network whose transmitters form a non-uniform Poisson point process and whose links are subject to arbitrary fading. Assuming interference from the $k-1$ strongest users is canceled, we derive the probability of decoding the k-th strongest user in closed-form. As a special case, when $k=1$, this probability is the *standard* coverage probability.**

This analytical result has immediate applications in networks with successive interference cancellation (SIC) capability. We use it to find closed-form upper and lower bounds on the probability of decoding at least k users and the mean number of successively decodable users. These bounds show that transmitter clustering is beneficial in exploiting SIC.

I. Introduction

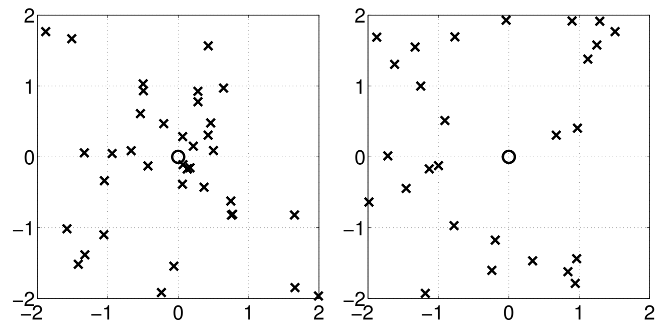
In random wireless networks, the *standard* coverage probability is the probability that a typical receiver in the network successfully decodes the message from the strongest transmitters in the presence of interference from the other transmitters. It has been studied extensively in the literature [1], [2].

This paper considers a more general type of coverage probability, *i.e.,* the probability of decoding the kth strongest user (transmitter) given that all the $k-1$ stronger users are canceled[1]. When $k=1$, it reduces to the standard coverage probability.

More specifically, we consider a d-dimensional interference-limited wireless network, where the user distribution is governed by a non-uniform Poisson point process (PPP) with a power-law density function. The links are subject to (spatially) iid fading with arbitrary distribution. We show that the coverage probability can be written in closed-form when the decoding signal-to-interference ratio (SIR) threshold is at least one.

Among the many possible applications of this general result, this paper provides a straightforward yet non-trivial one in wireless networks with successive interference cancellation (SIC) capability. In particular, we consider a Poisson network where all the users transmit at the same power and the same rate and the receivers attempt to decode as many users as possible. We demonstrate how the general coverage result can lead to much sharper system performance estimate than the ones in the prior works when the decoding SIR threshold is above or close to one.

[1]The cancellation can be carried out by successive interference cancellation, spatial interference cancellation, or simply local coordination.



(a) $b=-1$ (b) $b=1$

Fig. 1: Realizations of two non-uniform PPP with intensity function $\lambda(x)=3\|x\|^b$ with different b, where × denotes an active transmitter and o denotes the receiver at the origin.

II. System Model and Metrics

A. The Power-law Poisson Network with Fading (PPNF)

Let the receiver be at the origin o and the active transmitters be represented by a marked Poisson point process (PPP) $\hat{\Phi}=\{(x_i,h_{x_i})\}\subset\mathbb{R}^d\times\mathbb{R}^+$, where x is the location of a user, h_x is the iid (power) fading coefficient associated with the link from x to o, and d is the number of dimensions of the space. When the density function of the ground process $\Phi\subset\mathbb{R}^d$ is $\lambda(x)=a\|x\|^b,\ a>0,\ b\in(-d,\alpha-d)$, where $\|x\|$ is the distance from $x\in\mathbb{R}^d$ to the origin and α is the path-loss exponent, we refer this network as a *power-law Poisson network with fading (PPNF).* The condition $b\in(-d,\alpha-d)$ is needed in order to maintain a finite total received power at o and will be revisited later.

Fig. 1 shows realizations of two 2-d PPNFs with different b; Fig. 1a represents a network clustered around o whereas the network in Fig. 1b is sparse around the receiver at o. In general, the smaller b, the more clustered the network is at the origin, and $b=0$ refers to uniform networks.

