Successive Interference Cancellation in Downlink Heterogeneous Cellular Networks

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***Abstract*—Using a multi-tier Poisson model, this paper studies the performance gain of successive interference cancellation (SIC) in the downlink of K-tier heterogeneous cellular networks (HCNs). For each tier, a fraction of base stations (BSs) is *non-accessible*. By using a framework based on the *marked path loss process with fading* and calculating the *equivalent access probability*, we analytically characterize the coverage probability, *i.e.,* the probability of successfully connecting to at least one accessible BS, for a typical user equipment with finite or infinite SIC capability. The results show how the performance gain of SIC depends on many system parameters including path loss exponent, coding rate, fading distributions and BS accessibilities and densities. We show for contemporary OFDM-based HCNs, infinite SIC capability is often unnecessary. In fact, under typical system parameters, most of the gain of SIC comes from the ability of canceling only a single non-accessible BS.**

I. Introduction

In heterogeneous cellular networks (HCNs), macrocell base stations (BSs) are overlaid with many tiers of low power nodes, *e.g.,* femtocell BSs and picocell BSs. HCNs are believed to be the solution to the exponentially growing data demands from mobile users in contemporary cellular networks [1], [2]. However, the unplanned nature of HCNs incurs concerns about the interference among different tiers of the network, especially when some of the tiers have closed subscriber groups (CSGs). Therefore, novel techniques need to be exploited to mitigate the inter- and intra-tier interference in order to fully harness the benefits of network heterogeneity [2], [3].

One of the promising techniques in this context is successive interference cancellation (SIC). Although being suboptimal in general, SIC is known to be able to achieve much better performance than the naive approach of treating interference as noise in interference-limited networks. In addition, SIC is more amenable to implementation than its capacity-achieving alternatives such as joint decoding [4]. The performance gain of SIC in HCNs has been demonstrated in both the uplink (see *e.g.,* [5]) and downlink (see *e.g.,* [6]). However, due to inherent complexity and randomness of HCNs, the network performance is typically evaluated by system/link level simulation.

In contrast, this paper provides an analytical framework to quantify the baseline performance of SIC (at the user equipment (UE) side) in the downlink of a K-tier interference-limited HCN with *accessible* and *non-accessible* BSs[1]. Using a stochastic geometry-based model, we characterize how the coverage probability behaves as a function of many system parameters including path loss exponent, coding rate, fading distributions and BS accessibilities and densities. We show that such a characterization can be elegantly carried out by using a *marked path loss process with fading* (PLPF)-based framework and by calculating the *equivalent access probability* (EAP).

Our analysis suggests that for contemporary OFDM-based HCNs, infinite SIC capability is often unnecessary. In fact, under typical system parameters, most of the gain of SIC comes from the ability of canceling only a single non-accessible BS.

II. System Models and the Coverage Probability without SIC

We model the BSs in a K-tier HCN as a family of marked Poisson point processes (PPP)[2] $\{\hat{\Phi}_i,\ i \in [K]\}$, where $\hat{\Phi}_i = \{(x_j, h^{(i)}_{x_j}, t^{(i)}_{x_j})\}$ represents the BSs of the i-th tier, the ground process $\Phi_i = \{x_j\} \subset \mathbb{R}^2$ are uniform[3] PPPs with intensity λ_i, $h^{(i)}_x$ is the iid (subject to distribution $f^{(i)}_h(\cdot)$) (power) fading coefficient of the link from x to o, and $t^{(i)}_x$ is the *type* of the BS and is iid Bernoulli with $\mathbb{P}(t^{(i)}_x = 1) = \pi^{(i)}$ and $\mathbb{P}(t^{(i)}_x = 0) = 1 - \pi^{(i)}$. If $t^{(i)}_x = 1$, we call the BS x *accessible* and otherwise *non-accessible*. For a typical receiver (UE) at o, the received power from BS $x \in \Phi_i$ is $P^{(i)} h^{(i)}_x \|x\|^{-\alpha}$, where $P^{(i)}$ is the transmit power at BSs of tier i, and α is the path loss exponent. An example of a two tier HCN is shown in Fig. 1.

As an important quantity in the analysis of cellular networks, the coverage probability is the probability of a typical UE successfully connecting to one of the accessible BSs. Defining the event of coverage as the received signal-to-interference ratio (SIR) at a typical receiver at o being above a threshold θ, the *standard* coverage probability (without SIC)

[1]The non-accessible BSs can be interpreted as overloaded/biased BSs [1], femtocell BSs with closed-access configuration, or simply interferers outside the cellular system.

