Performance Analysis of Rayleigh Fading Ad Hoc Networks with Regular Topology

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***Abstract*— For wireless ad hoc networks with stationary and deterministically placed nodes, finding the optimal placement of the nodes is an interesting and challenging problem, especially under energy and QoS constraints. We study and compare the performance of several networks with regular topologies utilizing a Rayleigh fading link model. For nearest neighbor and shortest path routing, analytical expressions of the path efficiency, delay, and energy consumption for a given end-to-end reception probability are derived. For the interference analysis, the maximum throughput and optimum transmit probability are determined, and a simple MAC scheme is compared with an optimum scheduler, yielding lower and upper performance bounds.**

I. Introduction

In certain wireless ad hoc networks, in particular in wireless sensor network [1], many nodes will be stationary for most of the time after deployment. For example, in many applications of environmental monitoring, chemical/biological detection, security in a shopping mall or parking lot, the sensors are fixed. Moreover, to guarantee high exposure of the events of interest [2], uniform coverage is beneficial, suggesting the use of regular node placement schemes. Finding the optimal placement of nodes for a good trade-off between energy consumption, throughput, and delay is an important and challenging problem. In this paper, we investigate networks with regular topologies (square, triangle, hexagon) in which each node has the same number of nearest neighbors and the distance between all pairs of nearest neighbors is the same. We call them square, triangle, and hexagon networks.

Our analysis is based on a Rayleigh fading channel model, which includes both large-scale path loss and stochastic small-scale variations in the channel. As a result, all communication-related properties of the network become random variables, in particular the signal-to-noise-and-interference ratio (SINR) that determines the success of a transmission — in contrast to the "disk model" or "protocol model", where a deterministic transmission radius is assumed [3]. Note that even with static nodes as assumed in this paper, the channel quality varies because of movement in the environment, which is easily confirmed experimentally [4].

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For the performance analysis of multi-hop wireless ad hoc networks, the key issues are energy consumption, end-to-end reliability, delay and throughput. In the "disk model", the end-to-end reliability p_{EE} is not normally considered since links are assumed either 100% or 0% reliable. The Rayleigh fading model permit the characterization of packet reception probabilities and, consequently, the evaluation of the end-to-end reliability of a path. While Rayleigh fading was considered in earlier work [5], its impact on p_{EE} and the associated energy issues have not been studied. In this paper we are comparing all these characteristics for the three networks. In Section II, the Rayleigh fading link model is introduced, and it is shown that the noise analysis and interference analysis can be carried out separately. Section III presents the noise analysis of zero-interference networks. The energy consumption, path efficiency and delay of a connection for a given end-to-end reception probability is investigated. Section IV provides the interference analysis for different MAC schemes. The simulation results of a simple MAC scheme and an optimum MAC scheme determine the lower and upper bound of the achievable throughput. Section V concludes the paper.

II. The Rayleigh Fading Link Model

We assume a narrowband Rayleigh block fading channel. A transmission from node i to node j is successful if the signal-to-noise-and-interference ratio (SINR) γ_{ij} is above a certain threshold Θ that is determined by the communication hardware and the modulation and coding scheme. The SINR γ is given by $\gamma = \frac{Q}{N_0+I}$, where Q is the received power, which is exponentially distributed with mean $\bar{Q}$. Over a transmission of distance with an attenuation d^{α}, we have $\bar{Q} = P_0 d^{-\alpha}$, where P_0 denotes the transmit power, N_0 the noise power, and I is the interference power affecting the transmission, *i.e.*, the sum of the received power from all the undesired transmitters. The analysis is simplified by the following Theorem [6]:

Theorem 1: In a Rayleigh fading network, where nodes transmit at power level $P_i(\ i = 0, \dots, k)$, the reception probability $\mathbb{P}[Q_0 \geqslant \Theta(I + N_0)]$ of a transmission over a link distance d_0 with transmit power P_0 and k other nodes at distance d_i can be factorized into the reception probability of a zero-noise network and the reception probability of a

