The Performance of Successive Interference Cancellation in Random Wireless Networks

Xinchen Zhang and Martin Haenggi
Dept. of EE, Univ. of Notre Dame
Notre Dame, IN 46556, USA
{xzhang7,mhaenggi}@nd.edu

***Abstract*—This paper provides a unified framework to study the performance gain of successive interference cancellation (SIC) in d-dimensional interference-limited networks with arbitrary fading distribution and power-law path loss. We derive bounds on the mean number of users that can be successively decoded and the probability of successively decoding k users. Our results suggest that, without power control, the marginal benefit of enabling the receiver to successively decode k users diminishes very fast with k, especially in networks of high dimensions and small path loss exponent. On the other hand, SIC is more beneficial when the users are clustered around the receiver, or very low-rate codes are used.**

I. Introduction

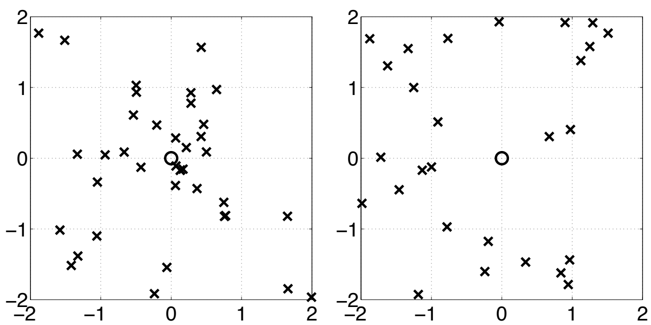
Successive interference cancellation (SIC) is a promising technique to improve the efficiency of the wireless networks with relatively small additional complexity [1]. However, in a network without centralized power control, *e.g.,* ad hoc networks, the use of SIC hinges on the ordering of the received power from different users (active transmitters), which depends on the spatial distribution of the users as well as many other network parameters. Therefore, it is important to quantify the gain of SIC with respect to different system parameters.

This paper provides a unified framework to study the performance of SIC in d-dimensional wireless networks. Modeling the active transmitters in the network by a Poisson point process (PPP) with power-law density function (which includes uniform PPP as a special case), we show how the effectiveness of SIC depends on the path loss exponent, fading, coding rate and user distribution.

II. System Model and Metrics

A. The Power-law Poisson Network with Fading (PPNF)

Consider a receiver at the origin o and the active transmitters are represented by a marked Poisson point process (PPP) $\hat{\Phi} = \{(x_i, h_{x_i})\} \subset \mathbb{R}^d \times \mathbb{R}^+$, where x is the location of the users, h_x is the iid fading coefficient associated with the link from x to o, and d is the number of dimensions of the space. When the density function of the ground process $\Phi \subset \mathbb{R}^d$ is $\lambda(x) = a\|x\|^b,\ a > 0,\ b \in (-d, \alpha - d)$, where $\|x\|$ is the distance from $x \in \mathbb{R}^d$ to the origin and α is the path-loss exponent, we refer this network as a *power-law Poisson network with fading (PPNF)*. Here, the condition $b \in (-d, \alpha - d)$ is put in order to maintain a finite received power at o and will be revisited later.



(a) $b = -1$ (b) $b = 1$

Fig. 1: Realizations of two non-uniform PPP with intensity function $\lambda(x) = 3\|x\|^b$ with different b, where × denotes an active transmitter and o denotes the receiver at the origin.

Fig. 1 shows realizations of two 2-d PPNFs with different b, where Fig. 1a represents a network clustered around o and the network in Fig. 1b is inversely clustered, *i.e.,* the network is clustered away from o. In general, the smaller b, the more clustered the network is at the origin, and $b = 0$ refers to uniform networks.

B. SIC Model and Metrics

Consider the case where all the nodes (users) transmit with unit power. Then, with an SIR model, a particular user at $x \in \Phi \subset \mathbb{R}^d$ can be successfully decoded (without SIC) iff

$$\mathrm{SIR}_x = \frac{h_x\|x\|^{-\alpha}}{\sum_{y\in\Phi\setminus\{x\}} h_y\|y\|^{-\alpha}} > \theta,$$

where θ is the SIR decoding threshold.