[2]The PPP model for HCN downlink is recently justified by comparison with the conventional Wyner model and with real BS placement. The reasonable accuracy of this model together with its tractability makes it particularly interesting in studying the HCN downlink [3].

[3]Although we only consider uniformly distributed BSs in this paper, with the results in [7], generalizing the results to non-uniform (power-law density) HCN is straightforward.

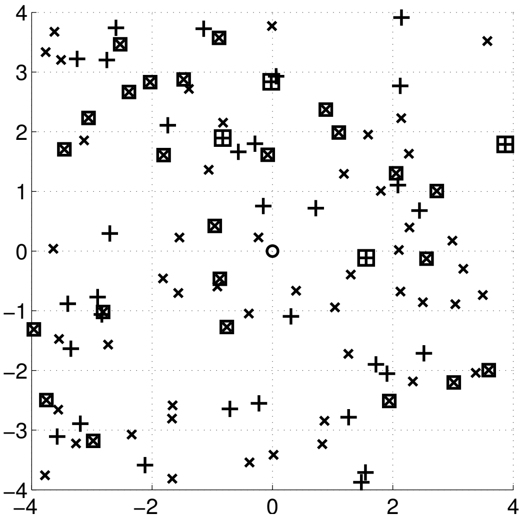

Fig. 1: A 2-tier HCN with 10% of Tier 1 (macrocell) BSs (denoted by $+$) overloaded and 30% of Tier 2 (femtocell) BSs (denoted by $\times$) configured as closed. A box is put on the BS whenever it is non-accessible (*i.e.,* either overloaded or configured as closed). The o at origin is a typical receiver.

can be written as

$$P_c = \mathbb{P}\left(\frac{P^{(j)} h_y^{(j)} \|y\|^{-\alpha}}{\sum_{i=1}^{K}\sum_{x\in\Phi_i\setminus\{y\}} P^{(i)} h_x^{(i)}\|x\|^{-\alpha}} > \theta\right), \quad (1)$$

where $(j,y) \triangleq \arg\max_{(i,x)} t_x^{(i)} P^{(i)} h_x^{(i)} \|x\|^{-\alpha}$. In words, P_c is the probability that the received SIR from the strongest accessible BS is above θ.

III. Successive Interference Cancellation

A. The (Marked) Path Loss Process with Fading (PLPF)

For the sake of notational convenience, we use a path loss process with fading (PLPF) based framework to study the downlink coverage in HCNs. First introduced in [8], this framework has been shown to be quite amenable to analyzing the performance of SIC [7].

Definition 1. *The* marked PLPF *corresponding to the tier i network is $\hat{\Xi}_i = \{(\frac{\|x\|^\alpha}{h_x^{(i)}P^{(i)}}, t_x) : x \in \Phi_i\}$, with $\Xi_i \triangleq \{\frac{\|x\|^\alpha}{h_x^{(i)}P^{(i)}} : x \in \Phi_i\}$ being the (ground) PLPF.*

Furthermore, we denote the union of the K marked PLPFs and (ground) PLPFs as $\hat{\Xi} \triangleq \bigcup_{i=1}^K \hat{\Xi}_i$ and $\Xi \triangleq \bigcup_{i=1}^K \Xi_i$, respectively. Then, the received powers from all the BSs are captured by Ξ. For example, we can rewrite the (standard) coverage probability defined (1) in terms of the marked PLPF, *i.e.,*

$$P_c = \mathbb{P}\left(\frac{\xi_*^{-1}}{\sum_{\xi\in\Xi\setminus\{\xi_*\}} \xi^{-1}} > \theta\right), \quad (2)$$

where $\xi_* \triangleq \arg\max_{\xi\in\Xi} t_\xi \xi^{-1}$.

B. Coverage Probability with SIC

Without loss of generality, we write $\Xi = \{\xi_i\}$ and $\hat{\Xi} = \{(\xi_i, t_i)\}$ where the index i is introduced in the way that $\{\xi_i\}$ is increasingly ordered, *i.e.,* $\xi_i < \xi_j$ for all $i < j$. In other words, ξ_i^{-1} is the received power from the ith strongest BS at a typical receiver at o.

In the case of perfect interference cancellation, once a BS is successfully decoded, its signal component can be completely subtracted from the received signal. Assuming the decoding order is always from the stronger users to the weaker users[4] and the UE can decode as many BSs as possible (infinite SIC capability), we can define the following event of coverage.