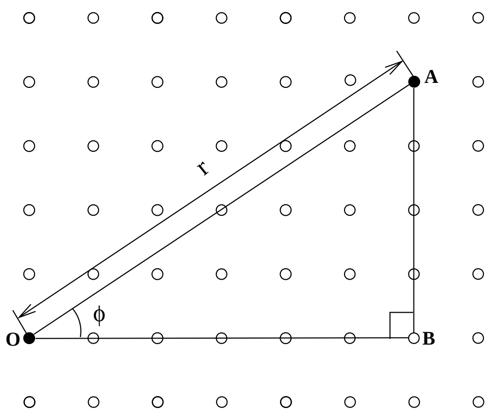


Fig. 1. Topology of a square network.

zero-interference network as follows:

$$p_r = \underbrace{\exp\Big(-\frac{\Theta N_0}{P_0 d_0^{-\alpha}}\Big)}_{p_r^N} \cdot \underbrace{\prod_{i=1}^{k} \frac{1}{1+\Theta \frac{P_i}{P_0}\big(\frac{d_0}{d_i}\big)^{\alpha}}}_{p_r^I} . \qquad (1)$$

p_r^N is the probability that the SNR $\gamma^N := Q_0/N_0$ is above the threshold Θ, *i.e.*, the reception probability in a zero-interference network as it depends only on the noise. The second factor p_r^I is the reception probability in a zero-noise network. This allows an independent analysis of noise and interference issues.

III. Noise Analysis

First, we study the performance of zero-interference networks, where only one node is transmitting at transmit power P_0 in every timeslot. For each connection, the source and destination are uniformly randomly chosen. It is assumed that the network is large and dense, which implies that the distributions of the Euclidean distance between the source and destination are identical for all three networks and that the direction is uniformly distributed in $[0, 2\pi)$. We route the packet via nearest neighbors along the shortest path toward its destination. In [7], this is called minimum-energy routing; we call it nearest neighbor and shortest path routing. It is assumed that all the three networks have the same node density $\lambda = 1$.

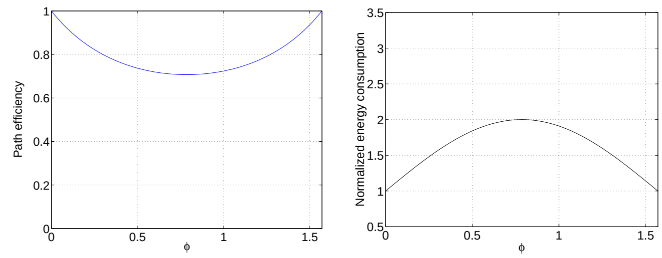
A. Square networks

We first analyze the square lattice network with $N = N_x \times N_y$ nodes and distance d_0 between all pairs of nearest nodes. The next-hop receiver of each packet is one of the four nearest neighbors (top, bottom, left and right). For square networks with unit density, the distance between nearest nodes $d_0 = 1$.

The optimality of a path can be measured by the ratio between the Euclidean distance r and the travelled distance d_T. So we define the path efficiency as

$$\eta = \frac{Euclidean\ distance}{travelled\ distance} = \frac{r}{d_T}, \qquad 0 < \eta \le 1. \qquad (2)$$

In Fig. 1, O is the source, A is the destination, and ϕ is the angle between $\overrightarrow{OA}$ and the horizontal axis. By using nearest



(a) Path efficiency (b) Energy consumption

Fig. 2. (a) Path efficiency and (b) normalized energy consumption as a function of ϕ for square networks.

neighbor and shortest path routing, the Euclidean distance r is $|OA|$ and the travelled distance d_T is $|OB| + |BA|$. We have

$$\eta(\phi) = \frac{r}{d_T} = \frac{r}{|r\cos\phi| + |r\sin\phi|} = \frac{1}{|\cos\phi| + |\sin\phi|}. \qquad (3)$$

If we move the destination along the line OA, the path efficiency will not change, so η is only a function of ϕ, and it is periodic with period $\pi/2$. Thus in the following analysis, we restrict ϕ between 0 and $\pi/2$. We can see that when $\phi = \pi/4$, $\eta_{\min} = 1/\sqrt{2}$; when $\phi = 0$ or $\pi/2$, $\eta_{\max} = 1$. ϕ is uniformly distributed based on the large and dense network assumptions, so the expected value of η is $\frac{2\sqrt{2}}{\pi}\arctan(\frac{\sqrt{2}}{2}) \approx 0.7935$. Fig. 2(a) displays the path efficiency as a function of ϕ between 0 and $\pi/2$.

