

The Impact of Power Amplifier Characteristics on Routing in Random Wireless Networks

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Abstract—Power amplifiers for wireless transmission provide a limited radiated power, and their efficiency depends highly on the operating point. We show that power control and routing strategies in multi-hop wireless networks are strongly affected by these non-ideal amplifier characteristics. For the analysis, we prove that the distances in random networks are governed by a generalized Rayleigh distribution, and we determine the power efficiency of different routing schemes. The main result is that nearest-neighbor routing is highly inefficient if the network has to be connected with high probability.

I. INTRODUCTION

Energy consumption in multi-hop wireless networks is a crucial issue that needs to be addressed at all the layers of the communication system, from the hardware up to the application. In this paper, we focus on the impact that the characteristics of the power amplifier has on energy-efficient routing strategies. The analysis is based on a Rayleigh fading channel model, and the results demonstrate that the properties of the hardware and the physical channel have a substantial impact on optimum protocol design at the network layer.

Two assumptions are generally made when assessing energy consumption in multi-hop networks: 1) the power consumption is equal (or proportional) to the radiated power; 2) reliable links exist if the receiver is within a certain distance of the transmitter, and interference is taken into account using the same geometric disk abstraction. Such a deterministic “disk model” is used for the analysis of multi-hop packet networks in [1]–[9], thereby ignoring the stochastic nature of the fading channel. Using such models, it is easy to show that, for a path loss exponent of α , there is an energy gain of $n^{\alpha-1}$ if a hop over a distance d is split into n hops of distance d/n . However, the volatility of the channel cannot be ignored in wireless networks [10], [11]; the inaccuracy of “disk models” has also been pointed out in [12] and is easily demonstrated experimentally [13].

To overcome some of these limitations of the “disk model”, we employ a simple Rayleigh fading link model that relates transmit power, large-scale path loss, and the success of a transmission.

While fading has been considered in the context of packet networks [14], [15], its impact on the network (and higher) layers is largely an open problem. Similarly, non-ideal characteristics of power amplifiers are not usually considered at higher layers. In this paper, we take both fading and amplifier properties into account, and we show that this cross-layer perspective sheds some new light on the routing problem.

II. THE RAYLEIGH NETWORK MODEL

A. The Rayleigh fading link model

We assume a narrowband Rayleigh block fading channel. A transmission from node i to node j is successful if the SINR γ_{ij} is above a certain threshold Θ that is determined by the communication hardware, and the modulation and coding scheme [10]. The SINR γ is a discrete random process given by $\gamma = \frac{R}{N_0+I}$. R is the received power, which is exponentially distributed with mean \bar{R} . Over a transmission of distance $d = \|x_i - x_j\|_2$ with an attenuation d^α , we have $\bar{R} = P_0 d^{-\alpha}$, where P_0 is proportional to the transmit power¹. N_0 denotes the noise power, and I is the interference power affecting the transmission, *i.e.*, the sum of the received power from all the undesired transmitters.

Theorem 1 *In a Rayleigh fading network, the reception probability $\mathbb{P}[\gamma \geq \Theta]$ can be factorized into the reception probability of a zero-noise network and the reception probability of a zero-interference network.*

Proof: Let R_0 denote the received power from the desired source and R_i , $i = 1, \dots, k$, the received power from k interferers. All the received powers are exponentially distributed, *i.e.*, $p_{R_i}(r_i) = 1/\bar{R}_i e^{-r_i/\bar{R}_i}$, where \bar{R}_i denotes the average received power $\bar{R}_i = P_i d_i^{-\alpha}$. The probability of correct reception is (a similar calculation has been carried out in the Appendix of [14] for a network with spreading gain and equal transmit powers for all nodes.)