Definition 2 (Coverage with infinite SIC capability)**.** *A UE with infinite SIC capability is* covered *iff there exists $l \in \mathbb{N}$ and $k \in \{i : t_i = 1\}$ such that $\xi_i^{-1} > \theta I_i$, $\forall i \le l$ and $\xi_k^{-1} > \theta I_l^{!k}$, where $I_i = \sum_{j=i+1}^{\infty} \xi_j^{-1}$ and $I_l^{!k} \triangleq \sum_{j\ge l+1}^{j\neq k} \xi_j^{-1}$.*

We will use $P_c^{\rm SIC}$ to denote the coverage probability for UEs with infinite SIC capability.

Analogously, we can define the coverage probability when the UE can only successively decode n BSs.

Definition 3 (Coverage with n-layer SIC capability)**.** *A UE with n-layer SIC capability is* covered *iff there exists $l \in [n]$ and $k \in \{i : t_i = 1\}$ such that $\xi_i^{-1} > \theta I_i$, $\forall i < l$ and $\xi_k^{-1} > \theta I_l^{!k}$.*

We will use $P_{c,n}^{\rm SIC}$ to denote the coverage probability for a typical UE with n-layer SIC capability. As two special cases, we have $P_{c,1}^{\rm SIC} = P_c$ and $P_{c,\infty}^{\rm SIC} = P_c^{\rm SIC}$.

IV. Statistical Properties of the Marked PLPF

A. Prior Results for the (Unmarked) PLPF

Lemma 1. *The unmarked (ground) PLPF Ξ_i, corresponding to the tier i network, is a one-dimensional PPP on $\mathbb{R}^+$ with intensity measure $\Lambda([0,r]) = \lambda_i \pi r^\delta \mathbb{E}[(h^{(i)})^\delta](P^{(i)})^\delta$, where $\delta \triangleq 2/\alpha$ and $h^{(i)}$ is a random variable with distribution $f_h^{(i)}(\cdot)$.*

Lemma 1 is a straightforward application of the results in [7, Lemma 1].

Furthermore, for an unmarked PLPF $\Xi = \{\xi_i\}$ where $\xi_i < \xi_j$, $\forall i < j$, we define

$$p_k \triangleq \mathbb{P}(\xi_i^{-1} > \theta I_i,\ \forall i \le k). \quad (3)$$

Then, we have the following lemma which also follows from [7, Proposition 1].

Lemma 2. *For all $C > 0$, if Ξ is a PLPF with intensity measures $\Lambda([0,r]) = Cr^\delta$, $\forall r \ge 0$, p_k is only a function of θ and δ.*

[4] It is straightforward to show that this stronger-to-weaker decoding order maximizes the coverage probability despite the fact that it is not necessarily the *only* optimal decoding order.

B. Equivalent Access Probability and the Marked PLPF

An important quantity that will simplify our analysis in the K-tier HCN is the *equivalent access probability* (EAP) defined as below.

Definition 4. *Let*

$$Z \triangleq \sum_{i=1}^{K} \lambda_i \mathbb{E}[(h^{(i)})^\delta](P^{(i)})^\delta.$$

The equivalent access probability *(EAP) is the following weighted average of the individual access probabilities $\pi^{(i)}$:*

$$\eta \triangleq \frac{1}{Z}\sum_{i=1}^{K} \pi^{(i)} \lambda_i \mathbb{E}[(h^{(i)})^\delta](P^{(i)})^\delta.$$

Then, we have the following lemma.

Lemma 3. *The marked PLPF corresponding to the K-tier heterogeneous cellular BSs is a marked inhomogeneous PPP $\hat{\Xi} = \{(\xi_i, t_i)\} \subset \mathbb{R}^+ \times \{0,1\}$, where the intensity measure of $\Xi = \{\xi_j\}$ is $\Lambda([0,r]) = Z\pi r^\delta$ and the marks t_i are iid Bernoulli with $\mathbb{P}(t_i = 1) = \eta$, $\forall i \in \mathbb{N}$.*

Based on Lemma 1, the independence between $t_x^{(i)}$ and the fact that the superposition of PPPs is still a PPP [9], the proof of Lemma 3 is straightforward and thus omitted from the paper. Despite the simplicity of the proof, the implication of Lemma 3 is significant: the effect of the different transmit powers, fading distributions and access probabilities of the K-tiers of the HCN can all be subsumed by the two parameters Z and η.

V. Infinite SIC Capability

Proposition 1. *In the K-tier heterogeneous cellular network, the coverage probability of a typical UE with (infinite) SIC capability is*

$$P_c^{\mathrm{SIC}} = \sum_{k=1}^{\infty} (1-\eta)^{k-1} \eta p_k,$$

where $p_k = p_k(\Xi)$ is the probability of successively decoding at least k users in a PLPF $\Xi \subset \mathbb{R}^+$ with intensity measure $\Lambda([0,r]) = Z\pi r^\delta$.

