Routing in Ad Hoc Networks—A Wireless Perspective

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Abstract

Traditionally, the routing problem is addressed at the network layer, an approach that has been extended to the wireless realm. In wireless multihop networks, however, strict layer-base protocol design leads to substantial inefficiencies. This paper addresses the routing problem for large wireless ad hoc networks from a fundamental point of view, not constrained by particular protocol implementations or layered architectures, but taking into account the properties of the wireless channel. First, an analytical channel model is presented that is based on Rayleigh fading. It demonstrates how noise and interference effects can be separated, and how each interfering transmission affects the packet reception probability. Second, the distribution of node distances in networks with uniformly randomly placed nodes is derived. These two ingredients are used to discuss the benefits of different routing strategies. In particular, short-hop and long-hop routing schemes are compared. Further, cooperative strategies such as multipath routing and cooperative diversity are briefly discussed as techniques that are enabled by the broadcast nature of the wireless channel.

1. Introduction

As often pointed out, routing is a critical issue for ad hoc[1] wireless networks. While it has certainly been addressed extensively by the research community (see, *e.g.*, [21,27,31] and references therein), the proposed algorithms are often derived from their wired counterparts, and the specific and peculiar properties of the wireless channel are not considered. This is reflected in the type of models that are prevalently chosen to evaluate the protocol performance. In particular, the so-called *disk model* or *protocol model* [7,9,14,20,22,23,25,26,29] is very frequently used. In this deterministic model, the radius for a successful transmission has a deterministic value, irrespective of the condition of the wireless channel, and interference is taken into account using the same geometric disk abstraction. The three main shortcomings of this model are:

- It ignores the accumulated interference of a large number of distant nodes. In particular, it does not capture the problem of diverging interference when the path loss exponent it close to two.
- It wrongly suggests that increasing the transmit power (by the same factor) of all the nodes in the network leads to more packet collisions. This misconception stems from the fact that the transmit radius grows with the transmit power and any interferer within a receiver's disk is assumed to cause a collision.
- Stochastic variations in the channel (fading) are neglected.

In networks with mobile terminals, as they are often considered in the context of ad hoc networks, the last point is as severe as the other ones, since fading is an immediate and fundamental consequence of node mobility.

A more suitable model is the so-called *physical model* [9], where a certain signal-to-noise-and-interference ratio (SINR) is required for successful packet reception. This model is more accurate, in particular for systems with strong channel coding. Yet it is still fully deterministic and does not take into account the distribution of the (Gaussian) noise or the stochastic nature of the wireless channel (fading, shadowing). As pointed out in [4, 6], the volatility of the channel cannot be ignored in wireless networks; it is also easily demonstrated experimentally [5, 19]. In addition, the "prevalent all-or-nothing model" [24] leads to the assumption that a transmission over a multihop path either fails completely or is 100% successful, ignoring the fact that end-to-end packet loss probabilities increase with the number of hops (unless the transmit power is adapted).

1 In this paper, the term *ad hoc* implies multihop routing.

Fig. 1 shows typical relationships between the (normalized) transmission distance and the packet reception probability for an AWGN channel. In particular for smaller path loss exponents (left plot), the curve does not resemble an “all-or-nothing-model”. It can also be seen how coding affects the shape of the curve. Already a rather weak code, the (127,120) Hamming code, causes the curve to be steeper; the reception probability at distance 1 increases from 75% to 95%.

To overcome some of these limitations of the disk and deterministic “physical” models, we suggest a Rayleigh fading link model that relates transmit power, large-scale path loss, and the success of a transmission over multipath channels. It is a physical model that incorporates fading. A typical distance-reception probability curve is plotted in Fig. 2. It is much flatter than the AWGN curve, but it too grows steeper with increasing path loss exponent α. In fact, both the AWGN and Rayleigh curves tend to the disk model as the path loss exponent goes to infinity.

This model captures the relevant effects of the wireless channel: large-scale path loss, the broadcast nature of omnidirectional antennas, interference, and fading. It will be described in detail in the next section. We use it to offer a comprehensive perspective at the routing problem, unconstrained of the traditional OSI layer architecture and particular routing protocols.