$$p_r = \mathbb{P}[R_0 \geq \Theta(I + N_0)] = \exp\left(-\frac{\Theta(I + N_0)}{\bar{R}_0}\right) \quad (1)$$

$$= \int_0^\infty \dots \int_0^\infty \exp\left(-\frac{\Theta(\sum_{i=1}^k r_i + N_0)}{\bar{R}_0}\right) \cdot \prod_{i=1}^k p_{R_i}(r_i) dr_1 \dots dr_k \quad (2)$$

$$= \underbrace{\exp\left(-\frac{\Theta N_0}{P_0 d_0^{-\alpha}}\right)}_{p_r^N} \cdot \underbrace{\prod_{i=1}^k \frac{1}{1 + \Theta \frac{P_i}{P_0} \left(\frac{d_0}{d_i}\right)^\alpha}}_{p_r^I} \quad (3)$$

¹This equation does not hold for very small distances. So, a more accurate model would be $\bar{R} = P'_0 \cdot (d/d_0)^{-\alpha}$, valid for $d \geq d_0$, with P'_0 as the average value at the reference point d_0 , which should be in the far field of the transmit antenna. At 916MHz, for example, the near field may extend up to 3-4ft (several wavelengths).

p_r^N is the probability that the SNR $\gamma_N := R_0/N_0$ is above the threshold Θ , *i.e.*, the reception probability in a zero-interference network as it depends only on the noise. The second factor p_r^I is the reception probability in a zero-noise network. \square

This allows an independent analysis of noise and interference. If the load is light (low interference probability), then $\text{SIR} \gg \text{SNR}$, and the noise analysis alone provides accurate results. For high load, a separate interference analysis has to be carried out [16]. Note that *power scaling*, *i.e.*, scaling the transmit powers of all the nodes by the same factor, does not change the SIR (p_r^I only depends on *power ratios*), but (slightly) increases the SINR. In any case, the noise analysis is relevant, so the main focus of this paper is on the noise.

In a zero-interference network, the reception probability over a link of distance d at a transmit power P_0 , is given by $p_r := \mathbb{P}[\gamma_N \geq \Theta] = e^{-\frac{\Theta N_0}{P_0 d^{-\alpha}}}$, therefore

$$P_0 = \frac{d^\alpha \Theta N_0}{-\ln p_r}. \quad (4)$$

Note that for high probabilities, the packet loss probability $1 - p_r$ is tightly upperbounded by the normalized mean NSR $\Theta N_0/R_0 = \Theta/\bar{\gamma}_N$ [17]. Since $-\ln p_r \approx 1 - p_r$, we can also say that the packet loss probability is inversely proportional to the transmit power for high p_r .

B. Random networks with uniform distribution

If nodes are distributed uniformly with a density λ in a large network, the probability of finding k nodes in an area A is given by the Poisson distribution [12]

$$\mathbb{P}[k \text{ nodes in } A] = e^{-\lambda A} \frac{(\lambda A)^k}{k!}. \quad (5)$$

Hence, the positions of the nodes constitute a Poisson point process in the plane². Without loss of generality, we can restrict ourselves to the case $\lambda = 1$ (unit density), since the product λA can always be scaled such that $\lambda = 1$.

For the routing schemes we consider, we need to determine the distance from one node to its neighboring nodes that lie within a sector ϕ , *i.e.*, within $\pm\phi/2$ of the source-destination axis (Fig. 1).

Proposition 1 *In a random network with uniform distribution and unit density, the distance R between a node and its nearest neighbor in a sector ϕ is Rayleigh distributed with mean $\sqrt{\pi/(2\phi)}$.*

Proof: Let R be the distance to the nearest neighbor in a sector ϕ . The probability that there is no neighbor in a sector ϕ up to a distance r is the complementary cumulative distribution $\mathbb{P}[R > r] = e^{-r^2\phi/2}$, thus the probability density is $p_R(r) = r\phi e^{-r^2\phi/2}$, which is a Rayleigh distribution with mean $\sqrt{\pi/(2\phi)}$ and variance $2/\phi - \pi/(2\phi) = (4 - \pi)/(2\phi)$. The distribution of the argument ψ is uniform between $-\phi/2$ and $\phi/2$. \square

²This can be generalized to higher dimensions if A is the Lebesgue measure of the subset considered.