Similarly, in the case of perfect interference cancellation, *i.e.,* once a user is successfully decoded, its signal component can be completely subtracted from the received signal, a user x can be decoded if all the users in $\mathcal{I}_c = \{y \in \Phi : h_y\|y\|^{-\alpha} > h_x\|x\|^{-\alpha}\}$ are successfully decoded and

$$\frac{h_x\|x\|^{-\alpha}}{\sum_{y\in\Phi\setminus\{x\}\setminus\mathcal{I}_c} h_y\|y\|^{-\alpha}} > \theta.$$

Consequently, consider the ordering of all nodes in Φ such that $h_{x_i}\|x_i\|^{-\alpha} > h_{x_j}\|x_j\|^{-\alpha},\ \forall i < j$. The number of users that can be successively decoded is N iff $h_{x_i}\|x_i\|^{-\alpha} > \theta\sum_{j=i+1}^{\infty} h_{x_j}\|x_j\|^{-\alpha},\ \forall j \le N$ and $h_{x_{N+1}}\|x_{N+1}\|^{-\alpha} \le$

$\theta\sum_{j=N+2}^{\infty} h_{x_j}\|x_j\|^{-\alpha}$. The goal of this paper is to evaluate $\mathbb{E}[N]$, *i.e.,* the mean number of users that can be successively decoded, with respect to different system parameters, and the distribution of N in the form

$$p_k \triangleq \mathbb{P}(N \geq k),$$

i.e., the probability of successively decoding at least k users at the origin.

III. The path loss Process with fading (PLPF)

We use the unified framework introduced in [2] to address the randomness from fading and random location of the nodes. In particular, we define the path loss process with fading (PLPF) as $\Xi \triangleq \{\xi_i = \frac{\|x\|^\alpha}{h_x}\}$, where the index i is introduced in the way such that $\xi_i < \xi_j$ for all $i < j$. Then, we have the following lemma, which follows from the mapping theorem [6].

Lemma 1. *The PLPF Ξ, corresponding to a PPNF, is a one-dimensional PPP on $\mathbb{R}^+$ with intensity measure $\Lambda([0,r]) = a\delta c_d r^\beta \mathbb{E}[h^\beta]/\beta$, where $\delta \triangleq d/\alpha$, $\beta \triangleq \delta + b/\alpha \in (0,1)$ and h is the iid fading coefficient.*

In Lemma 1, the condition $\beta \in (0,1)$ corresponds to the condition $b \in (-d, \alpha - d)$ in the definition of PPNF and is necessary in the sense that otherwise the aggregate received power at o is infinite almost surely.

Since for all $\xi_i \in \Xi \subset \mathbb{R}^+$, ξ_i^{-1} can be considered as the ith strongest received power component (at o) from the users in Φ, when studying the effect of SIC, it suffices to just consider the PLPF Ξ. For a PLPF Ξ mapped from $\hat{\Phi}$, if we let $p_k(\Xi)$ be the probability of successively decoding at least k users in the network $\hat{\Phi}$, we have the following proposition which significantly simplifies the analysis in the rest of the paper.

Proposition 1 (Scale-invariance)**.** *If Ξ and $\bar{\Xi}$ are two PLPFs with intensity measures $\Lambda([0,r]) = r^\beta$ and $\mu([0,r]) = Cr^\beta$ respectively, where C is any positive constant, then $p_k(\Xi) = p_k(\bar{\Xi}),\ \forall k \in \mathbb{N}$.*

Proof: Consider the mapping $f(x) = C^{-1/\beta}x$. Then $f(\Xi)$ is a PPP on $\mathbb{R}^+$ with intensity measure Cx^β over the set $[0,x]$. Let $\chi_k(\phi),\ k \in \mathbb{N}^+$ be a function with the domain of any countable subset of $\mathbb{R}^+$ and

$$\chi_k(\phi) = \begin{cases} 1, & \text{if } \xi_i^{-1} > \theta\sum_{j=i+1}^{\infty}\xi_j^{-1}, \forall i \leq k \\ 0, & \text{otherwise,} \end{cases}$$

where $\phi = \{\xi_i\}$ and $\xi_i < \xi_j,\ \forall i < j$. Note that $\chi_k(\cdot)$ is scale-invariant, *i.e.,* $\chi_k(\{\xi_i\}) = \chi_k(\{C'\xi_i\}),\ \forall C' > 0$. Then, we have

$$p_k(\Xi) = \mathbb{E}[\chi_k(\Xi)] \overset{\text{(a)}}{=} \mathbb{E}[\chi_k(f(\Xi))] \overset{\text{(b)}}{=} \mathbb{E}[\chi_k(\bar{\Xi})] = p_k(\bar{\Xi}),$$

where (a) is due to the scale-invariance of $\chi_k(\cdot)$ and (b) is because both $f(\Xi)$ and $\bar{\Xi}$ are PPPs on $\mathbb{R}^+$ with intensity measure $\mu([0,r]) = Cr^\beta$. ■

Proposition 1 shows that the absolute value of the density is not relevant as long as we restrict our analysis to the power-law density case. Combining it with Lemma 1, where it is shown that, in terms of the PLPF, the only difference introduced by different fading distributions is a constant factor in the density function, we immediately achieves the following corollary.