Proof: Without loss of generality, we consider the marked PLPF corresponding to the K-tier heterogeneous cellular BSs $\hat{\Xi} = \{(\xi_i, t_i)\}$, where the index i is introduced such that $\{\xi_i\}$ are increasingly ordered. Let $\mathcal{N}$ be the sample space of Ξ, *i.e.,* the family of all countable subsets of $\mathbb{R}^+$. Consider an indicator function $\vartheta_k : \mathcal{N} \to \{0,1\},\ k \in \mathbb{N}$, such that

$$\vartheta_k(\Xi) \triangleq \begin{cases} 1, & \text{if } \exists l \in \mathbb{N} \text{ s.t. } \chi_l(\phi) = 1,\ \xi_k^{-1} > \theta I_l^{!k} \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where $\Xi = \{\xi_i\}$ and

$$\chi_k(\Xi) \triangleq \begin{cases} 1, & \text{if } \xi_i^{-1} > \theta I_i,\ \forall i \le k \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Furthermore, we define a random variable $M = \min\{i : t_i = 1\}$, where t_i is the mark of the ith element in $\hat{\Xi}$. Note that since, according to Lemma 3, the t_i are iid (also independent from Ξ), M is geometrically distributed with parameter η and is independent of Ξ. Then, it is easy to check with Definition 2 that the coverage probability can be written as

$$P_c^{\mathrm{SIC}} = \mathbb{P}(\vartheta_M(\Xi)) = \mathbb{E}_M\left[\mathbb{P}(\vartheta_M(\Xi) \mid M)\right],$$

where the conditional probability is the probability of decoding the Mth strongest BS (with the help of SIC) conditioned on the fact that this BS is the strongest accessible BS.

Moreover, we have $\vartheta_k(\cdot) \equiv \chi_k(\cdot),\ \forall k \in \mathbb{N}$. To see this, we first notice that, by the definition of the two functions, $\chi_k(\phi) = 1 \ \Rightarrow\ \vartheta_k(\phi) = 1$. Conversely, assuming $\vartheta_k(\phi) = 1$, which by definition means $\exists l \in \mathbb{N}$ s.t. $\chi_l(\phi) = 1$ and $\xi_k^{-1} > \theta I_l^{!k}$, we immediately notice that $\chi_k(\phi) = 1$ if $l \ge k$. If $l < k$, we have $\xi_{l+1}^{-1} \ge \xi_k^{-1} > \theta I_l^{!k} \ge \theta I_{l+1}$, *i.e.,* $\chi_{l+1}(\phi) = 1$, which, by induction, leads to the fact that $\chi_k(\phi) = 1$. Since both $\chi_k(\cdot)$ and $\vartheta_k(\cdot)$ are indicator functions on the domain of all countable subsets of $\mathbb{R}^+$, we have established the equivalence of the two functions.

As is shown in [7], $p_k = \mathbb{E}[\chi_k(\Xi)]$, which is only a function of θ and δ (Lemma 2). Therefore, we have $P_c^{\mathrm{SIC}} = \mathbb{E}_M\left[\mathbb{P}(\chi_M(\Xi) \mid M)\right] = \mathbb{E}_M[p_M]$. ■

Thanks to Proposition 1 we can quantify the coverage probability of the HCN downlink using the bounds on p_k we obtained in [7]. In particular, based on [7, Proposition 2], a lower bound can be found as

$$P_c^{\mathrm{SIC}} \ge \sum_{k=1}^{K} (1-\eta)^{k-1} \eta (1+\theta)^{-\frac{\delta k(k-1)}{2}} \Delta_1(k),\ \forall K \ge 1 \tag{6}$$

where

$$\Delta_1(k) \triangleq \frac{1}{\Gamma(k)}\left(\gamma\left(k, \frac{1-\delta}{\theta\delta}\right) - \frac{\theta\delta}{1-\delta}\gamma\left(k+1, \frac{1-\delta}{\theta\delta}\right)\right),$$

$\gamma(\cdot,\cdot)$ is the *lower* incomplete gamma function and the choice of K affects the tightness of the bound. Furthermore, upper bounding the tail terms of the infinite sum and using [7, Proposition 4], we have that for all $K \ge 1$

$$P_c^{\mathrm{SIC}} \le \sum_{k=1}^{K} (1-\eta)^{k-1} \eta \bar{\theta}^{-\frac{\delta}{2}k(k-1)} \Delta_2(k) + (1-\eta)^{K+1}, \tag{7}$$

where $\bar{\theta} = \max\{\theta, 1\}$, $(1-\eta)^{K+1}$ bounds the residual terms in the infinite sum,

$$\Delta_2(k) \triangleq \bar{\gamma}(k, \frac{1}{c}) + \frac{e}{(1+c)^k}\bar{\Gamma}(k, 1+\frac{1}{c}),$$

$c = \theta^\delta \gamma(1-\delta, \theta) - 1 + e^{-\theta}$, $\bar{\gamma}(z,x) = \frac{\gamma(z,x)}{\Gamma(z)}$ and $\bar{\Gamma}(z,x) = \frac{\Gamma(z,x)}{\Gamma(z)}$ are the *normalized lower* and *upper* incomplete gamma function, respectively, and $\Gamma(\cdot,\cdot)$ is the *upper* incomplete gamma function.