We assume that every packet has a given end-to-end reception probability p_{EE}, dictated by the application (or the transport) layer. From Section II, we know that for a zero-interference network, the link reception probability over a link of distance d_0 is given by $p_r^N = e^{-\frac{\Theta N_0}{P_0 {d_0}^{-\alpha}}}$. Solving for P_0, we find the necessary transmit energy to achieve a link reliability p_r^N to be $E_L = \frac{{d_0}^{\alpha}\Theta N_0}{-\ln p_r^N}$. If there are h hops with equal distance d_0, the link reception probability p_r^N is $p_{EE}^{1/h}$. Then the transmit energy at each hop is

$$E_L = h\frac{{d_0}^{\alpha}\Theta N_0}{-\ln p_{EE}}. \qquad (4)$$

Using nearest neighbor and shortest path routing, the travelled distance is $d_T = r\cos\phi + r\sin\phi$, where r is the Euclidean distance. The number of hops d_T/d_0 is

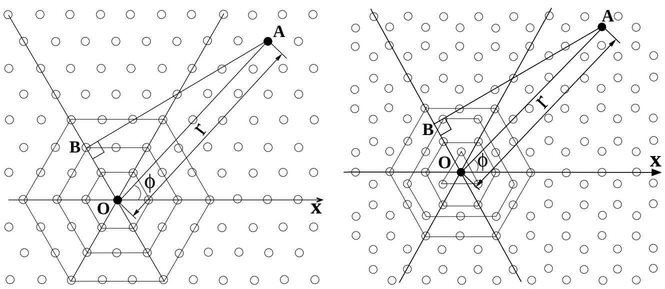
$$h = \frac{r}{d_0}(\cos\phi + \sin\phi). \qquad (5)$$

The total energy consumption of this route is

$$E_{tot}^s(\phi) = hE_L = \frac{r^2}{{d_0}^2}(\cos\phi + \sin\phi)^2 \frac{{d_0}^{\alpha}\Theta N_0}{-\ln p_{EE}}. \qquad (6)$$

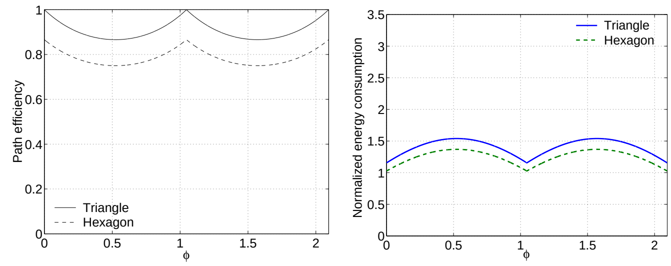
Let $E_0 := \frac{\Theta N_0}{-\ln p_{EE}}$. Considering the uniform distribution of ϕ, we can find the expected total energy consumption in units of E_0 as

$$\begin{aligned}\frac{\overline{E}_{tot}^s}{E_0} &= \frac{r^2}{{d_0}^{2-\alpha}}\frac{1}{\pi/2}\int_0^{\frac{\pi}{2}}(\cos\phi + \sin\phi)^2 d\phi = \frac{r^2}{{d_0}^{2-\alpha}}\left(1 + \frac{2}{\pi}\right)\\ &\approx 1.6366\frac{r^2}{{d_0}^{2-\alpha}}. \qquad (7)\end{aligned}$$



(a) Triangle network (b) Hexagon network

Fig. 3. Topologies of a triangle network and a hexagon network.



(a) Path efficiency (b) Energy consumption

Fig. 4. Path efficiency and normalized energy consumption as a function of ϕ for triangle and hexagon networks.

Fig. 2(b) displays the total energy consumption in units of $E_0 r^2$. Averaging the number of hops of (5) over ϕ, the expected number of hops $\overline{h}$ is $1.2732r$. For other routing schemes that permit longer hops, the transmit energy consumption is higher in most cases, in particular for high path loss exponents. We omit the details due to the limited space.

