ALOHA Performs Delay-Optimum Power Control

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***Abstract*—As a fundamental source of delay in wireless networks, the local delay is defined as the mean time, in number of time slots, until a packet is successfully received (decoded) over a link. This paper shows that with mean power and peak power constraints at each node, power control can significantly reduce the local delay. We show that, for links with Rayleigh fading and random length, there exists a simple power control strategy, which turns out to be optimal in reducing the local delay. This strategy acts as an ALOHA-type random on-off power control policy whose parameters depend on the link distance. The optimal power control policy as well as its variations are compared with constant power transmission and other simple random power control policies.**

I. Introduction

In wireless networks, the local delay is defined as the mean time, in number of time slots, for a packet to be successfully received (decoded) over a link. Although it is an important source of delay in wireless networks, the local delay is hardly mentioned before [1]–[3]. In [1], the author shows that different fading distributions can result in significantly different local delays, which suggests power control is helpful in reducing the local delay.

In this paper, focusing on the Rayleigh fading case, we show that an ALOHA-type random on-off power control policy minimizes the local delay in an interferenceless wireless network, where the link distances are random but fixed and the transmit power at each node is subject to a mean power as well as a peak power constraint.

Power control is well known to be a technique that can potentially benefit both the point-to-point wireless communication and wireless networks [4]–[7]. Among all the literature discussing power control, much takes into account the delay [5], [8]–[11]. However, the delay always enters the picture as a constraint when maximizing the throughput. Moreover, none of the existing literature, except for [1], [3], tries to characterize the relation between power control and the local delay. The optimal power control policy proved in this paper establishes this relation. We also present several suboptimal power control policies which are inspired by the optimal policy. The local delay performance of different power control policies as well as constant power transmission are compared and discussed at the end of the paper.

II. The Local Delay

The basic model we use in this paper is the one provided in [1]. We consider a collection of links, whose distances are iid randomly distributed and fixed over time (Fig. 1). These links do not interfere with each other. For any of the links, the received power is

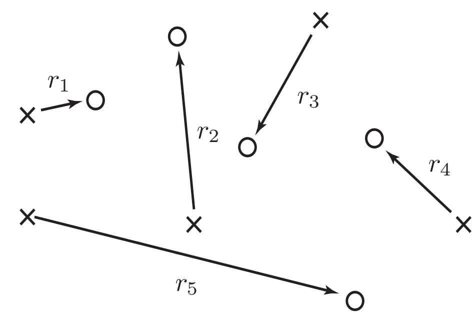

Fig. 1: A collection of links with random distances. Transmitters are denoted by × and receivers are denoted by o. The distances $r_k, k \in [5]$ are iid drawn from some distribution $f_R(x)$.

$$P_r = PHR^{-\alpha},$$

where P is the transmit power, H is the (power) fading factor, R is the link distance, and α is the path loss coefficient. We use an SNR condition to define whether a transmission is successful. A transmission is regarded successful if $P_r > \theta$, where θ incorporates both the SNR threshold and noise power. Then, we can write the success probability of a single transmission conditioned on R as

$$p_{s|R} = \mathbb{P}(PHR^{-\alpha} > \theta \mid R).$$

Although for each link the distance R is considered constant over time and can be learned by the transmitter as well as the receiver, the fading coefficient H is assumed iid over time and is unknown to both the transmitter and the receiver. In this paper, we focus on Rayleigh fading, and thus H is exponentially distributed with unit mean. Since the link distance R can be learned by the node pairs, the transmit power P can be a (stochastic) function of R.

For a particular link of distance r, the *conditional local delay*, defined as the mean number of time slots that the receiver needs to successfully decode the message conditioned on the link distance $R = r$ is

$$D_r = \frac{1}{p_s(r)}, \tag{1}$$

where $p_s(r) = p_{s|R=r} = \mathbb{P}(PHr^{-\alpha} > \theta)$. The mean of the conditional local delay is just the local delay $D = \mathbb{E}_R[\frac{1}{p_{s|R}}]$.

For the same power control policy and the same mean distance, *e.g.*, $\mathbb{E}R = 1$, different link distance distributions result in different local delays. Generally, for a particular power control policy, there exists some distribution of R which minimizes the local delay. For example, in the case of constant power transmission, it can be shown by Jensen's inequality that the local delay is minimized when the link distance is a constant. Finding the optimal distribution that minimizes the local delay for a specific power control policy is an interesting problem, but is beyond the content of this paper.