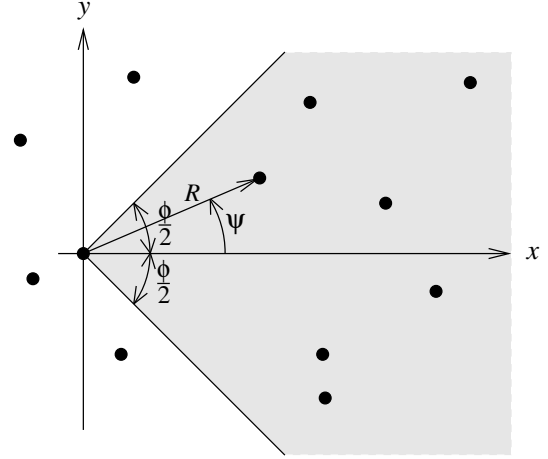


Fig. 1. Part of a Rayleigh network with the source at the origin and the x -axis pointing towards the destination node. R denotes the distance to the nearest neighbor within a sector ϕ around x , and ψ is its argument. Hence (R, ψ) are the polar coordinates of the nearest neighbor within a sector ϕ .

Definition 1 (Rayleigh network.) *A Rayleigh network is a large random network with uniformly distributed nodes where the physical channel is subject to Rayleigh fading.*

Proposition 2 *The probability density of the distance to the n -th nearest neighbor in a sector ϕ is*

$$p_{R_n}(r) = r^{2n-1} \left(\frac{\phi}{2}\right)^n \frac{2}{(n-1)!} e^{-r^2\phi/2} \quad (6)$$

Proof: Let S_k be the k -th coefficient in the Poisson distribution: $S_k := (r^2\phi/2)^k/k!$. The probability that there are less than n nodes closer than r in the sector ϕ is

$$P_n := \mathbb{P}[0 \dots n-1 \text{ nodes within } r] = \sum_{k=0}^{n-1} S_k e^{-r^2\phi/2}. \quad (7)$$

From $p_{R_n} = \frac{d}{dr} (1 - P_n)$ we get

$$p_{R_n} = \left(r\phi \sum_{k=0}^{n-1} S_k - \sum_{k=1}^{n-1} \underbrace{\frac{k(r^2\phi/2)^{k-1}}{k!}}_{S_{k-1}} r\phi \right) e^{-r^2\phi/2}. \quad (8)$$

The only term that is not cancelled in the two sums is the one at $n-1$, leading to

$$p_{R_n} = r\phi \cdot \underbrace{S_{n-1} e^{-r^2\phi/2}}_{\text{Erlang distribution}}, \quad (9)$$

which is identical to (6). \square

Since p_{R_n} is a Rayleigh distribution for $n = 1$, it can be considered a generalized Rayleigh distribution. Similarly, for a one-dimensional Poisson process, the Erlang distribution is a generalized exponential distribution. So, the transition from one dimension to two dimensions entails a multiplication by $r\phi$ (that comes from the inner derivative of the exponential part) in the distributions of the node distances. The distributions for $n = 1, \dots, 8$ are shown in Fig. 2.

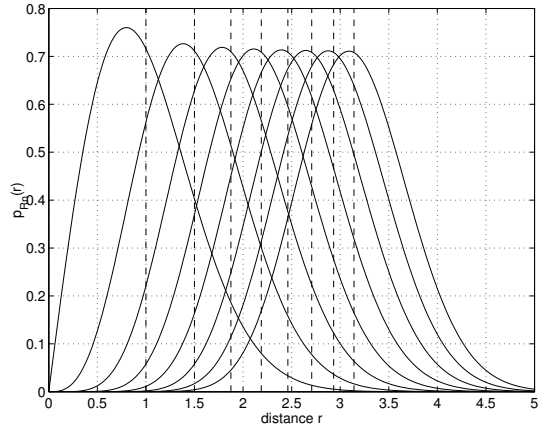


Fig. 2. The probability density function of the distances to the n -th nearest neighbor for $n = 1$ (leftmost curve) to $n = 8$ (rightmost curve) and $\phi = \pi/2$. For $n = 1$, this is a Rayleigh distribution. The mean values are indicated by the dashed lines. For $n = 1$ and $n = 2$, the mean values are 1 and $3/2$, respectively.