Corollary 1 (Fading-invariance)**.** *In a PPNF, the probability of successively decoding k users (at the origin) does not depend on the fading distribution.*

If we define the *Standard PLPF (SPLPF)* Ξ_β as a one-dimensional PPP with intensity measure $\Lambda([0,r]) = r^\beta$, where $\beta \in (0,1)$, we have the following fact which directly follows from Proposition 1 and Corollary 1 and significantly simplifies the analysis in the rest of the paper.

Fact 1. *The statistics of N in a PPNF can be fully captured by the study of Ξ_β which encompasses any fading distribution and any values of a, b, d, α, with $\beta = \delta + b/\alpha = (d+b)/\alpha$.*

IV. Bounds on the Probability of Successive Decoding

Despite the unified framework to analyze the PPNF in Section III, analytically evaluating p_k requires the joint distribution of the received powers from the k strongest users and the aggregate interference from the rest of the network, which is intimidating even for the simplest case (one-dimensional homogeneous PPP without fading). In this section, we derive bounds on p_k. These bounds provide us insights on how p_k depends on different system parameters.

A. Basic Bounds

The following lemma introduces basic upper and lower bounds on p_k in terms of the probability of decoding the kth strongest user if the $k-1$ strongest users did not exist. Although not being bounds in closed form, the bounds form the basis for the high-rate lower bound and the combined upper bound which will be introduced later.

Lemma 2. *In a PPNF, the probability of successively decoding k users is bounded as follows:*

- $p_k \geq (1+\theta)^{-\frac{\beta k(k-1)}{2}}\mathbb{P}(\xi_k^{-1} > \theta I_k)$
- $p_k \leq \theta^{-\frac{\beta k(k-1)}{2}}\mathbb{P}(\xi_k^{-1} > \theta I_k)$

where $\Xi_\beta = \{\xi_i\}$ is the corresponding SPLPF and $I_k \triangleq \sum_{j=k+1}^{\infty}\xi_j^{-1}$.

Proof: By Fact 1, p_k can be evaluated by considering Ξ_β. Define the following two events: $A_i = \{\xi_i^{-1} > \theta I_i\}$ and $B_i = \{\xi_i^{-1} > (1+\theta)\xi_i^{-1}\}$. Then, the probability of successively decoding at least k users can be written as $p_k = \mathbb{P}(\bigcap_{i=1}^{k} A_i)$.

Consider an arbitrary sample (realization) $\omega \in \bigcap_{i=1}^{k-1} B_i \cap A_k$. Again, assuming the increasing ordering of all $\xi_i(\omega) \in \Xi(\omega),\ i \in [k-1]$, we have

$$\xi_i^{-1}(\omega) \overset{\text{(a)}}{>} \xi_{i+1}^{-1}(\omega) + \theta\xi_{i+1}^{-1}(\omega) \overset{\text{(b)}}{>} \theta I_{i+1}(\omega) + \theta\xi_{i+1}^{-1}(\omega) = \theta I_i(\omega),$$

where (a) is due to $\omega \in B_i$, and (b) is due to $\omega \in B_{i+1}$ (if $i < k$) and $\omega \in A_k$ (if $i = k$). Therefore, $\omega \in \bigcap_{i=1}^{k} A_i$. Since

ω is arbitrarily chosen, we have $\left(\bigcap_{i=1}^{k-1} B_i \cap A_k\right) \subset \bigcap_{i=1}^{k} A_i$, and thus

$$\begin{aligned} p_k &\geq \mathbb{P}\left(\bigcap_{i=1}^{k-1} B_i \cap A_k\right) = \mathbb{E}_{\xi_k}\left[\mathbb{P}\left(\bigcap_{i=1}^{k-1} B_i \cap A_k \mid \xi_k\right)\right] \\ &= \mathbb{E}_{\xi_k}\left[\mathbb{P}\left(\bigcap_{i=1}^{k-1} B_i\right)\mathbb{P}\left(A_k\right) \mid \xi_k\right], \end{aligned} \tag{1}$$

where the last equality is because of the conditional independence between B_i, $\forall i \in [k-1]$ and A_k given ξ_k. Here, by definition, $\mathbb{P}\left(\bigcap_{i=1}^{k-1} B_i\right) = \mathbb{P}\left(\frac{\xi_i}{\xi_{i+1}} < (1+\theta)^{-1}, \forall i < k\right)$.