In this paper, we assume the link distance distribution $f_R(x)$ is given. Since the link distance $R = r$ can be learned by the transmitters, if there exists a power control policy that minimizes the conditional local delay D_r for all r, the local delay D is automatically minimized. The goal of the paper is to derive and analyze such a policy.

III. The Optimal Power Control Policy

A. Problem Formulation

As stated earlier, in fixed network, it is reasonable to assume a known link distance $R = r$. Thus, the optimal power control policy takes r as a parameter and is independent of the distribution of R.

We start from the general case where the transmit power P can be random. Without loss of generality, we consider a unit mean power constraint and a peak power constraint $P_{\max}$, with $P_{\max} > 1$ (otherwise, the mean power constraint will always be loose). Since except for P the only randomness in (1) is the fading factor H, which is iid in different time slots, there is no benefit in assigning different statistics to P in different time slots. Therefore, the unit mean power constraint and the peak power constraint above can be expressed as $\mathbb{E}P = 1$ and $P \leq P_{\max}$ respectively, and P is iid as well.

Let $\mathcal{P}$ be the class of probability density functions (pdf's) with support at most $[0, P_{\max}]$ and mean 1. Then, the general problem is to find the pdf $f^*_{P|r}$ of the transmit power $P(r)$, where

$$\begin{aligned} f^*_{P|r} &\triangleq \arg\min_{f_{P|r}\in\mathcal{P}} \mathbb{E}_R\left[\frac{1}{\mathbb{P}(P(R)HR^{-\alpha} > \theta \mid R)}\right] \\ &= \arg\max_{f_{P|r}\in\mathcal{P}} \mathbb{P}(P(r)Hr^{-\alpha} > \theta). \end{aligned}$$

B. ALOHA is the Optimal Policy

Conditioned on the distance $R = r$, the local delay is simply the inverse of the success probability $\mathbb{P}(HPr^{-\alpha} > \theta)$. Then,

$$\begin{aligned} \mathbb{P}(HPr^{-\alpha} > \theta) &= \int_0^\infty \bar{F}_P(\frac{\theta r^\alpha}{h})e^{-h}\mathrm{d}h \\ &= \theta r^\alpha \int_0^\infty \bar{F}_P(x^{-1})e^{-\theta r^\alpha x}\mathrm{d}x, \end{aligned}$$

where $\bar{F}_P(x)$ is the complementary cumulative distribution function (ccdf) of the randomly controlled power P. Thus it must be monotonically decreasing, $\bar{F}_P(x) = 0\ \forall x > P_{\max}$, and by the mean power constraint $\int_0^\infty \bar{F}_P(x)dx = 1$.

For the sake of simplicity, we define the following function

$$G(x) \triangleq \bar{F}_P(x^{-1}),\ \forall x > 0, \quad (2)$$

which is the cumulative distribution function (cdf) of P^{-1}. The constraints on $\bar{F}_P$ are then transferred into the constraint that $G(x)$ is monotonically increasing, $G(x) = 0\ \forall x < P_{\max}^{-1}$, $\lim_{x\to\infty} G(x) \leq 1$ and

$$\mathbb{E}P = \int_0^\infty x^{-2}G(x)\mathrm{d}x = 1.$$

The problem is to find the $G^*(x)$, defined as the optimal $G(x)$ satisfying all the requirements above and maximizing $\int_0^\infty G(x)e^{-\theta r^\alpha x}\mathrm{d}x$. Note that $\lim_{x\to\infty} G(x)$ stands for $\mathbb{P}(P \leq 0)$ which is non-zero whenever there is a positive probability of the event $\{P = 0\}$. Since the distribution of P is not necessarily continuous, in general, $\lim_{x\to\infty} G(x)$ does not have to be 1.

Lemma 1. *The desired function $G^*(x)$ satisfies*

$$G^*(x) = G^*(x_M),\ \forall x > x_M,$$

where $x_M \triangleq 1/\min\{P_{\max}, \theta r^\alpha\}$.