B. Triangle networks and hexagon networks

Other regular topologies of interest are the triangle topology and its dual, the hexagon topology. For each triangle, there are three vertices and six nearest neighbors for each vertex, while for the hexagon, there are six vertices for each hexagon and three nearest neighbors for each vertex. The distance between all pairs of nearest nodes is d_0. We use the same assumption as for the square networks, *i.e.*, the networks are large and dense such that the distribution of r and ϕ for all the topologies are identical. In the triangle network, each node is located in a hexagon with area $\frac{\sqrt{3}}{2}{d_0}^2$, so ${d_0^T}^2 = \frac{2}{\sqrt{3}}$ for unit density. Similarly, for hexagon networks, ${d_0^H}^2 = \frac{4}{3\sqrt{3}}$ for unit density. Note the superscript T, H denote the triangle network and hexagon network, respectively.

Fig. 3(a) shows a triangle network. O is the source and A is the destination. We want to find the number of hops h using nearest neighbor and shortest path routing. We can split the network into groups such that the members of one group are equidistant (in hops) to the source. These groups are the nodes in hexagons centered around the source as shown in Fig. 3(a). Thus, the first group will be 6 nodes that are one hop away in the hexagon with perimeter $6d_0$, the second group will be 12 nodes that are two hops away in the hexagon with perimeter $12d_0$, and so on. In Fig. 3(a), because the angle ϕ between $\overrightarrow{OA}$ and the horizontal axis Ox is between 0 and $\pi/3$, we draw the vertical $|AB|$ to the line with $2\pi/3$ to Ox. The hop number h is $|AB|$ divided by $d_0\sin(\pi/3)$. The travelled distance is hd_0. We will restrict ϕ within 0 and $\pi/3$ because h is a periodic function of ϕ with period $\pi/3$. Thus we have

$$
\begin{aligned}
h^T &= \frac{r\sin(2\pi/3-\phi)}{d_0\sin(\pi/3)} = \frac{r}{d_0^T}\left(\cos\phi + \frac{1}{\sqrt{3}}\sin\phi\right),\\
\eta^T(\phi) &= \frac{r}{h^T d_0^T} = \frac{\sqrt{3}}{2\sin(2\pi/3-\phi)} = \frac{\sqrt{3}}{\sqrt{3}\cos\phi+\sin\phi},\\
&\text{for} \qquad 0 \le \phi \le \pi/3. \qquad (8)
\end{aligned}
$$

The expected value of η^T is $\frac{3\sqrt{3}\ln 3}{2\pi} \approx 0.9085$.

TABLE I

COMPARISON OF SQUARE, TRIANGLE, HEXAGON NETWORKS FOR $\alpha = 3$ AND $\alpha = 4$ (NOISE ANALYSIS).

	Energy (α=3)	Energy (α=4)	Hop numbers $\overline{h}/r$	Path efficiency $\overline{\eta}$
Square	1.6366	1.6366	1.2732	0.7935
Triangle	1.3088	1.4064	1.0261	0.9085
Hexagon	1.4249	1.2502	1.4512	0.7868

For the hexagon topology, we use a similar method to find the group of nodes that are equidistant (in hops) from the source as shown in Fig. 3(b). We have

$$
\begin{aligned}
h^H &\approx \frac{2r\sin(2\pi/3-\phi)}{3/2 d_0^H} = \frac{2r}{\sqrt{3}d_0^H}\left(\cos\phi + \frac{1}{\sqrt{3}}\sin\phi\right),\\
\eta^H(\phi) &\approx \frac{3/2}{2\sin(2\pi/3-\phi)} = \frac{3}{2\sqrt{3}\cos\phi + 2\sin\phi}, \qquad (9)
\end{aligned}
$$

where $0 \le \phi \le \pi/3$. The expected value of η^H is $\frac{9\ln 3}{4\pi} \approx 0.7868$. Fig. 4(a) shows the relationship between the path efficiency and ϕ for triangle and hexagon networks for ϕ within 0 and $2\pi/3$.