Conditioned on ξ_k, $k \geq 2$, $\frac{\xi_i}{\xi_k} \overset{\text{d}}{=} X_{i:k-1}$, $\forall 1 \leq i \leq k-1$, where $\overset{\text{d}}{=}$ means equality in distribution, X is a random variable with cdf $F(x) = x^{\beta}1_{[0,1]}(x)$, and $X_{i:k-1}$ is the ith order statistics of $k-1$ iid random variables with the distribution of X, *i.e.*, the ith smallest one among $k-1$ iid random variables with the distribution of X.

Since $X^{\beta} \sim \text{Uniform}(0,1)$, we can apply the results from order statistics regarding uniform random variables [3]. In particular, if $U \sim \text{Uniform}(0,1)$, then $\left(\frac{U_{i:k-1}}{U_{i+1:k-1}}\right)^i \sim \text{Uniform}(0,1)$ and $\left(\frac{U_{i:k-1}}{U_{i+1:k-1}}\right)^i$ is iid for all $1 \leq i \leq k-2$. Therefore,

$$\begin{aligned} &\mathbb{P}\left(\frac{\xi_i}{\xi_{i+1}} < (1+\theta)^{-1}, \forall i < k \mid \xi_k\right) \\ &= \prod_{i=1}^{k-1} \mathbb{P}(X^{i\beta} < (1+\theta)^{-i\beta}) = (1+\theta)^{-\frac{\beta}{2}k(k-1)}, \end{aligned} \tag{2}$$

where the last inequality is due to $X^{i\beta} \overset{\text{d}}{=} U$. The lower bound is thus proved by combining (1) and (2).

Defining $\hat{B}_i = \{\xi_i^{-1} > \theta\xi_{i+1}^{-1}\}$ in the place of B_i, we can derive the upper bound in a very similar way. ■

B. The Lower Bounds

1) High-rate lower bound: Lemma 2 provides bounds on p_k as a function of $\mathbb{P}(\xi_k^{-1} > \theta I_k)$. In the following, we give the high-rate lower bounds by lower bounding $\mathbb{P}(\xi_k^{-1} > \theta I_k)$.

Lemma 3. *The kth smallest element in Ξ_β, ξ_k, has pdf*

$$f_{\xi_k}(x) = \frac{\beta x^{k\beta-1}}{\Gamma(k)}\exp(-x^{\beta}).$$

The proof of Lemma 3 is analogous to [4, Theorem 1].

Lemma 4. *For $\Xi_\beta = \{\xi_i\}$, $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ is lower bounded by*

$$\Delta_1(k) \triangleq \frac{1}{\Gamma(k)}\left(\gamma\left(k, \frac{1-\beta}{\theta\beta}\right) - \frac{\theta\beta}{1-\beta}\gamma\left(k+1, \frac{1-\beta}{\theta\beta}\right)\right),$$

where $\gamma(\cdot,\cdot)$ is the lower *incomplete gamma function.*

Proof: In order to prove the lower bound, we first calculate the mean of the interference I_k conditioned on $\xi_k = \rho$, and then derive the bound based on the Markov inequality. Denoting $I_k \mid \xi_k = \rho$ as I_ρ, we can calculate the conditional mean interference by Campbell's Theorem [5]

$$\mathbb{E}[I_\rho] = \mathbb{E}\Bigg[\sum_{x \in \Xi \cap [\rho,\infty)} x^{-1}\Bigg] = \int_\rho^\infty x^{-1}\Lambda(\mathrm{d}x) = \frac{a\beta}{1-\beta}\rho^{\beta-1}.$$

Thus, by the Markov inequality,

$$\mathbb{P}(\xi_k^{-1} > \theta I_k \mid \xi_k = \rho) = \mathbb{P}(\rho^{-1} > \theta I_\rho) \geq 1 - \theta\rho\mathbb{E}[I_\rho].$$

The lower bound can be refined as $[1 - \theta\rho\mathbb{E}[I_\rho]]^+$, where $[\cdot]^+ = \max\{0, \cdot\}$. Deconditioning over the distribution of ξ_k (given by Lemma 3) yields the stated lower bound. ■

Combining Lemmas 2 and 4, we immediately obtain the following proposition.