Besides these bounds, we can also use the approximation established in [10] to obtain an approximation on the coverage probability in closed-form. In particular, we showed in [10, Section III-C] that

$$p_k \approx \mathcal{L}_{\xi_k I_k}(s)|_{s=\theta} = \frac{1}{(c(\theta)+1)^k},$$

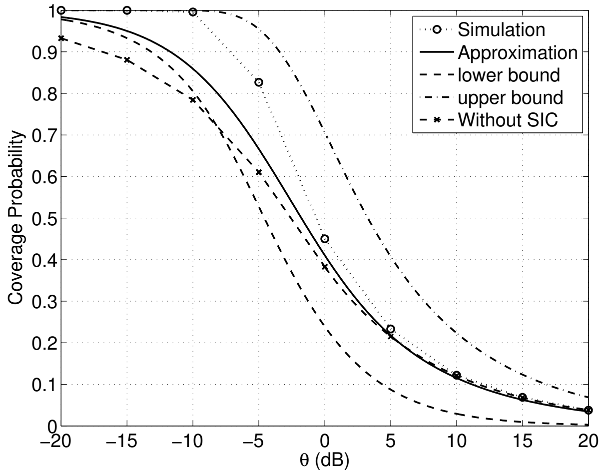

Fig. 2: The coverage probability (with infinite SIC capability) as a function of SIR threshold θ in HCN with $\eta = 0.6$ and $\alpha = 4$. The (Laplace-transform-based) approximation, lower bound and upper bounds of $P_c^{\rm SIC}$ is calculated according to (8), (6) and (7), respectively. The coverage probability in the case without SIC (a problem also studied in [3], [11]) is also plotted for comparison, where the $\theta \geq 0$dB part is analytically obtained by (9) and the $\theta < 0$dB part is based on simulation.

where $c(\theta) = \theta^\delta \gamma(1-\delta,\theta) - 1 + e^{-\theta}$. Combining this with Proposition 2, we have

$$P_c^{\rm SIC} \approx \frac{\eta}{1-\eta} \sum_{k=1}^{\infty} \left(\frac{1-\eta}{1+c(\theta)} \right)^k = \frac{\eta}{\eta + c(\theta)}. \tag{8}$$

In Fig. 2, we compare these bounds and the approximation with simulation results. These bounds give reasonably good estimates on the coverage probability throughout the full range of the SIR threshold θ. In comparison with the coverage probability when no SIC is available, we see that a significant gain can be achieved by SIC when the SIR threshold θ is between -10dB and -5dB. This conclusion is, of course, affected by η. The effect of η will be further explored in Section VI.

VI. Finite SIC Capabilty

A. Coverage Probability with 1-layer SIC Capability

By Definition 3, the coverage probability with 1-layer SIC capability is just the standard coverage probability (without SIC), *i.e.,* $P_{c,1}^{\rm SIC} = P_c$, which does not yield a closed-form expression in general [11]. However, for $\theta \geq 1$, we have

$$P_c = \eta p_1 = \frac{\eta \operatorname{sinc} \delta}{\theta^\delta}, \tag{9}$$

where the intuition behind $P_c = \eta p_1$ is that when $\theta \geq 1$, without SIC, the receiver can only be covered by the strongest BS [7, Lemma 6], and thus the coverage probability is just the probability that it is covered by the strongest BS and that the strongest BS is accessible, which, due to the independence of t_i, happens with probability η. Also, (9) utilizes the result that $p_1 = \mathbb{P}(\xi_1^{-1} > \theta I_1) = \frac{\operatorname{sinc}\delta}{\theta^\delta}$ [12, Corollary 2], which can be also derived using the results in [3].

B. Coverage Probability with n-layer SIC Capability

Unfortunately, even just for $\theta \geq 1$, finding a closed-form expression for $P_{c,n}^{\rm SIC}$ with general n seems hopeless. However, following a similar procedure as in the proof of Proposition 1, we can find a lower bound on $P_{c,n}^{\rm SIC}$ (in terms of p_k) which is exact when $\theta \geq 1$.