Similar to the square network, we determine the total transmit energy consumption for a given end-to-end reception probability provided the Euclidean distance from the source to the destination is fixed for all the topologies. The total energy consumption is $E_{tot}(\phi) = hE_L = h^2 \frac{{d_0}^\alpha \Theta N_0}{-\ln p_{EE}} = h^2 d_0^\alpha E_0$. Inserting the expressions of h for the triangle and hexagon topologies yields the energy consumption in both topologies:

$$
\begin{aligned}
E_{tot}^T(\phi) &= \frac{(\sqrt{3}\cos\phi+\sin\phi)^2}{3}E_0 r^2 {d_0}^{\alpha-2},\\
E_{tot}^H(\phi) &= \frac{4(\sqrt{3}\cos\phi+\sin\phi)^2}{9}E_0 r^2 {d_0}^{\alpha-2}, \qquad (10)
\end{aligned}
$$

where E_{tot}^T and E_{tot}^H denotes the energy consumption of triangle and hexagon networks for $0 \le \phi \le \pi/3$. The expected total energy consumption in units of $E_0 r^2$ is $(\frac{\sqrt{3}}{\pi} + \frac{2}{3}){d_0^T}^{\alpha-2}$ for the triangle topology and $(\frac{4}{\sqrt{3}\pi} + \frac{8}{9}){d_0^H}^{\alpha-2}$ for the hexagon topology. The expected total energy consumption for triangle and hexagon topology for $\alpha = 3$ and $\alpha = 4$ is listed in Table I. It is shown for $\alpha = 3$, triangle topology consumes less energy, while for $\alpha = 4$, hexagon topology gives the least energy consumption. The difference comes from the factor of $d_0^{\alpha-2}$. Fig. 4(b) displays the normalized energy consumption for

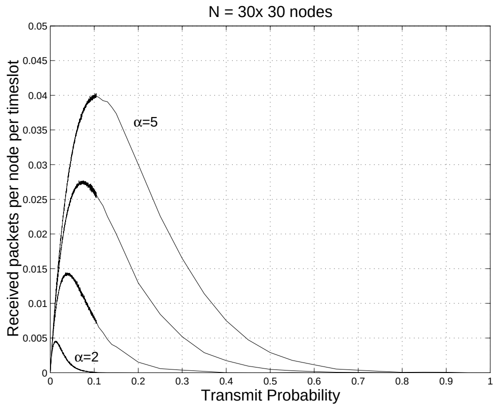


Fig. 5. Received packet per node per timeslot for $\Theta = 10$ and $\alpha = 2, 3, 4, 5$ for a square network with $N = 30 \times 30$ nodes.

$\alpha = 4$ for the two topologies. Averaging the number of hops of (8) and (9) over ϕ, the expected number of hops $\overline{h}$ for fixed r in units of r are $\frac{2\sqrt{3}}{\pi}/d_0^T \approx 1.0261$ and $\frac{4}{\pi}/d_0^H \approx 1.4512$ for triangle and hexagon networks. So for $\alpha = 2, 3$, in a zero-interference network, the triangle topology is the best one due to its lowest energy consumption, least delay and highest path efficiency. However, for $\alpha = 4, 5$, the hexagon topology has the least energy consumption.

IV. Interference Analysis

In this section, we consider a network of N nodes, where every node always has a packet to transmit (heavy traffic assumption). The reception is only corrupted by interference, not by noise.

A. A simple MAC scheme

First, we study a very simple MAC scheme, with the aim of finding a lower performance bound for more elaborate schemes. For the network, it is assumed that nodes are transmitting packets independently in every timeslot with transmit probability p at equal transmit power level and the next-hop receiver of every packet is one of its neighbors. The packets are of equal length and fit into one timeslot[1]. The performance measure is the throughput which is the expected number of successful packet transmissions in one timeslot.

Here we provide simulation results[2] of the three networks. A detailed analysis of the throughput for networks with different topologies can be found in [9]. Fig. 5 displays the simulation result of the relationship between the per-node throughput and transmit probability for $\Theta = 10$ and various α for a square network with 30×30 nodes, where for $\alpha = 4$, the maximum per-node throughput $g_{\max}/N = 0.0277$ is achieved at the optimum transmit probability $p_{\max} = 0.0748$. The throughput curve for other two topologies has a similar shape, but $g_{\max}$ and $p_{\max}$ are different.