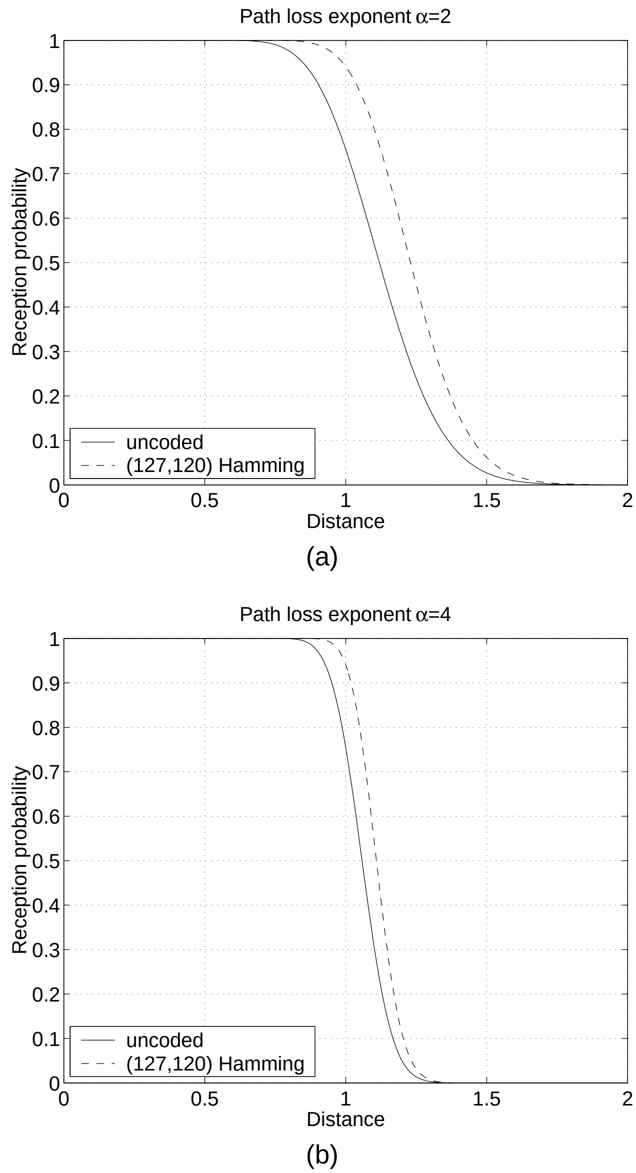


Figure 1. Reception probabilities of a 120 bit packet as a function of distance for an AWGN channel (BPSK modulation) for uncoded transmission and a (127,120) Hamming code. The path loss exponent is: (a) $\alpha = 2$, (b) $\alpha = 4$.

2. The Rayleigh Fading Link Model

We consider networks with Rayleigh block fading channel, which is justified not only in the case of TDMA or slow frequency hopping, but also when the packet transmission time approximately equals the coherence time of the channel (which is often the case for ad hoc networks) or when a random channel access scheme prevents a node from transmitting in multiple subsequent timeslots.

A transmission from node i to node j is successful if the SINR γ_{ij} is above a certain threshold Θ that is determined by the communication hardware, and the modulation and coding scheme [4]. The SINR γ is a discrete random process given by

$$\gamma = \frac{R}{N + I}\,. \tag{1}$$

R is the received power, which, in the case of Rayleigh fading, is exponentially distributed with mean $\bar{R}$. Over a transmission of distance $d = \|x_i - x_j\|_2$ with an attenuation d^α, we have $\bar{R} = P_0 d^{-\alpha}$, where P_0 is proportional to the transmit power[2]. N denotes the noise power, and I is the inter-

2 This equation does not hold for very small distances. So, a more accurate model would be $\bar{R} = P_0' \cdot (d/d_0)^{-\alpha}$, valid for $d \geqslant d_0$, with P_0' as the average value at the reference point d_0, which should be in the far field of the transmit antenna. At 916MHz, for example, the near field may extend up to 1m (several wavelengths).

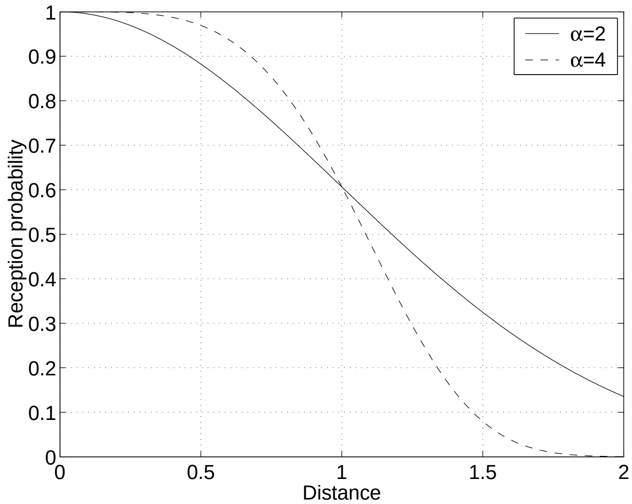


Figure 2. Reception probabilities over a Rayleigh fading channel for path loss exponents of 2 and 4.

ference power affecting the transmission, *i.e.*, the sum of the received power from all the undesired transmitters.

Theorem 1 *In a Rayleigh fading network, the mean reception probability* $p_r := \mathbb{E}[\mathbb{P}[\gamma \geqslant \Theta]]$ *can be factorized into the reception probability of a zero-noise network and the reception probability of a zero-interference network.*

Proof: Let R_0 denote the received power from the desired source and R_i, $i = 1, \dots, k$, the received power from k interferers. All the received powers are exponentially distributed, *i.e.*, $p_{R_i}(r_i) = 1/\bar{R}_i \, e^{-r_i/\bar{R}_i}$, where $\bar{R}_i$ denotes the average received power $\bar{R}_i = P_i d_i^{-\alpha}$. The probability of correct reception is

$$\begin{aligned} p_r =& \mathbb{E}\big[\mathbb{P}[R_0 \geqslant \Theta(I+N)]\big] && (2)\\ =& \mathbb{E}\Big[\exp\Big(-\frac{\Theta(I+N)}{\bar{R}_0}\Big)\Big] && (3)\\ =& \int_0^\infty \cdots \int_0^\infty \exp\Big(-\frac{\Theta(\sum_{i=1}^k r_i + N)}{\bar{R}_0}\Big) \cdot \\ & \prod_{i=1}^k p_{R_i}(r_i)\, \mathrm{d}r_1 \cdots \mathrm{d}r_k \\ =& \underbrace{\exp\Big(-\frac{\Theta N}{P_0 d_0^{-\alpha}}\Big)}_{p_r^N} \cdot \underbrace{\prod_{i=1}^k \frac{1}{1+\Theta \frac{P_i}{P_0}\big(\frac{d_0}{d_i}\big)^\alpha}}_{p_r^I}\,. && (4) \end{aligned}$$

p_r^N is the probability that the SNR $\gamma_N := R_0/N$ is above the threshold Θ, *i.e.*, the reception probability in a zero-interference network as it depends only on the noise. The second factor p_r^I is the reception probability in a zero-noise network. □

This allows an independent analysis of noise and interference. Note that (4) clearly shows that *power scaling*, *i.e.*, scaling the transmit powers of all the nodes by the same factor, does not change the SIR (p_r^I only depends on *power ratios*), but (slightly) increases the SINR.