B. Decoding the kth Strongest User

Without loss of generality, consider the case where all the nodes (users) transmit with unit power. Then, with an SIR model, a particular user at $x\in\Phi$ can be successfully decoded (without interference cancellation) iff

$$\text{SIR}_x = \frac{h_x\|x\|^{-\alpha}}{\sum_{y\in\Phi\setminus\{x\}} h_y\|y\|^{-\alpha}} > \theta,$$

where $h_x\|x\|^{-\alpha}$ is the received signal power from x, $\sum_{y\in\Phi\setminus\{x\}} h_y\|y\|^{-\alpha}$ is the aggregate interference from the other active transmitters, and θ is the SIR decoding threshold.

If we order the users in Φ by their received power at o, *i.e.,* $h_{x_i}\|x_i\|^{-\alpha} > h_{x_j}\|x_j\|^{-\alpha}$, $\forall i < j$, and consider the the case where the strongest $k-1$ users can be canceled. The kth strongest user can be decoded iff

$$\text{SrIR}_k \triangleq \frac{h_{x_k}\|x_k\|^{-\alpha}}{\sum_{i=k+1}^{\infty} h_{y_i}\|y_i\|^{-\alpha}} > \theta, \tag{1}$$

where SrIR_k denotes the signal to *residual* interference ratio at the kth strongest user. The first order goal of this paper is to quantify $\mathbb{P}(\text{SrIR}_k > \theta)$, a quantity useful in studying wireless networks with interference cancellation capability [3].

Note that $\mathbb{P}(\text{SrIR}_1 > \theta)$ is the *standard* coverage probability, *i.e.,* the probability of connecting to at least one of transmitters in the network, which has been extensively studied in the context of uniform Poisson networks [1], [2].

III. The path loss Process with fading (PLPF)

We use the unified framework introduced in [4] to jointly address the randomness from fading and random location of the nodes. In particular, we define the path loss process with fading (PLPF) as $\Xi \triangleq \{\xi_i = \frac{\|x_i\|^\alpha}{h_{x_i}}\}$, where $x_i \in \Phi$ are nodes in the PPNF, the indices i are introduced in the way such that $\xi_i < \xi_j$ for all $i < j$. Then, we have the following lemma, which directly follows from the mapping theorem [5].

Lemma 1. *The PLPF Ξ, corresponding to a PPNF, is a one-dimensional PPP on $\mathbb{R}^+$ with intensity measure $\Lambda([0,r]) = a\delta c_d r^\beta \mathbb{E}[h^\beta]/\beta$, where $\delta \triangleq d/\alpha$, $\beta \triangleq \delta + b/\alpha \in (0,1)$ and h is the iid fading coefficient.*

In Lemma 1, the condition $\beta \in (0,1)$ corresponds to the condition $b \in (-d, \alpha - d)$ in the definition of PPNF and is necessary in the sense that otherwise the aggregate received power at o is infinite almost surely[2].

Since for all $\xi_i \in \Xi \subset \mathbb{R}^+$, ξ_i^{-1} is the ith strongest received power component (at o) from the users in Φ, by definition, we can rewrite the decoding probability by

$$\mathbb{P}(\text{SrIR}_k > \theta) = \mathbb{P}(\xi_k^{-1} > \theta I_k),$$

where $I_k = \sum_{i=k+1}^{\infty} \xi_i^{-1}$, and in the following we will use $\mathbb{P}(\text{SrIR}_k > \theta)$ and $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ interchangeably. Furthermore, we have the following proposition which significantly simplifies the analysis in the rest of the paper.

Proposition 1 (Scale-invariance)**.** *If $\Xi = \{\xi_i\}$ and $\bar{\Xi} = \{\bar{\xi}_i\}$ are two PLPFs with intensity measures $\Lambda([0,r]) = r^\beta$ and $\bar{\Lambda}([0,r]) = Cr^\beta$ respectively, where C is any positive constant, then $\mathbb{P}(\xi_k^{-1} > \theta I_k) = \mathbb{P}(\bar{\xi}_k^{-1} > \theta \bar{I}_k)$, $\forall k \in \mathbb{N}$, where $\bar{I}_k = \sum_{i=k+1}^{\infty} \bar{\xi}_i^{-1}$.*