Proof: First, consider the case where $G^*(x)$ is a simple function. Since $G^*(x)$ is monotonically increasing, we can write it as

$$G^*(x) = \sum_{i=0}^N a_i 1_{[b_i, b_{i+1})}(x), \quad (3)$$

where $0 = a_0 < a_1 < a_2 < \cdots < a_N \leq 1$ and $0 = b_0 < b_1 < b_2 < \cdots < b_{N+1} = \infty$. Suppose there exists a $x_0 > x_M$, such that $G^*(x_0) \neq G^*(x_M)$, *i.e.*, $G^*(x_0) > G^*(x_M)$, and assume $x_0 \in [b_j, b_{j+1})$, $x_M \in [b_l, b_{l+1})$, for some $l, j \in \mathbb{N}$ such that $0 < l < j$. Then, let

$$\begin{aligned} \tilde{G}(x) \triangleq G^*(x) &- \sum_{n=l+1}^j (a_n - a_{n-1})1_{[b_n,\infty)}(x) \\ &+ x_M \sum_{n=l+1}^j \frac{a_n - a_{n-1}}{b_n} 1_{[x_M,\infty)}(x). \end{aligned} \quad (4)$$

It can be easily verified that $\int_0^\infty x^{-2}\tilde{G}(x)\mathrm{d}x = \int_0^\infty x^{-2}G^*(x)\mathrm{d}x$, and that $\tilde{G}(x)$ satisfies all the requirements for a valid $G(x)$ over $[0,\infty)$. Moreover,

$$\begin{aligned} &\int_0^\infty e^{-\theta r^\alpha x}\tilde{G}(x)\mathrm{d}x - \int_0^\infty e^{-\theta r^\alpha x}G^*(x)\mathrm{d}x \\ &= \int_0^\infty e^{-\theta r^\alpha x} x_M \sum_{n=l+1}^j \frac{a_n - a_{n-1}}{b_n} 1_{[x_M,\infty)}(x)\mathrm{d}x \\ &\quad - \int_0^\infty e^{-\theta r^\alpha x} \sum_{n=l+1}^j (a_n - a_{n-1})1_{[b_n,\infty)}(x)\mathrm{d}x \\ &= \sum_{n=l+1}^j \frac{a_n - a_{n-1}}{b_n \theta r^\alpha}\left(x_M e^{-\theta r^\alpha x_M} - b_n e^{-\theta r^\alpha b_n}\right) \\ &> 0, \end{aligned}$$

where the last inequality is due to the monotonicity of $xe^{-\theta r^\alpha x}$ in $[\frac{1}{\theta r^\alpha}, \infty)$ and the fact that $b_n > x_M \geq \frac{1}{\theta r^\alpha}$ $\forall n \geq l+1$. This contradicts the assumption that $G^*(x)$ is the function which maximizes $\int_0^\infty G(x)e^{-\theta r^\alpha x}\mathrm{d}x$ and satisfies all the constraints.

For general $G^*(x)$, consider a sequence of simple functions $(G_k^*)_1^\infty$ such that $G_i^* < G_j^* < G^*$, $\forall i < j$ and $\lim_{k\to\infty} G_k^* = G^*$. By the monotone convergence theorem, $\lim_{k\to\infty} \int_0^\infty x^{-2}G_k^*(x)\mathrm{d}x = \int_0^\infty x^{-2}G^*(x)\mathrm{d}x$ and $\lim_{k\to\infty} \int_0^\infty e^{-\theta r^\alpha x}G_k^*(x)\mathrm{d}x = \int_0^\infty e^{-\theta r^\alpha x}G^*(x)\mathrm{d}x$. Using the construction in the proof for the simple functions, we are able to produce another sequence of simple functions $(\tilde{G}_k)_1^\infty$, such that $\int_0^\infty e^{-\theta r^\alpha x}\tilde{G}_k(x)\mathrm{d}x > \int_0^\infty e^{-\theta r^\alpha x}G_k^*(x)\mathrm{d}x, \forall k$. Meanwhile, $\lim_{k\to\infty} \tilde{G}_k \neq G^*$, since $\tilde{G}_k(x_0) = \tilde{G}_k(\frac{1}{\theta r^\alpha})$. Thus, the limiting function of $\tilde{G}_k(x)$ is a strictly better candidate for $G(x)$ than $G^*(x)$. ■

Analogously, we have the following lemma.