Without interference, we would have $g_{\max}/N = 100\% p_{max}$. So we define the transmit efficiency as $T_{\text{eff}} = \frac{g_{\max}/N}{p_{\max}}$. For all the three networks, the transmission efficiency

TABLE II

Comparison of square, triangle and hexagon networks for $\alpha = 4$ (interference analysis), where $p_{\max}$, $g_{\max}/N$, T_{eff}, $\bar{l}_h$, Z denote optimum transmit probability, maximum throughput per node, transmit efficiency, effective hop length and effective transport capacity.

	$p_{\max}$	$g_{\max}/N$	T_{eff}	$\bar{l}_h$	Z
Square	0.0748	0.0277	0.37	0.7935	0.0220
Triangle	0.0642	0.0238	0.37	0.9085	0.0216
Hexagon	0.0990	0.0364	0.37	0.7868	0.0286

is about 0.37, which is similar to that of slotted ALOHA[3], namely e^{-1}. We define the effective hop length as $\bar{l}_h = E[r/h]$ for a fixed r from the expressions in (5), (8) and (9). The effective transport capacity is the distance-weighted throughput, defined as $Z := \frac{g_{\max}}{N} \cdot \bar{l}_h$.

The comparison of square, triangle and hexagon networks for $\alpha = 4$ is shown in Table II. The transmit efficiency is about 0.37 for three topologies. We see that the hexagon network has the highest transmit probability, throughput, and effective transport capacity.

B. Comparison with optimum scheduler

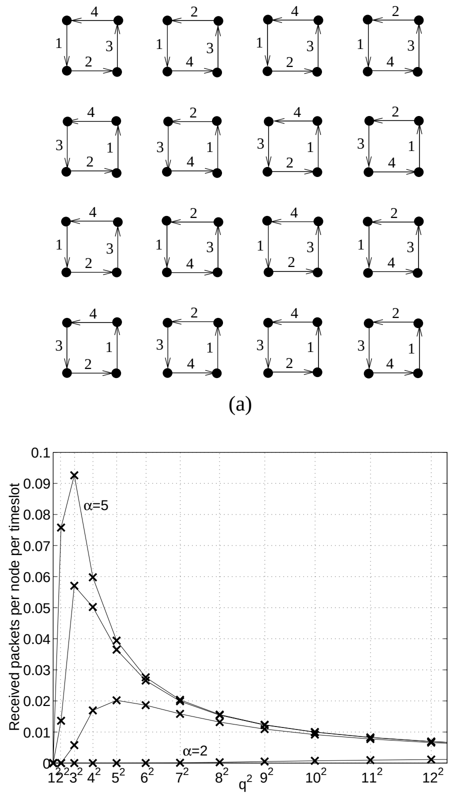
For large networks, nodes in different locations can use the channel simultaneously (spatial reuse) if they are sufficiently separated so that mutual interference will not prevent simultaneous successful transmissions. Exploiting spatial reuse, we can devise a scheduling scheme that maximizes the throughput. Here we will deal with the scheduling problem in a square network. Assume that in every square area with q^2 nodes, only one node is transmitting. Fig. 6(a) shows the optimum scheduling scheme for $q = 2$ for the first 4 phases (the number indicates the phase number). Shifting the four links connecting four nodes in the squares to their right and bottom squares, we can get another 4 phases. Since it is for bidirectional traffic, 16 phases are needed. The total number of phases is $2q(q-1)+4+2(q-2) = 4q^2$. In Fig. 6(b), the throughput as a function of q^2 is plotted. Optimum scheduling is achieved at $q = 16,\ 5,\ 3,\ 3$ for $\alpha = 2,\ 3,\ 4,\ 5$. The throughput ratio between the simple MAC scheme and the optimum one is $0.71,\ 0.48,\ 0.43$ for $\alpha = 3,\ 4,\ 5$, which shows that the relative performance of the simple MAC scheme is better for lower α than higher α.

Interestingly, the curve of $\alpha = 2$ is quite different from the simulation results of the simple MAC scheme shown in Fig. 5. The reason is that for $\alpha = 2$, the received interference power will be infinite for a receiver located in an infinite plane with a uniform and finite density of transmitters, as pointed out in [7]. The results of the simple MAC scheme is for a 30×30 network, while the result of the optimum scheduler is derived for a very large network. So, in the latter case, the SIR is much smaller, confirming that the per-node throughput for $\alpha = 2$ to converge to zero with increasing network size.