Proof: Consider the mapping $f(x) = C^{-1/\beta}x$. Then $f(\Xi)$ is a PPP on $\mathbb{R}^+$ with intensity measure Cx^β over the set $[0,x]$. Therefore, we have

$$\begin{aligned}\mathbb{P}(\xi_k^{-1} > \theta I_k) &= \mathbb{P}(C^{-1/\beta}\xi_k^{-1} > \theta \sum_{i=k+1}^{\infty} C^{-1/\beta}\xi_i^{-1}) \\ &\overset{\text{(a)}}{=} \mathbb{P}(\bar{\xi}_k > \theta \bar{I}_k)\end{aligned}$$

where (a) is because both $f(\Xi)$ and $\bar{\Xi}$ are PPPs on $\mathbb{R}^+$ with intensity measure $\mu([0,r]) = Cr^\beta$. ■

Proposition 1 shows that a constant prefactor in the density of the PLPF does not affect $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ in the PPNF. Combining it with Lemma 1, where it is shown that, in terms of the PLPF, the only difference introduced by different fading distributions is a constant factor in the density function, we immediately obtain the following corollary.

Corollary 1 (Fading-invariance)**.** *In a PPNF, $\mathbb{P}(\text{SrIR}_k > \theta)$ does not depend on the fading distribution.*

If we define the *Standard PLPF (SPLPF)* Ξ_β as a one-dimensional PPP with intensity measure $\Lambda([0,r]) = r^\beta$, where $\beta \in (0,1)$, we have the following fact which directly follows from Proposition 1 and Corollary 1.

Fact 1. $\mathbb{P}(\text{SrIR}_k > \theta)$ *in a PPNF can be determined by (only) studying of Ξ_β which encompasses any fading distribution and any values of a, b, d, α, with $\beta = \delta + b/\alpha = (d+b)/\alpha$.*

Thanks to Fact 1, in the rest of the paper, we will focus on the SPLPF, but the results hold for all PPNFs.

IV. Main Results

A. The Main Theorem

The main contribution of this paper is a closed-form characterization of $\mathbb{P}(\text{SrIR}_k > \theta)$. To derive the main theorem, we need the following lemma.

Lemma 2. *For an arbitrary k-element index set $\mathcal{K} \subset \mathbb{N}$ and an increasingly ordered PLPF $\Xi = \{\xi_i\}$, we have*

$$\xi_i^{-1} > \theta \sum_{j\notin\mathcal{K}} \xi_j^{-1},\ \forall i \in \mathcal{K} \Longrightarrow \xi_i^{-1} > \theta \sum_{j>k} \xi_j^{-1},\ \forall i \in [k].$$

Moreover, if $\theta \geq 1$,

$$\xi_i^{-1} > \theta \sum_{j\notin\mathcal{K}} \xi_j^{-1},\ \forall i \in \mathcal{K} \Longrightarrow \mathcal{K} = [k].$$

Proof: The first part of the lemma is obviously true when $\mathcal{K} = [k]$. If not, for any $l \in \mathcal{K}\setminus[k]$, we have $\xi_i^{-1} > \xi_l^{-1}$, $\forall i \in [k]$ by the ordering of Ξ. For the same reason, we have $\sum_{j\notin\mathcal{K}} \xi_j^{-1} > \sum_{j\notin[k]} \xi_j^{-1}$. As $\xi_l^{-1} > \theta\sum_{j\notin\mathcal{K}} \xi_j^{-1}$, we have $\xi_i^{-1} > \sum_{j\notin[k]} \xi_j^{-1}$, $\forall i \in [k]$.

To show the second part, consider an arbitrary $l \in \mathcal{K}$. Since all elements in Ξ are positive and $\theta \geq 1$, $\xi_l^{-1} > \theta\sum_{j\notin\mathcal{K}} \xi_j^{-1}$ implies $\xi_l < \xi_j$, $\forall j \notin \mathcal{K}$, and consequently $\mathcal{K} = [k]$. ■

[2]If $b < -d$, the PPP is not locally finite, which leads to infinite received power.

Next, we state the main theorem of this paper, which gives a closed-form expression for $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ when $\theta \geq 1$.