Lemma 2. *If $1 \leq \theta r^\alpha \leq P_{\max}$ and $G^*(x)$ is the desired function, we have*

$$G^*(x) = 0, \ \forall x < \frac{1}{\theta r^\alpha}.$$

Proof: The proof is essentially the same of that of the previous lemma. We start with the case where $G^*(x)$ is simple and then generalize to the case of any valid cdf.

Consider the case where $G^*(x)$ is a simple function and write it as (3). Assuming $\frac{1}{\theta r^\alpha} \in [b_l, b_{l+1})$, we can construct

$$\tilde{G}(x) = G^*(x) - \sum_{n=1}^{l} a_n 1_{[b_n,\infty)}(x) + \sum_{n=1}^{l} \frac{a_n}{b_n\theta r^\alpha} 1_{[\frac{1}{\theta r^\alpha},\infty)}(x).$$

Suppose that $G^*(x_0) > 0$ for some $x_0 < \frac{1}{\theta r^\alpha}$, we know $\tilde{G}(x) \neq G^*(x)$, since $\tilde{G}(x) = 0, \ \forall x < \frac{1}{\theta r^\alpha}$. Meanwhile, it can be verified that $\int_0^\infty x^{-2}\tilde{G}(x)\mathrm{d}x = \int_0^\infty x^{-2}G^*(x)\mathrm{d}x$. By Lemma 1, $G^*(x) = G^*(\frac{1}{\theta r^\alpha}), \ \forall x > \frac{1}{\theta r^\alpha}$ and thus $\tilde{G}(x) \leq 1$ (Because $\int_1^\infty x^{-2}\mathrm{d}x = 1$). All other constraints over $\tilde{G}(x)$ to be a valid candidate of $G(x)$ are automatically satisfied. Also,

$$\begin{aligned}
&\int_0^\infty e^{-\theta r^\alpha x}\tilde{G}(x)\mathrm{d}x - \int_0^\infty e^{-\theta r^\alpha x}G^*(x)\mathrm{d}x \\
&= \sum_{n=1}^{l} \int_0^\infty \left(\frac{a_n}{b_n\theta r^\alpha} 1_{[\frac{1}{\theta r^\alpha},\infty)} - a_n 1_{[b_n,\infty)} \right) e^{-\theta r^\alpha x}\mathrm{d}x \\
&= \sum_{n=1}^{l} \frac{a_n}{b_n\theta r^\alpha} \left(\frac{1}{\theta r^\alpha} e^{-\theta r^\alpha \frac{1}{\theta r^\alpha}} - b_n e^{-\theta r^\alpha b_n}\mathrm{d}x \right) \\
&> 0,
\end{aligned}$$

where the last inequality is due to the fact that $b_n \leq \frac{1}{\theta r^\alpha}$, $\forall n \leq l$ by assumption, and the monotonicity of $xe^{-\theta r^\alpha x}$ in $[0, \frac{1}{\theta r^\alpha}]$. Therefore, we found $\tilde{G}(x)$ as a better candidate than $G^*(x)$, contradicting the assumption that it is the desired function. The generalization from simple function to general functions is the same as that in the proof of Lemma 1. ■

Similar methods can be also applied to prove the following lemma.

Lemma 3. *If $\theta r^\alpha < 1$ and $G^*(x)$ is the desired function, we have*

$$G^*(x) = 0, \ \forall x < 1.$$

Although special care must be taken to make sure that $\tilde{G}(x) \leq 1$, the proof of Lemma 3 directly follows that of Lemma 2 and is therefore omitted.

Combining Lemmas 1, 2, 3 and the requirements we have for a valid $G(x)$, we directly deduce the form of $G^*(x)$. That is

$$G^*(x) = \begin{cases} 1_{[1,\infty)}(x), & \theta r^\alpha \leq 1 \\ \frac{1}{\theta r^\alpha} 1_{[\frac{1}{\theta r^\alpha},\infty)}(x), & 1 < \theta r^\alpha \leq P_{\max} \\ P_{\max}^{-1} 1_{[P_{\max}^{-1},\infty)}(x), & \theta r^\alpha > P_{\max}. \end{cases}$$

As stated earlier, there is a one-to-one mapping between $G(x)$ and $\bar{F}_P(x)$ (and thus $F_P(x)$). Then, the result above directly leads to the following theorem.