Proposition 2. *In the K-tier heterogeneous cellular network, the coverage probability of a typical UE with n-layer SIC capability is*

$$P_{c,n}^{\rm SIC} \geq \sum_{k=1}^{n} (1-\eta)^{k-1} \eta p_k, \tag{10}$$

where the equality holds when $\theta \geq 1$.

Different from Proposition 1, Proposition 2 only provides a lower bound on the coverage probability for general θ. Nevertheless, the proof is analogous (although a bit more tedious) and thus is omitted from the paper.

Comparing Propositions 1 and 2, it is obvious that the inequality in Proposition 2 is asymptotically tight as $n \to \infty$. More precisely, since $P_c^{\rm SIC} \geq P_{c,n}^{\rm SIC}$, we have

$$\sum_{k=1}^{n} (1-\eta)^{k-1} \eta p_k \leq P_{c,n}^{\rm SIC} \leq \sum_{k=1}^{\infty} (1-\eta)^{k-1} \eta p_k,$$

and the difference between the upper and lower bound decays (at least) exponentially with n. Thus, the lower bound in Proposition 2 converges to the true value (at least) exponentially fast with n.

Combining Propositions 1 and 2, we can estimate the performance gain of n-layer SIC capability in HCN downlink. First, let's consider the case with $\theta \geq 1$ and $\alpha = 4$. In this case, $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ can be written in a very simple closed-form expression [12, Corollary 3], which renders a tight bound on p_k. In particular, we have $p_k \leq (\pi\theta)^{-\frac{k}{2}}/\Gamma(k/2+1)$. Applying Proposition 2, we obtain a set of upper bounds on the coverage probability with finite SIC capability[5]

$$P_{c,n}^{\rm SIC} \leq \sum_{k=1}^{n} (1-\eta)^{k-1} \eta \frac{(\pi\theta)^{-\frac{k}{2}}}{\Gamma(k/2+1)}. \tag{11}$$

For infinite SIC capability, a closed-form upper bound on the coverage probability can also be obtained as

$$P_c^{\rm SIC} \leq \frac{\eta}{1-\eta} \left(\exp\left(\frac{(1-\eta)^2}{\pi\theta} \right) \left(1 + \operatorname{erf}\left(\frac{1-\eta}{\sqrt{\pi\theta}} \right) \right) - 1 \right). \tag{12}$$

Fig. 3 plots the coverage probability with different levels of SIC capability as a function of η for $\theta = 0$dB and 2dB. Here, we plot the upper bounds on $P_{c,n}^{\rm SIC}$ according to (11) for $n = 1, 2$, the upper bound on $P_c^{\rm SIC}$ according to (12), and simulated value of $P_{c,n}^{\rm SIC}$ for $n = 1, 2, 10$. The problem of $n = 1$ is already studied in [3].

Taking $n = 1$ in (11) and comparing it with (9) show that the upper bound in (11) is tight for $n = 1$. This explains why

[5] When $\theta > 1$, this bound can be further sharpened by introducing another term $\bar{\theta}^{-\frac{1}{4}k(k-1)}$ [7]. However, this simpler bound is already tight enough to give an accurate estimate on the usefulness of SIC.

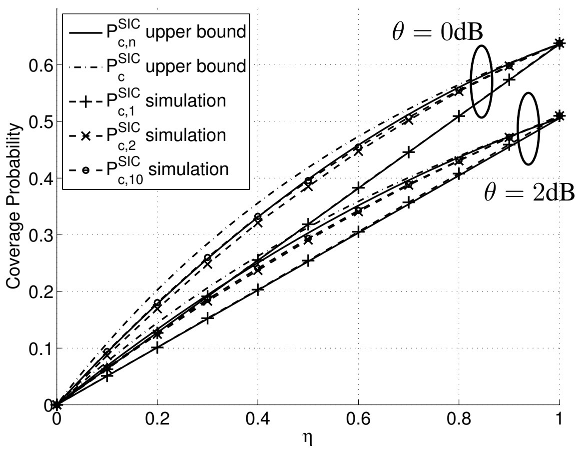

Fig. 3: Comparison between the upper bound and the simulated value of coverage probability in femto-cell networks with different layers of SIC capability when $\alpha = 4$. The upper bounds on $P_{c,n}^{\text{SIC}}$ is calculated according to (11) for $n = 1, 2$ (coverage probability is higher for larger n). The upper bound on P_c^{SIC} is calculated by (12). The simulated value of $P_{c,n}^{\text{SIC}}$ is plotted for $n = 1, 2, 10$. When $\theta = 2\text{dB}$, the curves for $n = 2$ and $n = 10$ almost overlap.