Theorem 1. *For $\theta \geq 1$,*

$$\mathbb{P}(\xi_k^{-1} > \theta I_k) = \frac{1}{\theta^{k\beta}\Gamma(1+k\beta)\big(\Gamma(1-\beta)\big)^k}, \quad (2)$$

where $\Gamma(\cdot)$ is the gamma function. Moreover, the RHS of (2) is an upper bound on $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ when $\theta < 1$.

Proof: Consider a 1-d Poisson (nonfading) network $\Phi \subset \mathbb{R}^+$ with intensity measure $\Lambda([0,r]) = r^\beta$ and its corresponding SPLPF $\Xi_\beta = \{\xi_i\}$[3]. For each element $x \in \Phi$ we introduce a mark h_x with iid exponential distribution with unit mean. In the following, we will refer this network as a Poisson networks with induced fading (PNIF) $\hat{\Phi} \subset \mathbb{R}^+ \times \mathbb{R}^+$. Similar as before, based on $\hat{\Phi}$, we can construct a PLPF $\hat{\Xi} \triangleq \Xi(\hat{\Phi}) = \{\hat{\xi}_i\}$ by collecting and ordering all the elements of $\{h_x x^{-1},\ \forall x \in \Phi\}$. By Proposition 1 and Corollary 1, we know

$$\mathbb{P}(\xi_k > \theta I_k) = \mathbb{P}(\hat{\xi}_k > \theta \hat{I}_k),\ \forall k \in \mathbb{N}, \quad (3)$$

where $\hat{I}_k = \sum_{i=k+1}^{\infty} \hat{\xi}_i^{-1}$. Therefore, in the following, we focus on the PNIF $\hat{\Phi}$ and the corresponding PLPF $\hat{\Xi}$.

First, considering a k-tuple of positive numbers $\mathbf{y} = (y_i)_{i=1}^k \in (\mathbb{R}^+)^k$, with slight abuse of notation, we say $(y_i)_{i=1}^k \subset \Phi$ if and only if $y_i \in \Phi,\ \forall i \in [k]$. Conditioned on $\mathbf{y} \subset \Phi$, we denote the interference from the rest of the network $\sum_{x\in\Phi\setminus\mathbf{y}} h_x x^{-1}$ as $I^{!\mathbf{y}}$. Since $\{y_i\}$ is a set of Lebesgue measure zero, by Slivnyak's theorem, we have $I^{!\mathbf{y}} \overset{\mathrm{d}}{=} I = \sum_{x\in\Phi} h_x x^{-1}$. Thus,

$$\begin{aligned}\mathcal{L}_I^{!\mathbf{y}}(s) &\triangleq \mathbb{E}[\exp(-sI^{!\mathbf{y}})] = \mathcal{L}_I(s)\\ &= \exp\left(-\mathbb{E}_h\left(\int_0^\infty \left(1-\exp(-shr^{-1})dr^\beta\right)\right)\right)\\ &= \exp\left(-\frac{s^\beta}{\operatorname{sinc}\beta}\right), \end{aligned} \quad (4)$$

where $\operatorname{sinc} x = \frac{\sin(\pi x)}{\pi x}$.

Second, let $\hat{\mathcal{N}}$ be the sample space of $\hat{\Phi}$ and consider the indicator function $\bar{\chi}_k : (\mathbb{R}^+ \times \mathbb{R}^+)^k \times \hat{\mathcal{N}} \to \{0,1\}$ defined as

$$\bar{\chi}_k\big((x_i,h_{x_i})_{i=1}^k, \hat{\phi}\big) \triangleq \begin{cases} 1, & \text{if } h_{x_i}x_i^{-1} > \theta\sum_{y\in\phi\setminus\{x_i\}} h_y y^{-1},\ \forall i \in [k] \\ 0, & \text{otherwise,}\end{cases}$$

where the k-element set $\{x_i\} \subset \phi$ and ϕ is the ground pattern of the marked point pattern $\hat{\phi}$. In words, $\bar{\chi}_k\big((x_i,h_{x_i})_{i=1}^k, \hat{\phi}\big)$ is one iff k of the users in the network $(x_i)_{i=1}^k$ all have received power larger than θ times the interference from the rest of the network. Then, for any $\hat{\phi}$ and $k \in \mathbb{N}$,