From an information-theoretic point of view, (4) can be viewed as an outage probability; it therefore permits the derivation of the outage capacity of the Rayleigh block fading/interference channel.

In a zero-interference network, the reception probability over a link of distance d at a transmit power P_0, is given by $p_r := \mathbb{P}[\gamma_N \geqslant \Theta] = e^{-\frac{\Theta N}{P_0\, d^{-\alpha}}}$, thus the transmit power P_0 that is necessary for a packet reception probability p_r is

$$P_0 = \frac{d^\alpha \Theta N}{-\ln p_r}\,. \qquad (5)$$

Note that for high probabilities, the packet loss probability $1 - p_r$ is tightly upper-bounded by the normalized mean noise-to-signal ratio (NSR) $\Theta N/\bar{R}_0 = \Theta/\bar{\gamma}_N$ [12]. Since $-\ln p_r \approx 1 - p_r$, we can also say that the packet loss probability is inversely proportional to the transmit power for high p_r.

Considering the interference part, it can be seen that for each interferer there is a corresponding factor in the product expression for p_r^I. So, the impact of a channel access scheme on the number and distances of interfering nodes is directly reflected in this expression. Since p_r^I only depends on power ratios, it permits the calculation of the power-independent throughput limits. For example, what is the throughput limit in a long line network with equidistant nodes where the leftmost node is the source and every node transmits to its right neighbor? The optimum MAC scheme is to have every q-th node in the chain transmit in a given timeslot. The optimum q is, from (4), given by

$$g_{\max} = \max_{q\in\mathbb{N}} \Bigg(q \prod_{i=1}^{\infty} \underbrace{\big(1+\Theta(qi-1)^{-\alpha}\big)}_{\text{right neighbors}} \underbrace{\big(1+\Theta(qi+1)^{-\alpha}\big)}_{\text{left neighbors}} \Bigg)^{-1}.$$

This maximization is easily carried out numerically, since the terms $(1+iq)^{-\alpha}$ approach zero very quickly with increasing i. For an SIR threshold of $\Theta = 10$, maximum throughput levels of 0.0758, 0.1599, 0.2174, 0.2504 are achieved at $q = 8, 5, 4, 3$ for $\alpha = 2, 3, 4, 5$ [18].

3. Distances in Networks with Random Node Distribution

One of the main distinguishing features of wireless networks from wired ones is the fact that distances matter, since the path loss over a wireless channel is huge compared with the signal attenuation over a cable. So, in order to evaluate the performance of ad hoc networks, we need some information on the internode distances. Often, due to random deployment or mobility, a uniformly random distribution is a reasonable assumption. For large networks, the uniform distribution is equivalent to a Poisson point process for all practical purposes, so we will focus on the latter due to its analytical simplicity.

We offer a unified treatment of m-dimensional random networks by determining distributions, expected values, and higher moments of the distances to n-th neighbors for any number of dimensions m. Comparing one- and higher-dimensional networks, it has been pointed out in [3] that one-dimensional networks do not percolate. So, connectivity is easier to guarantee for larger m; on the other hand, the interference problem becomes more severe, since the large-scale path loss exponent α has to be larger than the number of dimensions m to keep the interference finite[3]. A fundamental difference between regular (equidistant) networks and random networks is the variance of the internode distance, which causes imbalance in throughput and energy consumption (in schemes with power control) or link reliability (in constant-power schemes). So, as will be discussed in Section 4, it is beneficial to have all nodes in a route transmit over approximately the same distances.

3.1. The generalized Weibull distribution

If nodes are distributed according to a Poisson point process with a density λ in an $m-$dimensional network, the probability of finding k nodes in a subset of measure A is given by the Poisson distribution

$$\mathbb{P}[k \text{ nodes in } A] = e^{-\lambda A} \frac{(\lambda A)^k}{k!} \,, \qquad (6)$$

This permits the calculation of the distance to an n-th neighbor in a straightforward manner:

Theorem 2 (Distance to n-th neighbor.) *In an m-dimensional random network with uniformly distributed nodes and density λ, the distance R_n between a node and its n-th neighbor is distributed according to the generalized Weibull distribution*

$$f_{R_n}(r) = e^{-\lambda c_m r^m} \frac{m\,(\lambda c_m r^m)^n}{r(n-1)!} \,, \qquad (7)$$

where $c_m r^m$ is the volume of the m-sphere of radius r.

Proof: Let $A_m(r) := c_m r^m$ be the volume of the m-sphere of radius r, and let S_k be the k-th coefficient in the Poisson distribution: $S_k := (\lambda A_m(r))^k/k!$. The complementary cumulative distribution function (cdf) of R_n is the probability that there are less than n nodes closer than r:

$$\begin{aligned} P_n :&= \mathbb{P}[0 \ldots n-1 \text{ nodes within } r] \qquad (8) \\ &= \sum_{k=0}^{n-1} S_k \, e^{-\lambda A_m(r)} \,. \end{aligned}$$