Theorem 1. *Given a link distance r, the optimal distribution of the transmit power P that minimizes the local delay is*

$$F_P(x) = \begin{cases} 1_{[1,\infty)}(x), & \theta r^\alpha \leq 1 \\ (1 - \frac{1}{\theta r^\alpha}) 1_{[0,\theta r^\alpha)}(x) + 1_{[\theta r^\alpha,\infty)}(x), & 1 < \theta r^\alpha \leq P_{\max} \\ (1 - P_{\max}^{-1}) 1_{[0,P_{\max})}(x) + 1_{[P_{\max},\infty)}(x), & \theta r^\alpha > P_{\max}. \end{cases}$$

More concisely, we can define

$$\xi \triangleq \max\{1, \min\{P_{\max}, \theta r^\alpha\}\}.$$

Then, Theorem 1 says: the optimal random power control strategy is ALOHA-type random on-off policy with transmit probability ξ^{-1} and transmit power ξ.

Definition 1. *A link of distance r is said to be in the power-limited regime if $\theta r^\alpha > P_{\max}$.*

Definition 2. *A link of distance r is said to be in the short-distance regime if $\theta r^\alpha \leq 1$.*

Interestingly, although, in both regimes defined above, the optimal power control strategy can be interpreted as an ALOHA-type random on-off policy, the optimal strategy maximizes the variance of transmit power in the power-limited regime, while minimizing this variance in the short-distant regime.

Theorem 1 also indicates that in order to apply the optimal power control policy, we need to know either r and α or r^α, where given $\mathbb{E}H = 1$, r^α can be easily obtained by simply taking the average of the received power.

Corollary 1. *Without peak power constraint, but with the mean power limited to $\mathbb{E}P = 1$, the optimal random power control policy is*

$$F_P(x) = \begin{cases} 1_{[1,\infty)}(x), & \theta r^\alpha \leq 1 \\ (1 - \frac{1}{\theta r^\alpha}) 1_{[0,\theta r^\alpha)}(x) + 1_{[\theta r^\alpha,\infty)}(x), & \theta r^\alpha > 1. \end{cases}$$

The exact value of the local delay depends on the distribution of the link distance R. A Rayleigh distribution is often used for the distribution of R, since it is the distribution of the nearest-neighbor distance in a 2-dimensional network whose nodes are distributed as a Poisson point process (PPP) [12]. It is shown in [1] that with such distribution of R the local delay is unbounded if Rayleigh fading is considered and no power control is applied (except for the case of $\alpha = 2$). The natural question is whether random power control can make the local delay finite in the same scenario. In the case where only a mean power constraint is imposed, applying the result in Corollary 1, we have

$$\begin{aligned}D &= \mathbb{E}\left[\frac{1}{p_{s|R}}\right] \\ &= 2\pi\lambda\int_0^{\theta^{-\frac{1}{\alpha}}} re^{\theta r^\alpha-\lambda\pi r^2}\mathrm{d}r + 2\pi\lambda\theta e\int_{\theta^{-\frac{1}{\alpha}}}^{\infty} r^{\alpha+1}e^{-\lambda\pi r^2}\mathrm{d}r \\ &\le e(1-e^{-\lambda\pi\theta^{-\frac{2}{\alpha}}}) + \theta e(\lambda\pi)^{-\frac{\alpha}{2}}\Gamma(\frac{\alpha}{2}+1, \lambda\pi\theta^{-\frac{2}{\alpha}}) \\ &< \infty,\end{aligned}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function. In other words, power control can keep the local delay finite while keeping the mean transmit power at each node limited even if the link distance is Rayleigh distributed.