The mean of R_n is given by

$$\begin{aligned} \mathbb{E}[R_n] &= \frac{\sqrt{2}}{\sqrt{\phi}} \frac{\Gamma(n+1/2)}{\Gamma(n)} = \frac{\sqrt{2}}{\sqrt{\phi}} \frac{(2n)! \sqrt{\pi}}{n!(n-1)!4^n} \\ &\approx \sqrt{\frac{4(n-1) + \pi}{2\phi}}, \end{aligned} \quad (10)$$

where we have made use of some properties of the $\Gamma(\cdot)$ function [18] and then derived a highly accurate approximation that shows that the mean distance to the n -th neighbor increases with the square root of n . The second moment is $2n/\phi$, hence the variance is

$$\text{Var}[R_n] = \frac{2n}{\phi} - \mathbb{E}[R_n]^2 = \frac{4 - \pi}{2\phi}, \quad (11)$$

which is, interestingly, independent of n .

III. POWER AMPLIFIER CHARACTERISTICS

The most energy efficient operation of an RF power amplifier (PA) is near saturation as this is when the power added efficiency³ (PAE) is largest. Linear amplification is possible mainly by operating the power amplifier with a small input signal (large backoff) where the energy efficiency of the amplifier is smaller. This characteristic of nonlinear amplifiers makes large power efficiency and bandwidth efficiency hard to achieve.

Depending on the modulation scheme and the specific application, there are situations in which linearity, especially amplitude linearity, can be traded for efficiency and RF power output [19]. Such applications include constant-envelope schemes such as FSK and GMSK, which can tolerate high levels of amplitude distortion, and intermediate cases such as OQPSK and DQPSK systems, which can tolerate significant amounts of amplitude distortion. On the other hand, for modulations with non-constant envelope, amplifier linearity is important. In order to obtain higher PAE, a higher class of amplifiers

³There exist different definitions of PAE. We are considering the one that takes the drive power into account, which results in $\text{PAE} = P_{TX}/P_{dc}$.

such as class AB, class B or even class C or switched mode class E or F is often used. However, due to the I/V curves of devices operated in these classes, the amplifier becomes nonlinear. Therefore, high efficiency and high linearity are often contradictory objectives [20], [21].

To avoid discussions of amplifier classes, modulation schemes, different “overdrive” conditions, matching networks and other implementation details, we employ a simple piecewise linear model for the radiated power vs. bias power characteristics of a PA:

$$P_{dc} = \beta P_{TX} + P^* \quad 0 \leq P_{TX} \leq P_{\max}, \quad (12)$$

where P_{\max} is the maximum power that the modulation schemes tolerates (this can be close to saturation or well within the linear region of the PA), and P^* is the total static power consumption. The slope β normally ranges from 0 to 2. The maximum power efficiency $\kappa = P_{TX}/P_{dc}$ is reached at $P_{TX} = P_{\max}$, where $\kappa = \frac{1}{\beta + P^*/P_{\max}}$. The ratio P^*/P_{\max} ranges from $1/5$ to 1.

While such a model can be justified from a hardware point of view, it can also be derived from data sheets of existing PAs and single-chip transceivers manufactured by National Semiconductor, Maxim, RF Micro Devices, Motorola, TriQuist Semiconductor, Agilent, and Mitsubishi designed for 3G, WLAN, and Bluetooth applications [22].

To further simplify the discussion, we define the *generic amplifier*:

Definition 2 (Generic amplifier.) A generic amplifier is an amplifier with $\beta = 1$ and $P_{\max} = P^*$ in (12). Its maximum efficiency of $\kappa = 50\%$ is reached at $P_{TX} = P_{\max}$.

Note that the $P_{dc}(P_{TX})$ characteristics of a large number of commercial amplifiers are upperbounded by this generic amplifier curve, *i.e.*, most PAs have a smaller efficiency.