From $f_{R_n} = -\frac{dP_n}{dr}$, we have

$$\begin{aligned} f_{R_n} &= \lambda c_m m r^{m-1} \left(\sum_{k=0}^{n-1} S_k - \sum_{k=1}^{n-1} S_{k-1} \right) e^{-\lambda A_m(r)} \qquad (9) \\ &= \lambda c_m m r^{m-1} S_{n-1} e^{-\lambda A_m(r)} \qquad (10) \\ &= \frac{nm}{r} S_n e^{-\lambda A_m(r)} \,, \qquad (11) \end{aligned}$$

which is identical to (7). □

This distribution generalizes several well-known distributions. For $m = 1$, this is an Erlang distribution, for $n = 1$, this is a Weibull distribution, for $m = 1$ and $n = 1$, this is an exponential distribution (a special case of Erlang and Weibull), and for $m = 2$ and $n = 1$ (a very relevant case), this is a Rayleigh distribution (a special case of Weibull). If the factorial $(n-1)!$ is replaced by $\Gamma(n)$, the distribution is valid also for non-integer n, essentially generalizing the Γ distribution. The cdf of R_n can also be written as

$$F_{R_n}(r) = 1 - \frac{\Gamma_{\text{ic}}(n, \lambda c_m r^m)}{\Gamma(n)} \,, \qquad (12)$$

where $\Gamma_{\text{ic}}(\cdot,\cdot)$ is the incomplete Γ function.

Note that with the Poisson assumption, a simple statement can immediately be made on connectivity. Since the probability that no node lies on a disk of area πr^2 around a transmitter is $\exp(-\lambda \pi r^2)$, the transmit radius r has to grow at least with $\sqrt{\ln n}$ to keep a network with n nodes connected. This gives the right order for the power levels [8] and the number of neighbors [30] needed for connectivity.

Now, with the channel model and the knowledge about internode distances, we are ready to discuss the routing problem.

4. Routing in Rayleigh Networks

For efficient routing, *progress* should be made at each hop, *i.e.*, the next-hop neighbor should be closer to the destination. So, we have to determine the distance to a neighboring node that lies within an angle $0 < \phi \leqslant \frac{\pi}{2}$ of the source-destination axis[4]. For the two-dimensional case, this is illustrated in Fig. 3. In the distribution, this simply corre-

3 This is a straightforward generalization of a result in [24].

4 The angle between the source-destination vector and the vector to the next-hop neighbor must be smaller than ϕ.

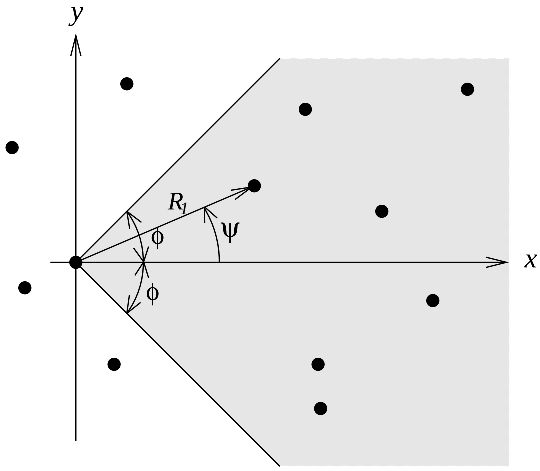


Figure 3. Part of a two-dimensional network with the source at the origin and the x-axis pointing towards the destination node. R_1 denotes the distance to the nearest neighbor within a sector ϕ around x, and ψ is its argument. Hence (R_1, ψ) are the polar coordinates of the nearest neighbor within a sector ϕ, and (X, Y) are its Cartesian coordinates.

sponds to a change of the volume from an m-sphere to an m-sector which has volume $c_{\phi,m} r^m$. For $m = 1, 2, 3$, we have $c_{\phi,1} = 1$, $c_{\phi,2} = \phi$, and $c_{\phi,3} = \frac{2\pi}{3}(1 - \cos\phi)$, respectively. Replacing c_m by $c_{\phi,m}$ in (7), the probability density function (pdf) of the distance to the n-th neighbor in a sector ϕ is given by

$$f_{R_n}(r) = e^{-\lambda c_{\phi,m} r^m} \frac{m\,(\lambda c_{\phi,m} r^m)^n}{r(n-1)!}\,. \tag{13}$$

The expected distance is

$$\begin{aligned}\mathbb{E}[R_n] &= \left(\frac{1}{\lambda c_{\phi,m}}\right)^{\frac{1}{m}} \frac{\Gamma\left(n + \frac{1}{m}\right)}{\Gamma(n)} \\ &= \left(\frac{1}{\lambda c_{\phi,m}}\right)^{\frac{1}{m}} (n)_{1/m}\,,\end{aligned} \tag{14}$$

where $(n)_{1/m}$ is the Pochhammer symbol notation. Irrespective of the particular channel model, the energy consumption is proportional to R^α. So, the expected energy consumption is given by the higher moments[5]

$$\begin{aligned}\mathbb{E}[R_n^\alpha] &= \left(\frac{1}{\lambda c_{\phi,m}}\right)^{\frac{\alpha}{m}} \frac{\Gamma\left(n + \frac{\alpha}{m}\right)}{\Gamma(n)} \\ &= \left(\frac{1}{\lambda c_{\phi,m}}\right)^{\frac{\alpha}{m}} (n)_{\alpha/m}\,.\end{aligned} \tag{15}$$

5 Note that α does not have to be an integer.

Remarks:
(a) m and α have complementary roles: m-dimensional networks with path loss exponent α require approximately the same energy for transmission to the n-th neighbor as km-dimensional networks with path loss exponent $k\alpha$, the difference coming from the different coefficients $c_{\phi,m}$.
(b) As a function of n, the Pochhammer sequence $(n)_{\alpha/m}$ grows as $n^{\alpha/m}$. So, for $m = 2$, the expected distance grows as $\sqrt{n}$.
(c) For $m = 1$, the variance is n/λ^2, *i.e.*, n times the variance of the exponential distribution, as expected from the variance of the Erlang distribution with parameter n.
(d) For $m = 2$, the variance is tightly bounded[6] for all n: $(1 - \pi/4)/\phi \leqslant \mathrm{Var}[R_n] < 1/\phi$ for all $n \geqslant 1$. For $m > 2$, the variance goes to 0 with increasing n.
(e) As a function of m, the variance is monotonically decreasing and approaches zero in the limit $m \to \infty$.
(f) Interestingly, $\lim_{m\to\infty} \mathbb{E}[R_n] = 1$ for any fixed λ and n.