Corollary 2. *With unit mean power constraint and peak power constraint, power control cannot reduce the local delay to a finite value, when the link distance is Rayleigh distributed.*

Proof: Applying the power control policy described in Theorem 1, we have

$$\begin{aligned}D &= \mathbb{E}\left[\frac{1}{p_{s|R}}\right] \\ &= 2\pi\lambda\int_0^{\theta^{-\frac{1}{\alpha}}} re^{\theta r^\alpha-\lambda\pi r^2}\mathrm{d}r + 2\pi\lambda\theta e\int_{\theta^{-\frac{1}{\alpha}}}^{\delta^{-\frac{1}{\alpha}}} r^{\alpha+1}e^{-\lambda\pi r^2}\mathrm{d}r \\ &\quad + 2\pi\lambda P_{\max}\int_{\delta^{-\frac{1}{\alpha}}}^{\infty} re^{\delta r^\alpha-\lambda\pi r^2}\mathrm{d}r \\ &\ge 2\pi\lambda P_{\max}\int_{\delta^{-\frac{1}{\alpha}}}^{\infty} re^{\delta r^\alpha-\lambda\pi r^2}\mathrm{d}r \\ &= \infty,\end{aligned}$$

for all $\alpha > 2$, where $\delta \triangleq \theta/P_{\max}$. The corollary then follows from the optimality stated in Theorem 1. ∎

IV. Comparison of Random Power Control Schemes

In this section, we compare the local delay performance of several power control policies. First, we define a few power control policies:

Definition 3. *The optimal power control (OPC) policy is the power control policy defined in Theorem 1.*

Definition 4. *The peak power control (PPC) policy transmits at power $P_{\max}$ with probability $P_{\max}^{-1}$ and does not transmit with probability $1 - P_{\max}^{-1}$ regardless of the value of r.*

Definition 5. *The uniform power control (UPC) policy transmits at power P each time with P uniformly distributed in $[1-\Delta, 1+\Delta]$. Here, $\Delta \triangleq \min\{1, P_{\max}-1\}$.*

Definition 6. *The hybrid uniform power control (HUPC) policy transmits with probability $\frac{2}{P_{\max}+1}$. If transmitting, the transmit power is uniformly distributed between 1 and $P_{\max}$.*

Definition 7. *The 1-bit power control (1BPC) policy transmits at constant power ($P = 1$) when $\theta r^\alpha \le \frac{\log P_{\max}}{1-P_{\max}^{-1}}$. When $\theta r^\alpha > \frac{\log P_{\max}}{1-P_{\max}^{-1}}$, the policy transmits at power $P_{\max}$ with probability $P_{\max}^{-1}$ and does not transmit with probability $1 - P_{\max}^{-1}$.*

While the peak power control (PPC) policy, uniform power control (UPC) policy, and hybrid uniform power control (HUPC) policy are all suboptimal, their complexity is lower than OPC's. In particular, they do not require the link distance information R. Their constructions are inspired by Theorem 1 in different ways. For example, in the power-limited regime PPC is as good as OPC. The intuition behind HUPC is that Theorem 1 implies that for all realizations of R it is non-optimal to transmit with power in $(0, 1)$.

The 1-bit power control (1BPC) policy is proposed as a trade-off between OPC and other kinds of power control policies that do not utilize the link distance information. In practice, although the link distance can always be measured, its precise value might be difficult to acquire, *e.g.*, it may take too long to accurately measure. In such occasions, the performance of OPC becomes difficult to realize. Meanwhile, 1BPC turns out to be more suitable, since it only requires 1 bit of information regarding the link distance, and its performance is identical to OPC's in two important regimes: the large r regime (where poor choice of power control policy can result in many orders of differences in the local delay) and the small r regime (which typically happens with high probability).

It is not difficult to find that if the link distance r is known and OPC is applied, the conditional local delay is

$$D_r = \begin{cases} P_{\max}e^{\frac{\theta r^\alpha}{P_{\max}}}, & \theta r^\alpha \ge P_{\max} \\ \theta r^\alpha e\ , & 1 < \theta r^\alpha < P_{\max} \\ e^{\theta r^\alpha}, & \theta r^\alpha \le 1. \end{cases}$$

In comparison, we can see that with constant power transmission the conditional local delay is always equal to $\exp(\theta r^\alpha)$. When $P_{\max} \ge 2$, the transmit power of UPC is uniformly distributed in $[0, 2]$. Its conditional local delay can be calculated as

$$\left(\exp(-\frac{1}{2}\theta r^\alpha) - \frac{1}{2}\theta r^\alpha\int_{\frac{1}{2}\theta r^\alpha}^{\infty}\frac{\exp(-x)}{x}\mathrm{d}x\right)^{-1}.$$

Straightforward (but tedious) manipulation reveals the conditional local delay for HUPC to be

$$\frac{P_{\max}^2-1}{2}\left(P_{\max}e^{-\frac{\theta r^\alpha}{P_{\max}}} - e^{-\theta r^\alpha} - \theta r^\alpha\int_{\frac{\theta r^\alpha}{P_{\max}}}^{\theta r^\alpha}\frac{e^{-x}}{x}\mathrm{d}x\right)^{-1}.$$

The calculation of the conditional local delay for 1BPC is similar to that of OPC.