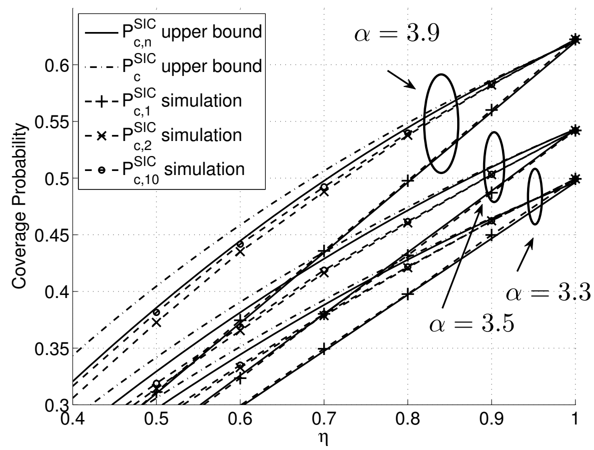

Fig. 4: Comparison between the upper bound and the simulated value of coverage probability in femto-cell networks with different path loss exponent α when $\theta = 0\text{dB}$. The upper bounds on $P_{c,n}^{\text{SIC}}$ is calculated according to (13) for $n = 1, 2$ (coverage probability is higher for larger n). The upper bound on P_c^{SIC} is calculated by (14). The simulated value of $P_{c,n}^{\text{SIC}}$ is plotted for $n = 1, 2, 10$. For $\alpha \leq 3.5$, the curves for $n = 2$ and $n = 10$ almost completely overlap.

the lowest solid lines (upper bound on $P_{c,1}^{\text{SIC}}$) and the lowest dashed lines (simulated $P_{c,1}^{\text{SIC}}$) in Fig. 3 overlap.

Fig. 3 shows that $P_{c,n}^{\text{SIC}} - P_{c,1}^{\text{SIC}}$, the absolute coverage probability gain of SIC, is much larger when η is close to $\frac{1}{2}$ than when η is close to 0 or 1. This phenomenon can be observed within a much wider range of system parameters.

Moreover, it is worth noting that with $\theta \geq 1$ and $\delta = \frac{1}{2}$, most of the gain of SIC is achieved by the ability of canceling only one non-accessible BS. This is consistent with observations reported in [13] where a different model for SIC is used and the transmission capacity is used as the metric. The fundamental reason of this observation can be explained by Proposition 2. The difference in coverage probability between infinite SIC capability and the capability of canceling $n-1$ BSs is $\sum_{k=n+1}^{\infty}(1-\eta)^{k-1}\eta p_k$, which, due to the exponential decay of p_k [7, Proposition 4], decays (at least) exponentially fast with n. Thus, most of the additional coverage probability comes from canceling a small number of non-accessible BSs. Since p_k decays faster for larger θ [7], we can expect that the ability of successively decoding more than one non-accessible BS becomes even less useful for larger θ, which is also demonstrated in Fig. 3.

Of course, with the same logic, we would expect that the ability to successively decode a large number of BSs does help if $\delta \to 0$ and/or $\theta \to 0$. $\delta \to 0$ could happen if the path loss exponent α is very large. $\theta \to 0$ happens when low-rate codes are used.

VII. Impact on Realistic Systems

Since the different values of θ and δ can result in different usefulness of the SIC capability at HCN downlink, it is worthwhile to discuss more realistic parameter choices in contemporary systems.

The exact values of θ and δ depends on many facts including modulation and coding schemes, receiver sensitivity, BS densities and propagation environment. However, in practical OFDM-type systems (*e.g.,* LTE and 802.11 networks), the SIR threshold θ is typically larger than -3dB and often more than 0dB [11][6]. For indoor propagation, α is typically between 3 and 4. Therefore, the system parameters used in the tractable case (Fig. 3) are reasonably realistic.

To have a closer look at the case with $\alpha < 4$, we fix $\theta = 1$ and invoke general expression provided in [12, Theorem 1]. Combining with Proposition 2, this gives us

$$P_{c,n}^{\text{SIC}} \leq \frac{\eta}{1-\eta}\sum_{k=1}^{n}\frac{1}{\Gamma(1+k\beta)}\left(\frac{1-\eta}{\Gamma(1-\beta)}\right)^k. \tag{13}$$

and in the limit (infinite SIC capability), we have

$$P_c^{\text{SIC}} \leq \frac{\eta}{1-\eta}\left(\mathbf{E}_{\beta,1}\left(\frac{1-\eta}{\Gamma(1-\beta)}\right)-1\right), \tag{14}$$

where $\mathbf{E}_{a,b}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(ak+b)}$ is the Mittag-Leffler function.