4.1. Maximum energy consumption in a route

The lifetime of a k-hop route is determined by the node that consumes the most energy. The expected maximum of the per-hop energy consumptions $E_1, \ldots, E_k$ is

$$\begin{aligned}\bar{E}_{\max} &= \mathbb{E}[\max\{E_1, \ldots, E_k\}] \\ &= \int_0^\infty (1 - F_E(r)^k)\,dr\,,\end{aligned} \tag{16}$$

where $E_i = (R_n^\alpha)_i$ and $F_E(\cdot)$ is the cdf of R^α. For nearest-neighbor routing ($n = 1$) and $\alpha = 2$, the node distances are exponentially distributed. The maximum of k i.i.d. exponential random variables with mean 1 is known to be given by the harmonic sum $\sum_{i=1}^k 1/i$, which is tightly lower-bounded by $\ln k + \gamma_{\mathrm{em}}$, where $\gamma_{\mathrm{em}} \approx 0.577$ is the Euler-Mascheroni constant. Now, from Jensen's inequality we can conclude that

$$\bar{E}_{\max} \gtrsim \left(\mathbb{E}[R_1^2](\ln k + \gamma_{\mathrm{em}})\right)^{\alpha/2}\,, \tag{17}$$

thus the maximum energy consumption over a k-hop connection grows with at least $\ln k$. For $\alpha = 3$ and different n, this (normalized) maximum energy consumption is displayed in Fig. 4.

Clearly, the main problem is the large variance in the expected energy consumption. To decrease this variance, the principle of nearest-neighbor routing has to be abandoned, and all nodes should transmit approximately over the same distance, thereby emulating a network with regular topology. With the framework we have developed, the performance of routing algorithms can be compared analytically.

6 The lower bound is the variance of the Rayleigh distribution, the upper bound can be derived from Stirling's approximation, letting $n \to \infty$.

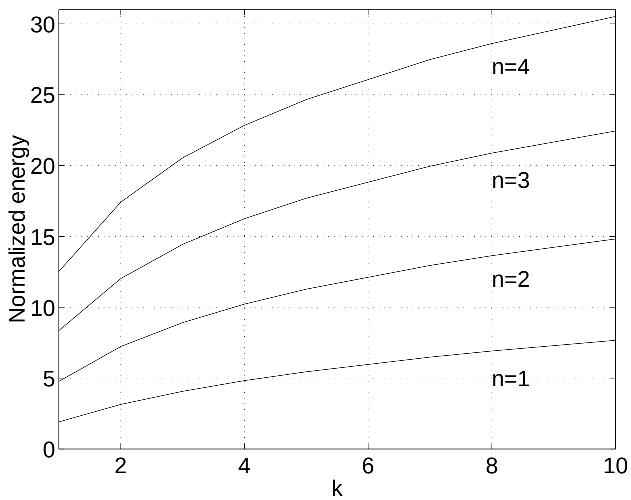


Figure 4. Maximum per-node energy consumption over a k-hop route in a two-dimensional network for different n with $\phi = \pi/4$, $\lambda = 1$, and $\alpha = 3$.

In [13], different strategies are studied, and one of the conclusions is that routing over many short hops is not as advantageous as it seems to be, in particular if the delay is taken into account: a routing scheme with long hops can benefit from time diversity in the form of retransmissions. In the following section, we will give a number of reasons why longer hops may be preferred.

4.2. Short-hop vs. long-hop routing

The question whether routing over many short hops or fewer but longer hops is more efficient is certainly very relevant. Intuitively, with path loss exponents ranging from 2 to 4, short-hop routing seems to be more energy-efficient while long-hop routing has the edge in terms of delay. But what if both schemes have the same delay constraints? Which one is more energy-efficient?

Short-hop routing has a lot of support, and its proponents mainly produce the following two arguments:

1. Energy consumption. If a long hop of distance d is divided into k hops of distance d/k, the energy benefit is often assumed to be $k^{\alpha-1}$.
2. Capacity. The shorter the hops, the higher the transport capacity in an interference-limited network [9]. This is not due to less interference but due to higher received power levels.

The first argument stems from an oversimplified analysis of the energy consumption and neglects important issues such as delay, end-to-end reliability, and bias power consumption. The second argument is only valid as long as the connectivity of the network is guaranteed, was derived for an increasingly dense network that takes advantage of the singularity of the attenuation $d^{-\alpha}$ at $d = 0$,[7] and also neglects delay.

Interference. According to [4], “It is unclear whether more interference is caused by a single transmission at higher power or multiple transmissions at lower power”. Indeed, a shorter transmission at higher power may permit a more efficient reuse of the communication channel. If the total radiated energy (product of power and duration) is a good indicator for interference, this boils down to an energy consumption problem. However, it must not be forgotten that *the SIR does not depend on absolute power levels*. If all nodes scale their power by the same factor $q > 1$, all the SIR levels remain constant, but the SINR levels will increase. So, increasing all transmit power levels does not have a negative impact on any packet reception probability in the network, in stark contrast to what is predicted by the disk model. On the contrary, the SINR levels will slightly increase. This indicates that *long-hop transmission does not inherently cause more interference*.