Fig. 2 and 3 compare all the power control policies defined above along with constant power transmission ($P \equiv 1$) in different scales.

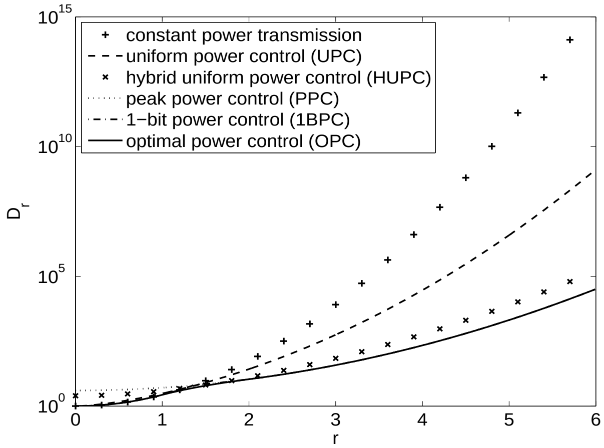

Fig. 2: Comparison of the conditional local delay for different power control schemes. Here, $P_{\max} = 4$, $\theta = 1$, $\alpha = 2$.

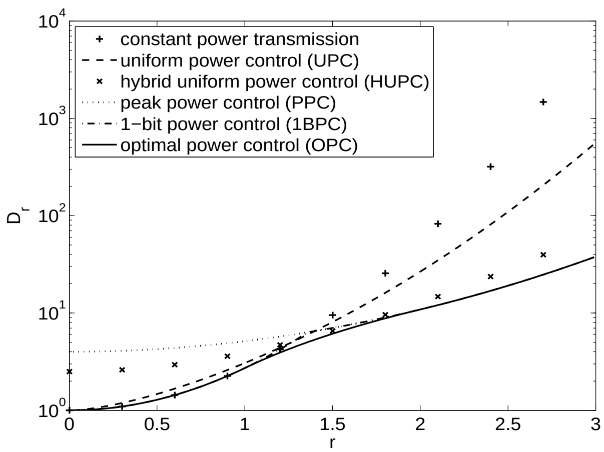

Fig. 3: Comparison of the conditional local delay for different power control schemes. Here, $P_{\max} = 4$, $\theta = 1$, $\alpha = 2$.

Fig. 2 shows that in the power-limited regime (large r) the local delay grows exponentially with r for all power control policies. This is mainly due to the peak power constraint. However, for different power control schemes the exponent is quite different, which results in many orders of difference in conditional local delay. As expected, in this regime, OPC, PPC and 1BPC perform the best among all schemes, and constant power transmission is the worst. Both UPC and HUPC appear to be good trade-offs between the best and the worst.

In the short-distance regime (small r), the difference in the local delay between different schemes can be at most by a factor of 4 (Fig. 3). Still, UPC and HUPC perform between the two extremes. Fig. 3 also shows that 1BPC is not considerably inferior to OPC even in its suboptimal regime ($1 < \theta r^{\alpha} < P_{\max}$), and thus appears to be a good substitute for OPC in some cases.

V. Conclusions and Future Work

This paper shows that an ALOHA-type random on-off power control policy is optimal in reducing the local delay (Theorem 1). This policy requires the knowledge of the link distance r, which is typically easy to obtain in a random but fixed wireless network. Inspired by this optimal policy, an even simpler and more practical 1-bit power control (1BPC) policy is constructed, which has the same performance as OPC in the short-distance and power-limited regime. Both power control strategies are compared with constant power transmission and a few basic power control policies, and a considerable reduction in local delay is observed.

The proof of Theorem 1 relies heavily on the monotonicity of the exponential distribution of Rayleigh fading. This prevents the theorem being easily extended to more general fading statistics, *e.g.*, Nakagami fading. Also, it is unknown how much loss there will be if the fading coefficient in different time slots are correlated. All these topics remain for future work.

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