$$\begin{aligned}1_{\{\hat{\xi}_k > \theta\hat{I}_k\}}(\hat{\phi}) &= 1_{\{\hat{\xi}_i > \theta\hat{I}_k,\ \forall i\in[k]\}}(\hat{\phi})\\ &\overset{\text{(a)}}{\leq} \frac{1}{k!}\sum_{x_1,\ldots,x_k\in\phi}^{\neq} \bar{\chi}_k\big((x_i,h_{x_i})_{i=1}^k, \hat{\phi}\big), \end{aligned} \quad (5)$$

[3]Clearly, Φ and Ξ are statistically equivalent. However, there is a subtle difference: Ξ_β is increasingly ordered while Φ is unordered.

where $\neq$ means $x_i \neq x_j,\ \forall i \neq j$ and (a) is due to the first part of Lemma 2. Also, the second part of Lemma 2 shows that when $\theta \geq 1$ the equality in (a) holds.

Therefore, we have

$$\begin{aligned}\mathbb{P}(\hat{\xi}_k^{\,-1} > \theta\hat{I}_k) &= \mathbb{E}[1_{\{\hat{\xi}_k > \theta\hat{I}_k\}}(\hat{\Phi})]\\ &\overset{\text{(b)}}{\leq} \frac{1}{k!}\mathbb{E}\left[\sum_{x_1,\ldots,x_k\in\Phi}^{\neq} \bar{\chi}_k\big((x_i,h_{x_i})_{i=1}^k, \hat{\Phi}\big)\right]\\ &= \frac{1}{k!}\mathbb{E}_\Phi\left[\sum_{x_1,\ldots,x_k\in\Phi}^{\neq} \mathbb{E}\left[\bar{\chi}_k\big((x_i,h_{x_i})_{i=1}^k, \hat{\Phi}\big)\right]\right]\\ &\overset{\text{(c)}}{=} \frac{1}{k!}\mathbb{E}_\Phi\left[\sum_{\mathbf{x}:\ x_1,\ldots,x_k\in\Phi}^{\neq} \mathcal{L}_I^{!\mathbf{x}}(\theta\sum_{i=1}^k x_i)\right]\\ &\overset{\text{(d)}}{=} \frac{1}{k!}\int_{(\mathbb{R}^+)^k} \mathcal{L}_I^{!\mathbf{x}}(\theta\sum_{i=1}^k x_i)\Lambda^{(k)}(\mathrm{d}\mathbf{x}),\end{aligned}$$

where (b) is due to (5) and the equality holds when $\theta \geq 1$, (c) holds since h_y are iid exponentially distributed with unit mean for all $y \in \Phi$, and (d) is due to the definition of $\Lambda^{(k)}(\cdot)$, the kth factorial moment measure of Φ. Since Φ is a PPP with intensity measure $\Lambda([0,r]) = r^\beta$, we have $\Lambda^{(k)}(\mathrm{d}\mathbf{x}) = \prod_{i\in[k]} \mathrm{d}x_i^\beta$. Applying (3) and (4), we have

$$\mathbb{P}(\xi_k^{-1} > \theta I_k) \leq \frac{1}{k!}\int_{(\mathbb{R}^+)^k} \exp\left(-\frac{\theta^\beta}{\operatorname{sinc}\beta}\|\mathbf{x}\|_{\frac{1}{\beta}}\right)\mathrm{d}\mathbf{x}.$$

where $\|\cdot\|_p$ denotes the L_p norm, and the equality holds when $\theta \geq 1$. The integral on the RHS can be further simplified into closed-form by using the general formulas in [6, eqn. 4.635], which completes the proof. ■

B. The Standard Coverage Probability

Taking $k = 1$, we obtain the following corollary of Theorem 1, which gives the exact standard coverage probability in a PPNF for $\theta \geq 1$ and a upper bound for general θ.