End-to-end Reliability. Under the disk model, reception probabilities are either 100% or 0%. If every receiver lies in its desired transmitter’s disk, the end-to-end reliability is always 100%, which is clearly not realistic, since packet errors or bit errors accumulate. So, to achieve a desired end-to-end reliability with short-hop routing, the relay nodes need to transmit at a higher power. This compensates, at least partially, for the loss in SNR. In the case of Rayleigh fading, this effect completely offsets the multihop benefit for $\alpha = 2$ [13]. If the same end-to-end delay is permitted, a number of transmissions (at lower power) are possible in the long-hop case, and the benefit vanishes even for higher path loss exponents, in particular for high end-to-end reliabilities or when channel state information is available at the transmitter.

Channel Coding. For long-hop routing, longer (stronger) channel codes can be used to satisfy the same delay constraint. Indeed, for optimum coding in channels with additive white Gaussian noise (AWGN), we find that a change from a nominal capacity $C_0 := \log_2(1 + P_0/N)/2$ to C_0/q results in an energy consumption of $E(q) = qP(q) = qN(2^{2C_0/q} - 1)$, which is strictly monotonically decreasing in q.[8] Note that $C_0/q =: R$ is the information theoretic rate (bits/symbol), and that the gain from using longer codes is higher for higher rates.

7 This is unrealistic since the received power can exceed the transmitted power if d is small enough.

8 The increase in transmission length is only linear in q, while the power can be reduced exponentially.

Total Energy Consumption. It is often assumed that a reduction of the *transmit energy* yields a proportional reduction of the total energy consumption. Even without taking into account *receive* energy, this is not true for any practical power amplifier. In particular in low-power transceivers, the local oscillators and bias circuitry will dominate, so that short-hop routing does not yield any energy benefit if a more distant relay node can be reached with sufficient reliability [11]. For random networks, relatively high peak power levels are necessary to keep the network connected [30], and short-hop routing would require a substantial backoff on the average, resulting in poor power efficiency.

Path Efficiency in Random Networks. Routes in random networks cannot follow straight lines. The path efficiency, defined as the ratio of Euclidean distance of the end nodes and the traveled distance, is higher if longer hops are used.

Sleep Modi or Cooperation. If neighboring nodes are not used as relays, they can either be put to sleep, or they can assist the transmission by cooperation [15] or retransmission (*e.g.*, if an ACK packet is lost).

Routing Overhead and Route Maintenance. In [4], it is pointed out that (when we replace a larger number of short hops by a smaller number of long hops) "It is far from clear what happens to the overall transmission energy, since to implement a nearest-neighbor policy, significantly augmented overhead control traffic will be required to coordinate the establishment of the routing paths and access control protocols across the entire network."

In a first order approximation, the control traffic for routing and route maintenance is proportional to the number of nodes in the route. Also, the probability of a route break due to energy depletion and node failure clearly increases with the number of nodes involved, as well as the memory requirements for the routing tables.

Route Longevity in Mobile Environments. The SNR of short-hop routes is more quickly affected by moving nodes. *Example:* If a node at distance 1 moves by 1 unit, the SNR change is 2^α, which causes the link to break (unless an unreasonably high SNR margin is applied). On the other hand, if the next-hop neighbor is 3 units away and moves by 1 unit, the SNR change is only $(4/3)^\alpha$, which can probably be tolerated.

Traffic Accumulation and Energy Balancing. For certain multihop networks, traffic accumulation around a base station or access point is a big problem. With strict short-hop routing, the relaying burden cannot be distributed among a sufficiently large number of nodes. The more nodes can reach the BS directly, the better distributed the load can be [10].

Further, in random networks, due to the variance in hop length, the variance in energy consumption is large when nearest-neighbor schemes are used, causing substantial imbalance in the energy consumption.

Bounded Attenuation. A path loss model with a singularity at distance $d = 0$ is not realistic for networks with high density [2]. Clearly, the received power cannot exceed the transmit power, so there is a bound on the received power. If we assume that this bound is achieved for distances $0 < d < R$, then there is no benefit in using shorter hops than R.

Multicast Advantage. So far, we have only discussed the unicast case. For multicast routing, more neighbors can be reached with a single transmission.

5. Node Cooperation and Diversity.

Thanks to the broadcast nature of wireless transmissions, ad hoc networks lend themselves to node cooperation. Nearby nodes that overhear a transmission may assist in relaying a packet if the direct path is obstructed or suffers from fading. The foundations to this *cooperative diversity*, a form of spatial diversity, are laid in [15–17]. Since then, node cooperation has been paid increased attention by the information theory and communications communities.

Since cooperative schemes are difficult to analyze, in particular when multiple nodes are involved, we suggest a simple formalism that is based on the Rayleigh fading link model and provides a simple but powerful framework for the analysis and design of diversity-based communication strategies [12].