With this model, we can state the following result on multi-hop vs. single-hop communication:

Proposition 3 For the generic amplifier, there is no energy benefit in using multiple hops, if the destination can be reached at $P_{TX} = P_{\max}$ directly with the desired reliability.

Proof: The power consumption for the one-hop case is⁴ $E_1 = 2P_{\max}$. For the n -hop case, we have $E_n = n(P_{\max} + n^{-\alpha}P_{\max})$, thus the ratio is

$$\frac{E_n}{E_1} = \frac{n}{2}(1 + n^{-\alpha}), \quad (13)$$

which is bigger than 1 for any α for $n \geq 2$. \square

IV. ROUTING SCHEMES

As shown in Fig. 1, we want to ensure that the source node is connected to a neighbor within a sector ϕ with (high) probability p_c .

⁴It is assumed that E expresses the energy required to send one packet at a power level P .

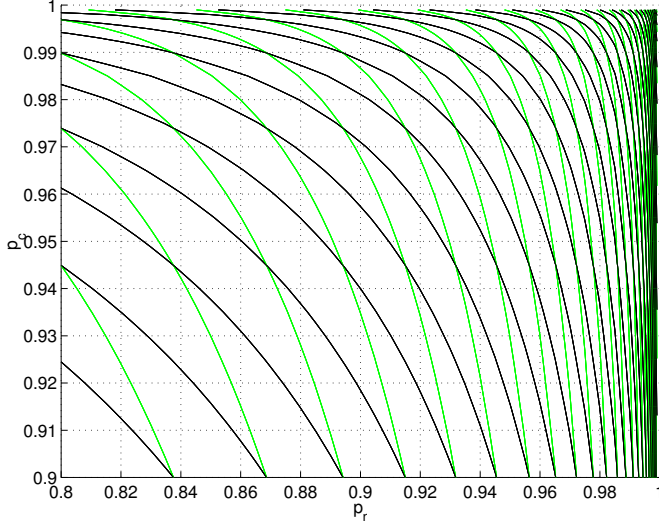


Fig. 3. Contour plot of P_{\max} as a function of p_r and p_c for $\alpha = 2$ (light) and $\alpha = 4$ (dark). The spacing between contour lines is 1dB, with increasing P_{\max} for higher p_r and/or p_c . The contours for the different path loss exponents have different absolute values.

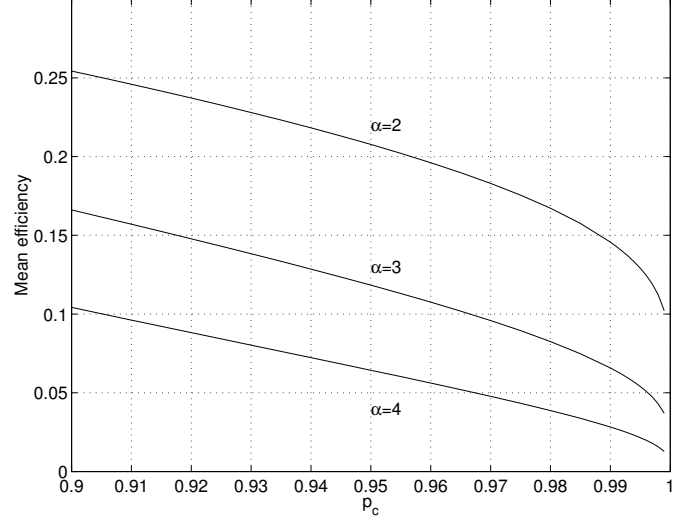


Fig. 4. Average efficiency of the power amplifier as a function of p_c for $\alpha = 2, 3, 4$.

Proposition 4 Let p_c denote the probability that there is a neighbor with a certain distance d in the sector ϕ . The necessary power to reach this nearest neighbor within a sector ϕ with probability p_r is

$$P_{\max} = \frac{\Theta N_0}{-\ln p_r} \left(\frac{-2 \ln(1 - p_c)}{\phi} \right)^{\alpha/2}. \quad (14)$$

Proof: From (4) follows $P_{\max} = -N_0 \Theta d_{\max}^{\alpha} / \ln p_r$, and we get $d_{\max}^2 = -2 \ln(1 - p_c) / \phi$ from $p_c = \mathbb{P}[R < d_{\max}] = 1 - e^{-d_{\max}^2 / \phi}$. \square

Figure 3 shows a contour plot of $P_{\max}(p_r, p_c)$.