[1]The same MAC scheme was considered in [8].

[2]We use MATLAB to simulate the MAC scheme and the Rayleigh fading channel.

[3]In fact, the simple MAC scheme is very similar to slotted ALOHA. In general, slotted ALOHA assumes Poisson traffic, whereas our simple MAC scheme assumes every node always has a packet to transmit in each timeslot (heavy traffic assumption).



(a)

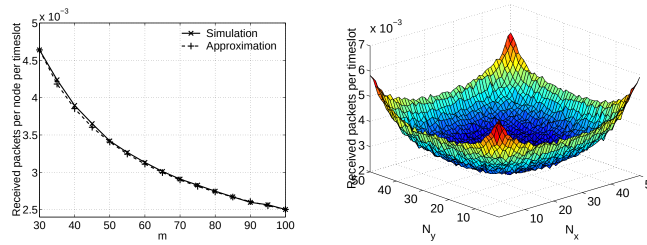
(b)

Fig. 6. (a) The optimum transmit scheduler for $q = 2$. (b) Received packets per node and timeslot for $\Theta = 10$ and $\alpha = 2$, 3, 4, 5 for a large square network where every q^2-th node is transmitting in every timeslot.

For square networks with $m \times m$ nodes, the simulation results in Fig. 7(a) show that the per-node throughput for $\alpha = 2$ decreases with m (solid line). We approximate the relationship between the per-node throughput and m by $a/\ln(bm)$ with $a = 0.0065$ and $b = 0.1366$. The approximation is plotted in Fig. 7(a) by dashed line. The perfect match confirms the result in [7], where it was shown that for $\alpha = 2$, the total interference power of a network of radius r is given by integrating c/r (for some constant c) from some R_0 to r. Hence, the SIR depends logarithmically on r. In Fig. 7(b), the throughput distribution over the location of the nodes is recorded for a network with 50×50 nodes at the optimal transmit probability $p_{\max} = 0.0101$ with the per-node throughput $g_{\max}/N = 0.0034$. We notice that nodes at the boundary of the network, especially at the corner, contribute the most to the throughput, since they are subject to less interference. This shows that even for fairly large networks, the boundary effects cannot be neglected.

V. Conclusions

The Rayleigh fading link model does not only characterize the wireless link more accurately than the "disk model", but it also permits a separate analysis of noise and interference. The resulting packet reception probabilities simply have to be multiplied, yielding the total reception probability $\mathbb{P}[\text{SINR} > \Theta]$. The noise analysis can be carried out independently of the MAC scheme, and the interference analysis is invariant to a common factor in the transmit power, as the SIR does not change when all the nodes scale their power by the same factor.



(a) (b)

Fig. 7. For $\alpha = 2$, (a) The simulation result and approximation of the relationship between the per-node throughput and m. (b) Distribution of throughput over 50×50 nodes.

In the case of light traffic, the noise analysis alone provides accurate results, whereas in the case of heavy traffic, the network will be interference-limited, and both parts are relevant. The characterization of the link reception probabilities permits the evaluation of end-to-end packet reception probabilities p_{EE}; in our analysis, p_{EE} is assumed to be dictated by the application as part of the QoS specification.

In the noise analysis, for $\alpha = 2, 3$, the triangle network gives the best performance due to its lowest energy consumption, delay and highest path efficiency. However, for $\alpha = 4, 5$, the hexagon network has the least energy consumption.

In the interference analysis, the hexagon network exhibits the highest transmit probability, throughput and effective transport capacity. By comparing the topologies and results, we find that connecting with less nearest neighbors can improve the throughput. For square networks, for $\alpha > 2$, the throughput ratio between the simple MAC scheme and the optimum one is between $[0.40, 0.75]$ – the simple MAC is closer to the optimum for lower α. The performance of any practical MAC layer will lie between the bounds provided by these two MAC schemes. For $\alpha = 2$ (free space propagation), spatial reuse is not possible for the interior nodes in large and dense networks, since the interference power diverges to infinity as the network size is scaled.

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