5.1. The erristor framework

Multihop connections From (4), the end-to-end reception probability over a chain of n nodes is

$$p_{\mathrm{EE}} = e^{-\Theta \sum_{i=1}^{n} \frac{1}{\bar{\gamma}_i}}, \tag{18}$$

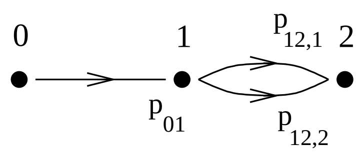
where $\bar{\gamma}_i$ denotes the mean SNR at receiver i. Let p_{D} denote the desired end-to-end reception probability, and R the normalized average noise-to-signal ratio (NSR) at the receiver, *i.e.*, $R := \Theta/\bar{\gamma}$. So we have $-\ln p_{\mathrm{EE}} = \sum_{i=1}^{n} R_i = R_{\mathrm{tot}}$, hence the condition $p_{\mathrm{EE}} \geqslant p_{\mathrm{D}}$ translates into the condition that the sum (or the *series connection*) of the NSR values R_i is at most $R_{\mathrm{D}} := -\ln p_{\mathrm{D}}$. So, the individual R_i's can be replaced by an equivalent R_{tot}. For a single link, we have

$$R = -\ln p_r \quad \Longleftrightarrow \quad p_r = e^{-R}. \tag{19}$$

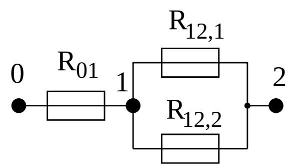
For probabilities close to 1 (or $R \ll 1$), the following first-order approximations are accurate:

$$\hat{R} := 1 - p_r \lessapprox R \quad \Longleftrightarrow \quad \hat{p}_r := 1 - R \lessapprox p_r \tag{20}$$

As pointed out before, for small values, the NSR can be considered equivalent to the packet error probability. To emphasize this fact and the resistor-like series connection property



(a) Transmission diagram.



(b) Erristor diagram.

Figure 5. Example for erristor framework. (a) There is one transmission over the first link with reception probability p_{01}, and there are two transmissions over the second link with probabilities $p_{12,1}$ and $p_{12,2}$. (b) shows the corresponding erristor diagram.

of the NSR, we denote R as an "erristor" and its value as its "erristance". The transmit power is inversely proportional to R.

Retransmissions For n transmissions over one link at NSR levels R_i, we have

$$p_n = 1 - \prod_{i=1}^{n}(1 - e^{-R_i}) \,. \qquad (21)$$

To derive a general rule for the simplification of these expressions, we apply the following theorem.

Theorem 3 *For* $(x_1, x_2, \ldots, x_n) \in (\mathbb{R}_0^+)^n$,

$$1 - \prod_{i=1}^{n}(1 - e^{-x_i}) \geqslant e^{-\prod_{i=1}^{n} x_i} \,. \qquad (22)$$

The identity holds if and only if $\prod_{i=1}^{n} x_i = 0$.

The elementary proof is based on induction. So, multiple transmissions over a single link result in (at least) a *multiplication* of the packet loss probabilities.

Example Consider a simple two-hop scheme with two transmission over the second hop and its erristor representation, as shown in Fig. 5. The precise analysis yields $p_{\text{EE}} = p_{01} \cdot \big(1 - (1 - p_{12,1})(1 - p_{12,2})\big)$. This is equivalent to $p_{12} = 1 - \big(1 - e^{-R_{12,1}}\big)\big(1 - e^{-R_{12,2}}\big)$. Applying Theorem 3, we find that $e^{-R_{12,1}R_{12,2}}$ is a lower bound for p_{12}. For $R_1 \ll 1$ and $R_2 \ll 1$ (high reception probabilities), the bound is tight. This greatly simplifies the design and analysis for large networks.

5.2. Multipath routing

Multipath routing is another form of spatial diversity [1, 28, 29]. The main problem is to identify suitable disjoint routes. It needs to be emphasized that while *node-disjoint* routes are sufficient for wired multipath routing, *interference-disjoint* routes are needed in the wireless case for a full diversity benefit. Interference-disjointness is much harder to achieve, since the routes need to be spatially separated and may vary substantially in length. The area around the destination is critical as all routes need to converge. The contention in this area has to be traded off against the gain in reliability.

6. Concluding Remarks

For the design of routing protocols for wireless ad hoc networks, the characteristics of the wireless channel cannot be ignored. Most current routing protocols are derived from wired versions and may perform suboptimally, in particular in mobile scenarios where fading needs to be considered. This problem is also reflected in the models that are frequently used for the analysis and design of protocols, since these models do not capture essential properties of the wireless channel such as interference, fading, and noise. Further, in contrast to wired links, distances matter greatly, so we need to know the internode distances, or at least their distribution. Having only statistical or estimated information on distances adds another layer of uncertainty to the network that the protocols need to be able to deal with. Strategies based on cooperation and diversity are promising approaches to achieve good performance in such uncertain environments.

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References

[1] J. Chen, P. Druschel, and D. Subramanian. An Effi cient Multipath Forwarding Method. In *INFOCOM'98*, volume 3, pages 1418–1425, 1998.

[2] O. Dousse and P. Thiran. Connectivity vs Capacity in Dense Ad Hoc Networks. In *IEEE INFOCOM*, Hongkong, Mar. 2004.

[3] O. Dousse, P. Thiran, and M. Hasler. Connectivity in ad-hoc and hybrid networks. In *IEEE INFOCOM*, New York, NY, June 2002.

[4] A. Ephremides. Energy Concerns in Wireless Networks. *IEEE Wireless Communications*, 9(4):48–59, Aug. 2002.

[5] D. Ganesan, B. Krishnamachari, A. Woo, D. Culler, D. Estrin, and S. Wicker. An Empirical Study of Epidemic Algorithms in Large Scale Multihop Wireless Networks, 2002. Intel Research Report IRB-TR-02-003. Available at www.intel-research.net/Publications/Berkeley/050220021703_19.pdf.

[6] A. J. Goldsmith and S. B. Wicker. Design Challenges for Energy-Constrained Ad Hoc Wireless Networks. *IEEE Wireless Communications*, 9(4):8–27, Aug. 2002.