Proposition 2 (High-rate lower bound)**.** *In the PPNF, $p_k \geq (1+\theta)^{-\frac{\beta k(k-1)}{2}}\Delta_1(k)$.*

Since $\Delta_1(k)$ is monotonically decreasing with k, the lower bound in Proposition 2 decays super-exponentially with k^2.

2) Low-rate lower bound: The lower bound in Proposition 2 is tight for large θ. However, it becomes loose when $\theta \to 0$. This is because Proposition 2 estimates p_k by approximate the relation between ξ_i and I_i with the relation between ξ_i and ξ_{i+1}. This approximation is accurate when $\xi_{i+1}^{-1} \approx \theta I_{i+1}$. Yet, when $\theta \to 0$, $\xi_{i+1}^{-1} \gg \theta I_{i+1}$ happens frequently, making the bound loose. The following proposition provides an alternative lower bound particularly designed for the small θ regime.

Proposition 3 (Low-rate lower bound)**.** *In the PPNF, for $k < 1/\theta + 1$, p_k is lower bounded by*

$$\underline{p}_k^{\text{LR}} \triangleq \frac{1}{\Gamma(k)}\left(\gamma\left(k, \frac{1-\beta}{\tilde{\theta}\beta}\right) - \frac{\tilde{\theta}\beta}{1-\beta}\gamma\left(k+1, \frac{1-\beta}{\tilde{\theta}\beta}\right)\right),$$

where LR *means* low-rate *and* $\tilde{\theta} \triangleq \frac{\theta}{1-(k-1)\theta}$.

Proof: Using Fact 1, we work with $\Xi_\beta = \{\xi_i\}$. For an arbitrary integer $n \in [k-1]$, we have

$$\begin{aligned} &\mathbb{P}\left(\left\{\xi_n^{-1} > \frac{\theta I_n}{1-(n-1)\theta}\right\} \cap \{\xi_i > \theta I_i,\ n < i \leq k\}\right) \\ &\overset{\text{(a)}}{\geq} \mathbb{P}\left(\left\{\xi_{n+1}^{-1} > \frac{\theta I_n}{1-(n-1)\theta}\right\} \cap \{\xi_i > \theta I_i,\ n < i \leq k\}\right) \\ &\overset{\text{(b)}}{=} \mathbb{P}\left(\left\{\xi_{n+1}^{-1} > \frac{\theta I_{n+1}}{1-n\theta}\right\} \cap \{\xi_i > \theta I_i,\ n \leq i \leq k\}\right) \\ &\overset{\text{(c)}}{=} \mathbb{P}\left(\left\{\xi_{n+1}^{-1} > \frac{\theta I_{n+1}}{1-n\theta}\right\} \cap \{\xi_i > \theta I_i,\ n+2 \leq i \leq k\}\right), \end{aligned}$$

where (a) is because of the ordering of Ξ, (b) is due to $I_n = \xi_{n+1}^{-1} + I_{n+1}$, and (c) is due to $\theta > 0$ and thus

$$\left\{\xi_{n+1}^{-1} > \frac{\theta I_{n+1}}{1-n\theta}\right\} \subset \left\{\xi_{n+1}^{-1} > \theta I_{n+1}\right\}.$$

Using this inequality sequentially for $n = 1, 2, \cdots, k-1$ yields

$$p_k \geq \mathbb{P}\left(\xi_k^{-1} > \frac{\theta I_k}{1-(k-1)\theta}\right),$$

where a lower bound for the RHS is given by Lemma 4 (substituting θ with $\tilde{\theta}$). ■

The bound in Proposition 3 is nontrival only for $k < 1/\theta+1$. However, as will be shown in Section V, when $\theta \to 0$, this bound behaves much better than the one in Proposition 2.

C. The Upper Bound

Similar to the high-rate lower bound, we derive the upper bound by upper bounding $\mathbb{P}(\xi_k^{-1} > \theta I_k)$.