If the maximum transmit power is given, the sector ϕ has to satisfy

$$\phi \geq -2 \ln(1 - p_c) \left(\frac{\Theta N_0}{P_{\max} (-\ln p_r)} \right)^{2/\alpha}. \quad (15)$$

The average distance to the nearest neighbor is $\bar{d} = \sqrt{\pi / (2\phi)}$. So, $d_{\max} / \bar{d} = 2 \sqrt{-\ln(1 - p_c) / \pi}$, independent of ϕ or p_r .

A. Nearest-neighbor routing

Under optimum power control, the mean *backoff* of the generic PA from the optimum operating point is $(d_{\max} / \bar{d})^{\alpha}$. For $\alpha = 2$, this is about 4 at $p_c = 95\%$ and 10 at $p_c = 99.9\%$.

We define the *average efficiency* $\bar{\kappa}$ to be the efficiency at a transmit power of $P = (d_{\max} / \bar{d})^{\alpha} P_{\max}$. We find

$$\bar{\kappa} = \frac{1}{1 + \left(2 \sqrt{\frac{-\ln(1 - p_c)}{\pi}} \right)^{\alpha}}. \quad (16)$$

This relationship is shown in Fig. 4 for $\alpha = 2, 3, 4$.

B. Routing to the n -th nearest neighbor

We have already determined the distance to the n^{th} nearest neighbor in (6). The corresponding cumulative density cannot be analytically solved for the distance, but we notice that all the curves have similar shapes (which is also corroborated by the fact that all distributions have the same variance), and the mean is increasing with \sqrt{n} . Is it therefore reasonable to assume that, with $p_c = \mathbb{P}[R_n < d_n]$, for any given p_c , $q := d_n - \bar{d}_n$ does not depend on n . Hence, the backoff $d_n / \bar{d}_n \approx 1 + q / \bar{d}_n$ is getting smaller with increasing n . So, it is a good strategy to design the PA such that it can reach over longer distances and transmit to n -th nearest neighbors instead of just nearest neighbors. Due to the additional terms in (7), it is clear that q is slightly decreasing with increasing n . So, if we insert the value for $n = 1$, we get a lower bound for the efficiency for larger n .

$$\frac{d_n}{\bar{d}_n} < 1 + \frac{2 \sqrt{-\ln(1 - p_c)} - \sqrt{\pi}}{\sqrt{4(n - 1) + \pi}} \approx 1 + \frac{\sqrt{-\ln(1 - p_c)}}{\sqrt{n}}. \quad (17)$$

The resulting average efficiency is

$$\bar{\kappa}_n > \frac{1}{1 + \left(1 + \sqrt{\frac{-\ln(1 - p_c)}{n}} \right)^{\alpha}}. \quad (18)$$

This bound gets tight with increasing n , and it shows that the efficiency (slowly) approaches 1/2 as n increases.

C. Optimum routing

Since the efficiency of the PA is optimum at P_{\max} , the routing scheme should try to identify nodes that are as far away as possible within the radius that allows a reception with probability p_r . This might be the first, second, third, . . . or n -th neighbor within the sector. In other words, if there are exactly n neighbors within the sector, then the routing algorithm should use the n -th nearest one. Hence the probability that the n -th nearest neighbor is chosen is given by the Poisson