Corollary 2. *For $\theta \geq 1$, we have*

$$p_1 = \mathbb{P}(\xi_1^{-1} > \theta I_1) = \frac{\operatorname{sinc}\beta}{\theta^\beta}, \quad (6)$$

and the RHS is an upper bound on $\mathbb{P}(\xi_1^{-1} > \theta I_1)$ when $\theta < 1$.

It is worth noting that the closed-form expression in Corollary 2 has been discovered in several special cases. For example, [1] derived the equality part of (6) for the Rayleigh fading case in 2-d uniform Poisson networks, and [2] further showed that the equality is true for arbitrary fading distribution. However, none of the existing works derives the results in Corollary 2 in as much generality as here. Using the PLPF-based framework, we proved that (6) holds for arbitrary fading (including the no-fading case) in d-dimensional PPNFs (which includes non-uniform user distribution).

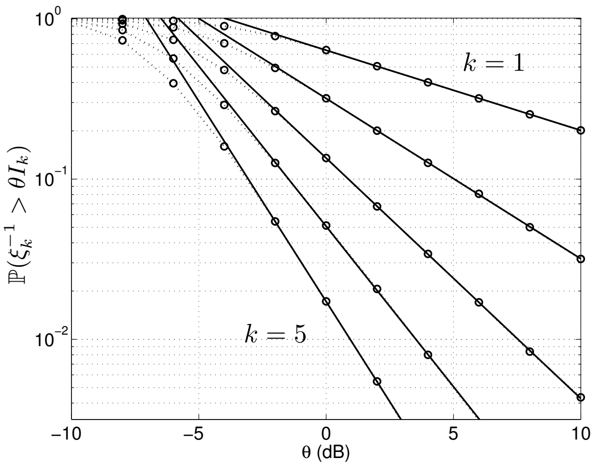


Fig. 2: Comparison between simulation and the upper bound in Theorem 1 for $\beta = \frac{1}{2}$ and $k = 1, 2, 3, 4, 5$ (upper to lower).

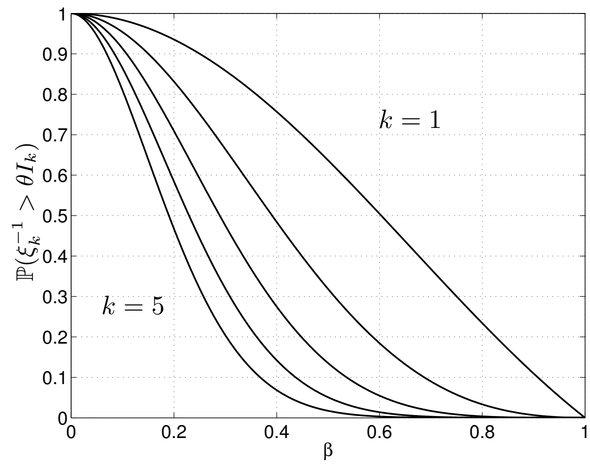


Fig. 3: $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ given by (2) as a function of β for $\theta = 1$ and $k = 1, 2, 3, 4, 5$ (upper to lower).

C. Numerical Evaluation

Corollary 3. *When $\beta = 1/2$,*

$$\mathbb{P}(\xi_k^{-1} > \theta I_k) = \frac{1}{(\pi\theta)^{\frac{k}{2}}\Gamma(\frac{k}{2}+1)}, \tag{7}$$

and the RHS is an upper bound on $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ when $\theta < 1$.

Fig. 2 compares (7) with simulation results for $-10\text{dB} < \theta < 10\text{dB}$ and $k = 1, 2, 3, 4, 5$. We found that (7) is not only exact for $\theta \geq 1$ but also quite accurate for $\theta > -4\text{dB}$, which is consistent with the observation in [1]. Moreover, we can see that with larger k, the regime where (2) is accurate extends to smaller θ, which gives us further confidence in applying Theorem 1 in the context of successive interference cancellation in Section V.

Fig. 3 plots $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ for the case $\theta = 1$ and for $k = 1, 2, 3, 4, 5$. This figure demonstrates how the decoding probability changes as a function of β.