Lemma 5. *For $\Xi_\beta = \{\xi_i\}$, $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ is upper bounded by*

$$\Delta_2(k) \triangleq \frac{\gamma(k, 1/c)}{\Gamma(k)} + \frac{e}{(1+c)^k}\frac{\Gamma(k, 1+1/c)}{\Gamma(k)},$$

where $c = \theta^\beta \gamma(1-\beta, \theta) - 1 + e^{-\theta}$, and $\Gamma(\cdot,\cdot)$ is the upper *incomplete gamma function.*

Proof: For a non-fading 1-d network, the Laplace transform of the aggregate interference from $[\rho, \infty)$ can be calculated by the probability generating functional (PGFL) of the PPP [6]. Similarly, the Laplace transform of $I_\rho \triangleq I_k \mid \xi_k = \rho$ is

$$\begin{aligned}\mathcal{L}_{I_\rho}(s) &= \exp\left(-\int_\rho^\infty (1 - e^{-sr^{-1}})\Lambda(\mathrm{d}r)\right) \\ &= \exp\left(-\left(s^\beta \int_0^{s\rho^{-1}} r^{-\beta} e^r \mathrm{d}r - \rho^\beta (1 - e^{-s\rho^{-1}})\right)\right).\end{aligned} \tag{3}$$

Considering an *artificial* fading coefficient H, where H is exponentially distributed with mean 1, we can relate $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ with $\mathcal{L}_{I_k}(s)$ as $\mathbb{P}(\xi_k^{-1} > \theta I_k)$

$$\begin{aligned}&= e\mathbb{P}(H > 1)\mathbb{P}(\xi_k^{-1} > \theta I_k) \overset{\text{(a)}}{=} e\mathbb{P}(\xi_k^{-1} > \theta I_k, H > 1) \\ &\leq e\mathbb{P}(H\xi_k^{-1} > \theta I_k) \overset{\text{(b)}}{=} e\mathbb{E}_{\xi_k}[\mathcal{L}_{I_k|\xi_k}(s)|_{s=\theta\xi_k}] \\ &\overset{\text{(c)}}{=} \mathbb{E}_{\xi_k}\left[\exp\left(-[c\xi_k^\beta - 1]^+\right)\right],\end{aligned}$$

where (a) is due to the independence between H and Ξ, (b) is due to the well-known relation between the Laplace transform of the interference and the success probability over a link subject to Rayleigh fading [6], (c) makes use of the PGFL in (3), taking into account the fact that $\mathbb{P}(\xi_k^{-1} > \theta I_k) \leq 1$. With the distribution of ξ_k given by Lemma 3, the proposition is then proved by straightforward but tedious manipulation. ■

Combining Lemmas 5 and 2 yields the following proposition.

Proposition 4 (Combined upper bound)**.** *In the PPNF, we have $p_k \leq \bar{\theta}^{-\frac{\beta}{2}k(k-1)}\Delta_2(k)$, where $\bar{\theta} = \max\{\theta, 1\}$.*

For $\theta > 1$, similar to the high-rate lower bound in Proposition 2, the upper bound in Proposition 4 decays super-exponentially with k^2, which suggests that, in this regime, the marginal gain of adding SIC capability (*i.e.,* ability successively cancelling more users) diminishes very fast.

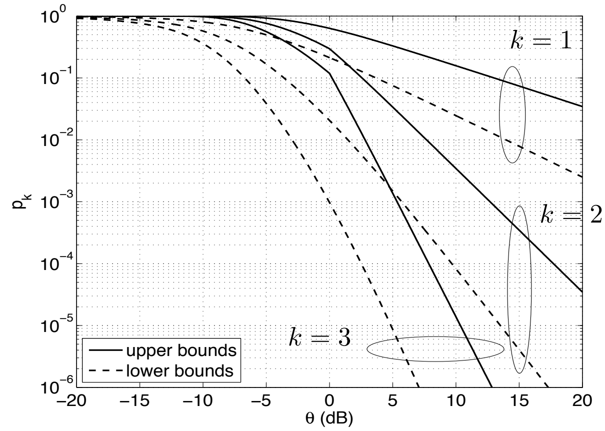


Fig. 2: Combined upper bound and high-rate lower bound for p_k ($k = 1, 2, 3$) in a 2-d uniform network with path loss exponent $\alpha = 3$.