Fig. 4 compares the coverage probabilities with different levels of SIC capability for different path loss exponents α when $\theta = 1$. As expected, as α decreases, both the coverage probability and the gain of additional SIC capability decrease. The former is due to the fact that with a smaller α the far users contribute more to the interference. The latter can be explained by the fact that when α is smaller, the received power from different users are more comparable, leaving less structure in the received signal that can be exploited by SIC.

Similarly, we can apply the bounds in (13) and (14) to even smaller α which may apply to outdoor environments, and

[6] The small θ regime is more applicable to wide-band systems, *e.g.,* CDMA or UWB systems.

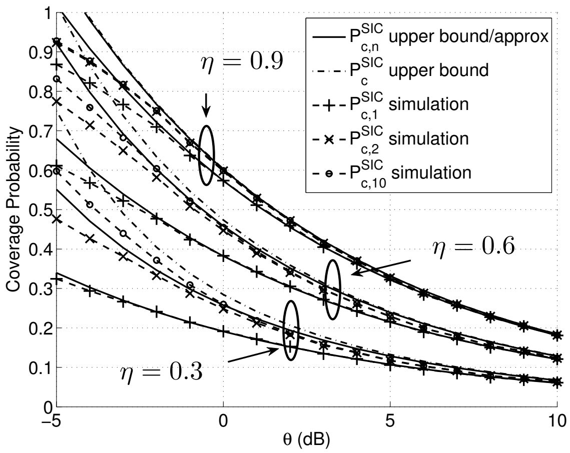

Fig. 5: Coverage probability of femto-cell networks with SIR threshold $\theta \geq -5$dB with $\alpha = 4$. The solid lines are calculated for $n = 1, 2$ according to (13) (the lines are higher for larger n), which are an upper bounds on $P_{c,n}^{\mathrm{SIC}}$ when $\theta \geq 0$dB. For $\theta \leq 0$dB, these lines should be considered as approximations. The upper bound on P_c^{SIC} is calculated by (14). The simulated value of $P_{c,n}^{\mathrm{SIC}}$ is plotted for $n = 1, 2, 10$. For $\eta \leq 0.9$, the curves for $n = 2$ and $n = 10$ almost completely overlap throughout the simulated SIR range.

conceivably the gain of SIC will becomes even more marginal. Therefore, *SIC is more useful in an indoor environment*.

In general, accurately estimating $P_{c,n}^{\mathrm{SIC}}$ is more difficult when $\theta < 1$. One of the reasons is that the upper bounds (on p_k) in finding (13) and (14) become increasingly loose as θ decreases. However, within the range of realistic parameters, *i.e.,* $\theta > -3$dB, the values calculated by (13)[7] and (14) are still informative as is shown in Fig. 5. This figure shows the coverage probability as a function of $\theta \geq -5$dB for $\eta = 0.3, 0.6, 0.9$. We found that most of the the conclusions we made for $\theta \geq 1$ still hold when $\theta \geq -5$dB. For example, we can still see that most of the gain of SIC comes from canceling a single non-accessible BS and that the gain is larger when η is close to 0.5.

Quantitatively, we found that when η is relatively small ($\eta = 0.3, 0.6$) the analytical results still track the results obtained by simulation closely for $\theta > -3$dB. The analytical results are less precise when η is large. However, large η characterizes a regime where most of the BSs are accessible. In this case, it is conceivable that SIC is often unnecessary, which can be verified by either the simulation results or the analytical results in Fig. 5. Therefore, the analytical results generate enough quantitative insights for the most interesting set of parameters.

VIII. Conclusions

This paper analyzes the coverage probability in heterogeneous cellular networks (HCNs) with SIC capability at the UE side. We show that the code rate can significantly impacts the usefulness of successively canceling a large number of non-accessible BSs. In particular, SIC in combination with low rate codes can boost the coverage probability of the HCN to a large extent. We also observe that, for contemporary OFDM-based cellular systems, most of the gain of SIC comes from canceling a single non-accessible BS.

An important contribution of this paper is the demonstration of a general approach to analyze HCNs based on constructing the (marked) PLPF and calculating the *equivalent access probability* (EAP). We show that the complexity introduced by the network heterogeneity can be elegantly addressed through this approach and the coverage probability with SIC can be evaluated based on the knowledge about the same problem in the homogeneous networks. In addition to SIC, this approach can be used to analyze many other techniques in HCNs and has the potential to generalize many known results from homogeneous networks to heterogeneous networks.

[7](13) can only be considered as an approximation on $P_{c,n}^{\mathrm{SIC}}$ when $\theta < 1$ since Proposition 2 only gives a lower bound in this regime.

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