V. Successive Interference Cancellation

An immediate application of Theorem 1 is to provide estimates on the performance of successive interference cancellation (SIC). Under the PLPF-based framework, [3] shows that the usefulness of SIC in random wireless networks is fundamentally limited by the network geometry, *i.e.,* number of dimensions, path loss exponent, density function. This fundamental limit can be characterized by

$$\begin{aligned} p_k &\triangleq \mathbb{P}(\xi_i^{-1} > \theta I_i,\ \forall i \leq k), \\ &= \mathbb{P}(\xi_i^{-1} > \theta I_i,\ \forall i < k \mid \xi_k > \theta I_k)\mathbb{P}(\xi_k^{-1} > \theta I_k) \end{aligned} \tag{8}$$

the probability of successively decoding at least k users in the network. Furthermore, the conditional probability in (8) can be estimated using properties of the 1-d PPP and the order statistics of uniform random variables, and thus p_k can be bounded in terms of $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ [3, Lemma 2]. Therefore, we have the following proposition, which follows immediately from Theorem 1.

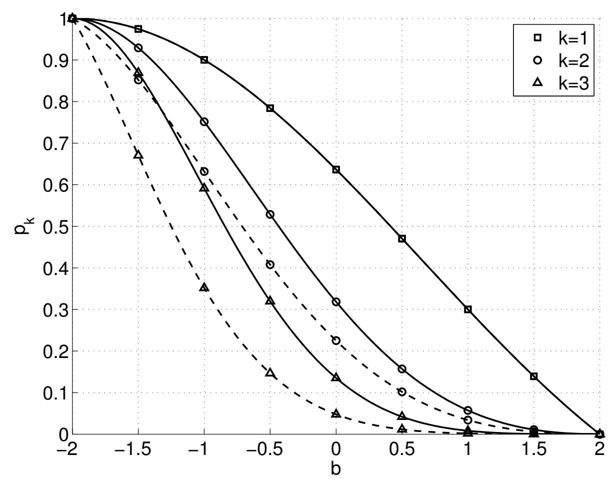


Fig. 4: Upper and lower bounds (in Proposition 2) on p_k ($k = 1, 2, 3$) in a 2-d network with path loss exponent $\alpha = 4$, $\theta = 1$ and density function $\lambda(x) = a\|x\|^b$. $b = 0$ is the uniform case. Here, solid lines are the upper bounds and dashed lines are the lower bounds.

Proposition 2. *For $\theta \geq 1$ and $\Xi_\beta = \{\xi_i\}$, we have*

$$p_k \geq \frac{1}{(1+\theta)^{\frac{\beta}{2}k(k-1)}\theta^{k\beta}\Gamma(1+k\beta)\big(\Gamma(1-\beta)\big)^k}$$

and

$$p_k \leq \frac{1}{\theta^{\frac{\beta}{2}k(k+1)}\Gamma(1+k\beta)\big(\Gamma(1-\beta)\big)^k}.$$

More generally, for all $\theta > 0$, we have

$$p_k \leq \frac{1}{\bar{\theta}^{\frac{\beta}{2}k(k-1)}\theta^{k\beta}\Gamma(1+k\beta)\big(\Gamma(1-\beta)\big)^k}, \tag{9}$$

where $\bar{\theta} = \max\{\theta, 1\}$.

Fig. 4 plots the upper and lower bounds in Proposition 2 as a function of b for $k = 1, 2, 3$. Here, the upper and lower bounds for the case $k = 1$ are both tight and overlapping. A similar plot is given in [3], which was produced based on a set of bounds constructed by completely different ideas. Nevertheless, the bounds in Fig. 4 are significantly sharper than the bounds there[4]. Since b determines the density scaling of the active transmitters, Fig. 4 shows networks clustering (small b) is desirable if SIC is allowed.

Since one of the critical issues in implementing successively decoding is the latency, Proposition 2 provides quantitative insights on the value of building stronger SIC at the receiver. For example, if the decoding SIR threshold is 0dB, Fig. 4 shows that in a uniform 2-d network ($b = 0$), the ability of successive canceling 2 or more users is only useful in at most about 10% of the cases (see p_3). This suggests building such SIC hardware in such networks may not be cost-effective.