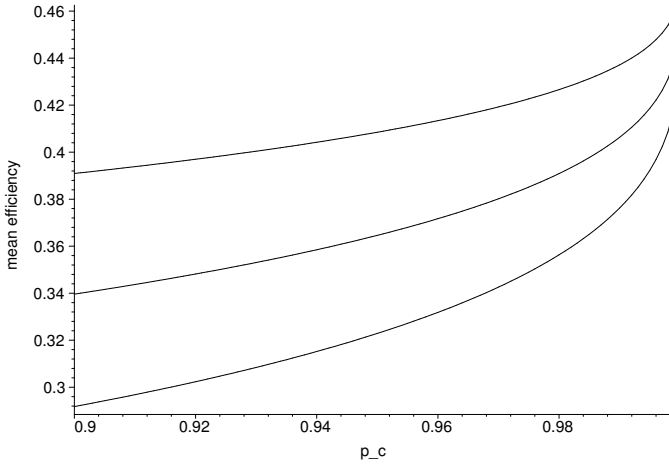


Fig. 5. Average efficiency for the optimum scheme for $\alpha = 2$ (top curve) $\alpha = 3$ and $\alpha = 4$ (bottom curve).

term $(d^2\phi/2)^k/k!e^{-d^2\phi/2}$. We need the density function of the distance to this neighbor:

Proposition 5 *The probability density of the distance to the furthest neighbor within distance d_{\max} in a sector ϕ , given that there is a least one neighbor in the sector, is*

$$p_R(r) = \frac{r\phi e^{r^2\phi/2}}{e^{d_{\max}^2\phi/2} - 1}, \quad r \in [0, d_{\max}]. \quad (19)$$

Proof: The complementary cumulative distribution $\mathbb{P}[R > r]$, conditioned on having at least one node in the sector within distance d_{\max} , is given by the probability that there is (at least) one node with distance $r < R \leq d_{\max}$:

$$\mathbb{P}[R > r] = \frac{1 - e^{-(d_{\max}^2 - r^2)\phi/2}}{1 - e^{-d_{\max}^2\phi/2}}. \quad (20)$$

For the mean distance, we get

$$\bar{d} = \mathbb{E}[R] = \frac{d_{\max}e^{d_{\max}^2\phi/2} - c}{e^{d_{\max}^2\phi/2} - 1} \quad (21)$$

with $c := \sqrt{\frac{\pi}{2\phi}} \operatorname{erfi}\left(\frac{d_{\max}}{2}\sqrt{2\phi}\right)$, where $\operatorname{erfi}(\cdot)$ is the imaginary error function, *i.e.*, $\operatorname{erfi}(x) = 2/\sqrt{\pi} \cdot \int_{t=0}^x e^{t^2} dt$. \bar{d} tends to d_{\max} with increasing d_{\max} , hence the efficiency increases with increasing p_c , in contrast to the other schemes. This is shown in Fig. 5.

V. CONCLUDING REMARKS

Using a Rayleigh fading channel model and a simple power amplifier model that takes into account that the power added efficiency strongly depends on the transmit power, we have shown that the benefits of multi-hop routing vanish completely if the maximum radiated power allows to reach a destination in a single hop. In the case of random networks with uniform distribution, routing schemes that transmit as far as possible clearly outperform nearest-neighbor routing. The optimum strategy is to choose the furthest neighbor (within a certain sector of the axis to the destination) that can be reached with sufficient reliability.

ACKNOWLEDGMENTS

The author would like to thank Stefan Haenggi (Advanced Wireless Technology Group, National Semiconductor) for helpful information on RF power amplifiers. The partial support of the DARPA/IXO-NEST Program (AF-F30602-01-2-0526), and NSF (ECS02-25265) is gratefully acknowledged.