[7] M. Grossglauser and D. Tse. Mobility Increases the Capacity of Ad-hoc Wireless Networks. In *IEEE INFOCOM*, Anchorage, AL, 2001.

[8] P. Gupta and P. R. Kumar. *Stochastic Analysis, Control, Optimization and Applications*, chapter "Critical Power for Asymptotic Connectivity in Wireless Networks", pages 547–566. Birkhauser, Boston, 1998. ISBN 0-8176-4078-9.

[9] P. Gupta and P. R. Kumar. The Capacity of Wireless Networks. *IEEE Transactions on Information Theory*, 46(2):388–404, Mar. 2000.

[10] M. Haenggi. Energy-Balancing Strategies for Wireless Sensor Networks. In *IEEE International Symposium on Circuits and Systems (ISCAS'03)*, Bangkok, Thailand, May 2003. Available at http://www.nd.edu/~mhaenggi/iscas03.pdf.

[11] M. Haenggi. The Impact of Power Amplifier Characteristics on Routing in Random Wireless Networks. In *IEEE Global Communications Conference (GLOBECOM'03)*, San Francisco, CA, Dec. 2003. Available at http://www.nd.edu/~mhaenggi/globecom03.pdf.

[12] M. Haenggi. Analysis and Design of Diversity Schemes for Ad Hoc Wireless Networks. *IEEE Journal on Selected Areas in Communications*, 2004. Accepted for publication. Available at http://www.nd.edu/~mhaenggi/jsac_adhoc.pdf.

[13] M. Haenggi. On Routing in Random Rayleigh Fading Networks. *IEEE Transactions on Wireless Communications*, 2004. Accepted for publication. Available at http://www.nd.edu/~mhaenggi/routing.pdf.

[14] L. Hu. Topology Control for Multihop Packet Networks. *IEEE Transactions on Communications*, 41(10):1474–1481, 1993.

[15] J. N. Laneman, D. N. C. Tse, and G. W. Wornell. Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior. *IEEE Transactions on Information Theory*. Accepted for publication. Available at: http://www.nd.edu/~jnl/pubs/it2002.pdf.

[16] J. N. Laneman and G. W. Wornell. Energy-Efficient Antenna Sharing and Relaying for Wireless Networks. In *IEEE Wireless Comm. and Netw. Conf. (WCNC'00)*, pages 7–12, Chicago, IL, 2000.

[17] J. N. Laneman and G. W. Wornell. Distributed Space-Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks. *IEEE Transactions on Information Theory*, 59(10):2415–2525, Oct. 2003.

[18] X. Liu and M. Haenggi. Throughput Bounds and Energy Consumption of Mobile Multihop Networks. In *IEEE Vehicular Technology Conference (VTC'04 Fall)*, Los Angeles, CA, Sept. 2004.

[19] D. A. Maltz, J. Broch, and D. B. Johnson. Lessons from a Full-Scale Multihop Wireless Ad Hoc Network Testbed. *IEEE Personal Communications*, 8(1):8–15, Feb. 2001.

[20] G. Németh, Z. R. Turányi, and A. Valkó. Throughput of Ideally Routed Wireless Ad Hoc Networks. *ACM Mobile Computing and Communications Review*, 5(4):40–46, 2001.

[21] C. E. Perkins, editor. *Ad Hoc Networking*. Addison Wesley, 2000. ISBN 0-201-30976-9.

[22] M. Sanchez, P. Manzoni, and Z. Haas. Determination of Critical Transmission Range in Ad-Hoc Networks. In *Multiaccess, Mobility and Teletraffic for Wireless Communications (MMT'99)*, Venice, Italy, Oct. 1999.

[23] C. Schurgers, V. Tsiatsis, S. Ganeriwal, and M. Srivastava. Optimizing Sensor Networks in the Energy-Latency-Density Design Space. *IEEE Transactions on Mobile Computing*, 1(1):70–80, 2002.

[24] T. J. Shepard. A Channel Access Scheme for Large Dense Packet Radio Networks. In *ACM SIGCOMM*, Stanford, CA, Aug. 1996. Available at: http://www.acm.org/sigcomm/sigcomm96/papers/shepard.ps.

[25] J. A. Silvester and L. Kleinrock. On the Capacity of Multihop Slotted ALOHA Networks with Regular Structure. *IEEE Transactions on Communications*, COM-31(8):974–982, Aug. 1983.

[26] H. Takagi and L. Kleinrock. Optimal Transmission Ranges for Randomly Distributed Packet Radio Terminals. *IEEE Transactions on Communications*, COM-32(3):246–257, Mar. 1984.

[27] C.-K. Toh. *Ad Hoc Mobile Wireless Networks - Protocols and Systems*. Prentice-Hall, 2002. ISBN 0-13-007817-4.

[28] A. Tsirigos and Z. Haas. Analysis of Multipath Routing—Part I: The Effect on the Packet Delivery Ratio. *IEEE Transactions on Wireless Communications*, 3(1):138–146, Jan. 2004.

[29] J. L. Wang and J. A. Silvester. Maximum Number of Independent Paths and Radio Connectivity. *IEEE Transactions on Communications*, 41(10):1482–1493, Oct. 1993.

[30] F. Xue and P. R. Kumar. The Number of Neighbors Needed for Connectivity of Wireless Networks. *Wireless Networks*, 10(2):169–181, Mar. 2004. Available at http://black.csl.uiuc.edu/~prkumar/ps_files/connect.pdf.

[31] H. Y. Youn, C. Yu, and B. Lee. *Routing Algorithms for Balanced Energy Consumption in Ad Hoc Networks*. CRC Press, 2003. The Handbook of Ad Hoc Wireless Networks.