D. Numerical Results

Focusing on $k = 1, 2, 3$, Fig. 2 plots the combined upper and high-rate lower bounds as a function of θ. We see that p_k decays very rapidly with θ, especially when k is large, which suggests that the benefit of decoding many users can be very small under high-rate codes.

Note that the combined upper bound behaves slightly differently for $\theta > 1$ and $\theta < 1$ when $k > 1$. This is because the combined upper bound in Proposition 4 becomes $\Delta_2(k)$ when $\theta < 1$. More precisely, the combined upper bound ignores the ordering among the k strongest users and only considers $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ when $\theta < 1$. Therefore, the combined upper bound is, in most cases, strictly tighter when $\theta > 1$, with only one exception: for $k = 1$, $p_k = \mathbb{P}(\xi_k > \theta I_k)$, which is upper bounded by $\Delta_2(1)$ irrespective to θ.

The bounds derived above also provide a good method to study the impact of clustering on the effectiveness of SIC. Fig. 3 compares the probability of successively decoding 1, 2 and 3 users under different network clustering parameter b, which shows the more clustered the network is, the more useful SIC will be.

V. Bounds on the Expected Number of Successively Decoded Users

With the bounds on p_k, we are able to derive bounds on $\mathbb{E}[N]$, the expected number of users that can be successively decoded in the system, since $\mathbb{E}[N] = \sum_{k=1}^\infty p_k$.

Proposition 5. *In the PPNF, we have $\mathbb{E}[N] \geq \sum_{k=1}^K (1+\theta)^{-\frac{\beta}{2}k(k-1)}\Delta_1(k)$ for all $K \in \mathbb{Z}^+$.*

On the one hand, Proposition 5 follows directly from Proposition 2 when $K \to \infty$. On the other hand, since for large θ, p_k decays very fast with k, a tight approximation can be obtained for small integer K. In fact, the error term $\sum_{k=K+1}^\infty (1+\theta)^{-\frac{\beta}{2}k(k-1)}\Delta_1(k)$ can be upper bounded by

$$\frac{(1+\theta)^{\frac{\beta}{8}}\Delta_1(K)}{\sqrt{2\beta\log(1+\theta)}}\Gamma\left(\frac{1}{2}, \frac{\beta}{2}\left(K - \frac{1}{2}\right)^2 \log(1+\theta)\right). \tag{4}$$

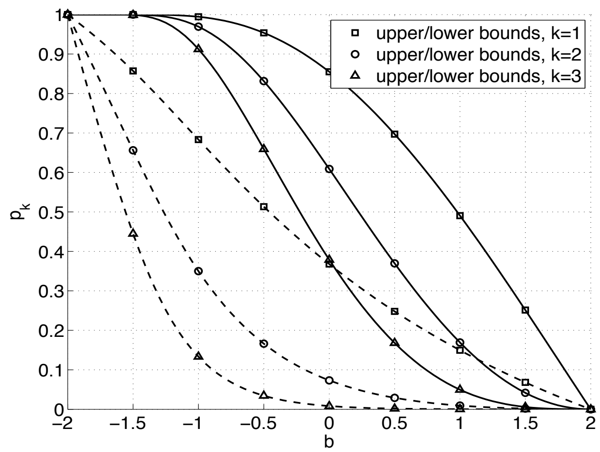


Fig. 3: Upper and (high-rate) lower bounds for p_k ($k = 1, 2, 3$) in a 2-d network with with path loss exponent $\alpha = 3$, $\theta = 1$ and density function $\lambda(x) = a\|x\|^b$. $b = 0$ is the uniform case.

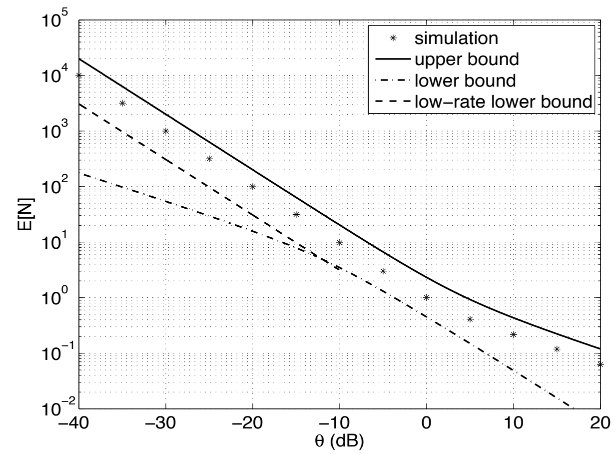


Fig. 4: The mean number of users that can be successively decoded in a 2-d uniform network with path loss exponent $\alpha = 4$.