Another important metric on the performance of SIC is the aggregate throughput [7]. In interference-limited networks, it is defined as the sum rate at the receiver,

$$R \triangleq \log(1+\theta)\mathbb{E}N, \tag{10}$$

where N is the number of users that can be successively decoded, *i.e.,* $\mathbb{E}N = \sum_{k=1}^{\infty} p_k$. Naturally, Proposition 2 gives rise to an upper bound on the aggregate throughput.

Proposition 3. *The mean number of decodable users is upper bounded by*

$$\mathbb{E}N \leq \sum_{k=1}^{K-1} \left(\frac{C(k)}{\Gamma(1-\beta)}\right)^k \frac{1}{\Gamma(1+k\beta)} + \frac{1}{\Gamma(1+K\beta)} \left(\frac{C(K)}{\Gamma(1-\beta)}\right)^K \frac{\Gamma(1-\beta)}{\Gamma(1-\beta)-C(K)},$$

where $C(k) \triangleq \theta^{-\beta}\bar{\theta}^{-\frac{\beta}{2}(k-1)}$.

Proof: Since $\mathbb{E}N = \sum_{k=1}^{\infty} p_k$, an upper bound on $\mathbb{E}N$ can be obtained by summing the bound in (9). The proposition follows by summing the bound for $k < K$ and then upper bounding the terms for $k \geq K$. ■

Due to the definition of aggregate throughput in (10), Proposition 3 leads to an upper bound on the aggregate throughput.

Fig. 5 compares this bound with the simulation. This figure shows that the bounds based on Proposition 3 are generally tight for $\theta > 1$. For larger β, the bound is also tight even if θ is slightly less than 1. As smaller β correspond to more clustered networks, Fig. 5 shows (again) that network clustering is desirable in terms of the aggregate throughput. Moreover, we observe that the aggregate throughput is a monotonically decreasing function of θ, suggesting that lower per user rate leads to higher sum rate.

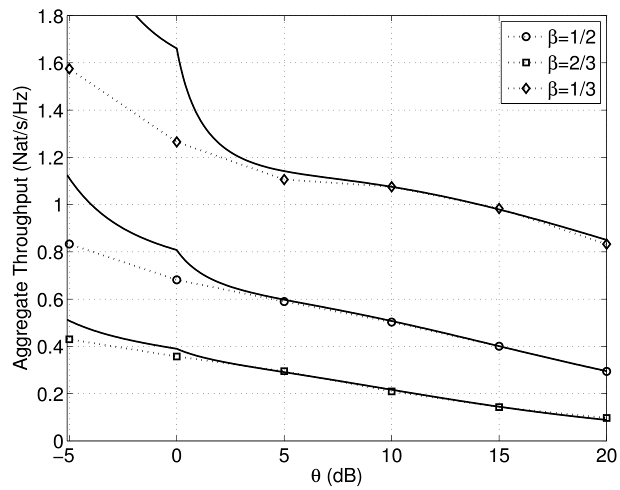


Fig. 5: Comparison between the aggregate throughput upper bound (based on Proposition 3) and the simulated aggregate throughput for $\beta = \frac{1}{3},\ \frac{1}{2},\ \frac{2}{3}$. In a 2-D uniform networks, the curves correspond to $\alpha = 6,\ 4,\ 3$. The bound is calculated choosing $K = 20$.

VI. Conclusions

This paper derives a closed-form expression for the probability of decoding the k-th strongest user in wireless networks for SIR threshold no less than 1. Derived under the PLPF-based framework, this expression encompasses all possible cases in d-dimensional power law Poisson networks with arbitrary fading distribution and path loss exponent.

The derived decoding probability is a generalized version of the conventional coverage probability and is useful in analyzing wireless networks with interference cancellation capabilities. In the case of SIC, we demonstrate how to use the result to obtain accurate system performance estimates. An important insight obtained is that transmitter clustering is desirable in networks with SIC capability. This points to a quite different MAC design paradigm compared with the one without SIC.

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[4]This does not mean that the bounds in Proposition 2 are better in general. The bounds in [3] are tighter when $\theta \ll 1$.