REFERENCES

- [1] J. A. Silvester and L. Kleinrock, "On the Capacity of Multihop Slotted ALOHA Networks with Regular Structure," *IEEE Transactions on Communications*, vol. COM-31, pp. 974–982, Aug. 1983.
- [2] H. Takagi and L. Kleinrock, "Optimal Transmission Ranges for Randomly Distributed Packet Radio Terminals," *IEEE Transactions on Communications*, vol. COM-32, pp. 246–257, Mar. 1984.
- [3] L. Hu, "Topology Control for Multihop Packet Networks," *IEEE Transactions on Communications*, vol. 41, no. 10, pp. 1474–1481, 1993.
- [4] J. L. Wang and J. A. Silvester, "Maximum Number of Independent Paths and Radio Connectivity," *IEEE Transactions on Communications*, vol. 41, pp. 1482–1493, Oct. 1993.
- [5] M. Sanchez, P. Manzoni, and Z. Haas, "Determination of Critical Transmission Range in Ad-Hoc Networks," in *Multiaccess, Mobility and Teletraffic for Wireless Communications (MMT'99)*, (Venice, Italy), Oct. 1999.
- [6] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, Mar. 2000.
- [7] M. Grossglauser and D. Tse, "Mobility Increases the Capacity of Ad-hoc Wireless Networks," in *IEEE INFOCOM*, (Anchorage, AL), 2001.
- [8] G. Németh, Z. R. Turányi, and A. Valkó, "Throughput of Ideally Routed Wireless Ad Hoc Networks," *ACM Mobile Computing and Communications Review*, vol. 5, no. 4, pp. 40–46, 2001.
- [9] C. Schurgers, V. Tsiatsis, S. Ganeriwal, and M. Srivastava, "Optimizing Sensor Networks in the Energy-Latency-Density Design Space," *IEEE Transactions on Mobile Computing*, vol. 1, no. 1, pp. 70–80, 2002.
- [10] A. Ephremides, "Energy Concerns in Wireless Networks," *IEEE Wireless Communications*, vol. 9, pp. 48–59, Aug. 2002.
- [11] A. J. Goldsmith and S. B. Wicker, "Design Challenges for Energy-Constrained Ad Hoc Wireless Networks," *IEEE Wireless Communications*, vol. 9, pp. 8–27, Aug. 2002.
- [12] E. S. Sousa and J. A. Silvester, "Optimum Transmission Ranges in a Direct-Sequence Spread-Spectrum Multihop Packet Radio Network," *IEEE Journal on Selected Areas in Communications*, vol. 8, pp. 762–771, June 1990.
- [13] D. A. Maltz, J. Broch, and D. B. Johnson, "Lessons from a Full-Scale Multihop Wireless Ad Hoc Network Testbed," *IEEE Personal Communications*, vol. 8, pp. 8–15, Feb. 2001.
- [14] M. Zorzi and S. Pupolin, "Optimum Transmission Ranges in Multihop Packet Radio Networks in the Presence of Fading," *IEEE Transactions on Communications*, vol. 43, pp. 2201–2205, July 1995.
- [15] Y. Y. Kim and S. Li, "Modeling Multipath Fading Channel Dynamics for Packet Data Performance Analysis," *Wireless Networks*, vol. 6, pp. 481–492, 2000.
- [16] M. Haenggi, "Probabilistic Analysis of a Simple MAC Scheme for Ad Hoc Wireless Networks," in *IEEE CAS Workshop on Wireless Communications and Networking*, (Pasadena, CA), Sept. 2002.
- [17] M. Haenggi, "A Formalism for the Analysis and Design of Time and Path Diversity Schemes in Wireless Sensor Networks," in *The 2nd International Workshop on Information Processing in Sensor Networks (IPSN'03)*, (Palo Alto, CA), Apr. 2003. Available at <http://www.nd.edu/~mhaenggi/ipsn03.pdf>.
- [18] G. Arfken, *Mathematical Methods for Physicists*. Orlando, FL: Academic Press, 3rd ed., 1985.
- [19] S. C. Cripps, *RF Power Amplifiers for Wireless Communications*. Artech House, 1999. ISBN 0-89006-989-1.
- [20] "WTEC Panel Report on Wireless Technologies and Information Networks," July 2000. International Technology Research Institute, Baltimore, MD. Available at: <http://www.wtec.org/loyola/pdf/wireless.pdf>.
- [21] P. Lentini and J. DeFalco, "Adaptive Power Management Increases Cell Phone Talk Time," *Applied Microwave & Wireless*, vol. 14, pp. 32–41, Oct. 2002.
- [22] "RF Power Amplifier Characteristics." <http://www.rfmd.com>, <http://para.maxim-ic.com>, <http://www.spectrian.com/wa/whites.htm>.