We do not provide a proof due to space constraints. By inverting (4), one can control the numerical error introduced by choosing an finite K. As the *upper* incomplete gamma function has an exponential tail and $\Delta_1(k)$ is monotonically decreasing, the error term decays super-exponentially with K^2 when $K \gg 1$ and thus a finite K is a good approximation for $K \to \infty$ case.

On the other hand, (4) also implies that when $\theta \to 0$, for any finite K, the error blows up quickly. Therefore, in the small θ regime, we need another, tighter, bound, and this is where the low-rate lower bound in Proposition 3 helps.

Proposition 6. *In the PPNF, we have* $\mathbb{E}[N] \geq \sum_{k=1}^{\lfloor 1/\theta \rfloor} \underline{p}_k^{\mathrm{LR}}$.

A rigorous upper bound needs to be derived with more caution as we cannot discard a number of terms in the sum.

Proposition 7. *In the PPNF,* $\mathbb{E}[N]$ *is upper bounded by*

$$\frac{e^{1+K}}{\sqrt{2\pi}} \frac{(cK)^{1-K}}{cK-1} + \frac{e}{c}(1+c)^{1-K} + \sum_{k=1}^{K-1} \Delta_2(k),$$

for all $K \in \mathbb{Z}^+ \cap [e/c, \infty)$.

Proof: By Proposition 4, we have $\mathbb{E}[N] \leq \sum_{k=1}^{\infty} \Delta_2(k)$. The proposition then follows by upper bounding $\sum_{k=K}^{\infty} \Delta_2(k)$. (Due to the space constraints, the full proof is omitted from the paper.) ■

Fig. 4 compares the bounds provided in Propositions 5, 6 and 7 with simulation results in a uniform 2-d network with $\alpha = 4$. We only plot the low-rate lower bound for $\theta < -10$dB because this is the regime where the high-rate lower bound fails to capture the rate at which $\mathbb{E}[N]$ grows with the decreasing of θ. As is shown in the figure, $\mathbb{E}[N]$ increases unbounded with decreasing of θ, which further confirms that SIC is particularly beneficial for low-rate applications in wireless networks, such as node discovery, channel control, *etc*.

VI. Conclusions

Using a unified PLPF-based framework, this paper analyzes the performance of SIC in d-dimensional fading networks with power law density functions. We show that the probability of successively decoding at least k users decays super-exponentially with k^2 if high-rate codes are used, and it decays especially fast under small path loss exponent in high dimensional networks, which suggests the marginal gain of adding more SIC capability diminishes very fast. On the other hand, SIC is shown to be especially beneficial if very low-rate codes are used or the active transmitters are clustered around the receiver.

Since the clustering of the active transmitters can be the result of location-dependent independent thinning of the transmitter process [5], our results also suggest a simple, distributed MAC scheme can improve the performance of SIC by granting users closer to the receiver with larger access probabilities.

Acknowledgment

The partial support of the NSF (grants CCF 728763 and CNS 1016742) and the DARPA/IPTO IT-MANET program (grant W911NF-07-1-0028) is gratefully acknowledged.

References

[1] S. Weber, J. Andrews, X. Yang, and G. D. Veciana, "Transmission capacity of wireless ad hoc networks with successive interference cancellation," *IEEE Transactions on Information Theory*, vol. 53, no. 8, pp. 2799–2814, Aug. 2007.

[2] M. Haenggi, "A geometric interpretation of fading in wireless networks: theory and applications," *IEEE Transactions on Information Theory*, vol. 54, no. 12, pp. 5500–5510, Dec. 2008.

[3] B. C. Arnold, N. Balakrishnan, and H. N. Nagaraja, *A First Course in Order Statistics (Classics in Applied Mathematics)*. SIAM, 2008.

[4] M. Haenggi, "On distances in uniformly random networks," *IEEE Transactions on Information Theory*, vol. 51, no. 10, pp. 3584–3586, Oct. 2005.

[5] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*. John Wiley & Sons, 1995, 2nd Ed.

[6] M. Haenggi and R. K. Ganti, "Interference in large wireless networks," *Foundations and Trends in Networking*, vol. 3, no. 2, pp. 127–248, 2008, available at http://www.nd.edu/~mhaenggi/pubs/now.pdf.