Coordinated Multipoint in Heterogeneous Networks: A Stochastic Geometry Approach

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Abstract—Motivated by the ongoing discussion on coordinated multipoint in wireless cellular standard bodies, this paper considers the problem of base station cooperation in the downlink of heterogeneous cellular networks. The focus of the paper is the *joint transmission* scenario, where an ideal backhaul network allows a set of randomly located base stations, possibly belonging to different network tiers, to jointly transmit data, so as to mitigate intercell interference and hence improve coverage and spectral efficiency. Using tools from stochastic geometry, an exact integral expression for the network coverage probability is derived in the scenario where a typical user can non-coherently combine the received signal from a pool of base stations, that are selected based on their average received power strengths or distance from the receiver. In the special case where cooperation is limited to two base stations, numerical evaluations illustrate relative gains in coverage probability of up to about 30% compared to the non-cooperative case.

I. INTRODUCTION

The wireless industry is currently facing an increasing demand for data traffic over cellular networks, just as the performance of modern point-to-point communication schemes are fast approaching the fundamental information-theoretic limits. Therefore, to address this increasing demand, one of the solutions for increasing network coverage and capacity is the deployment of heterogeneous networks—networks of small base stations (BSs) along with the existing macro ones. In order to address the additional intercell interference caused by such deployments, the most recent discussions in the LTE cellular standards bodies are around the proposals of coordinated multipoint (CoMP) techniques [1], where BSs communicate with each other over a backhaul link to limit intercell interference and exploit the benefits of distributed multiple antenna systems [2], hence increasing the network throughput.

The concept of base station cooperation in wireless networks has been extensively studied in the past few decades. In the information-theoretic literature, several studies including [3]–[6] analyzed the advantages of cooperation within the framework of the Wyner model [7] for downlink communication—a widely used model to analyze the capacity of cellular systems, which is also known to trade off simplicity
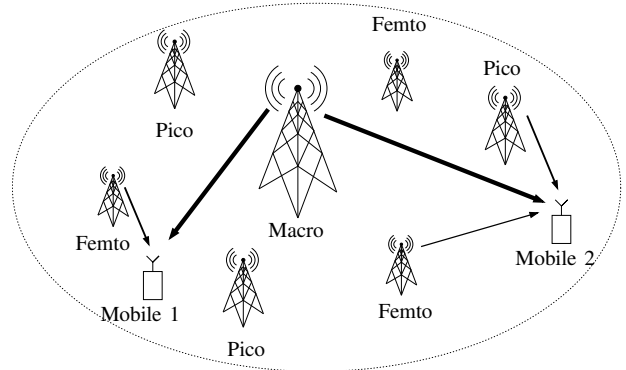
Fig. 1. A heterogeneous network composed by three tier networks: a macro, a femto, and a pico tier. Terminal node 1 is connected to the two BSs that result in maximum received power (Case 1), while user 2 is connected to the closest base station from each tier network (Case 2).

and analytic tractability at the expense of accuracy [8]. We refer the reader to [9] for an overview on the information-theoretic techniques to study multi-cell MIMO cooperation in wireless networks. Another approach that has been recently followed by several authors is to assume that the BSs are *randomly* located, so that tools from stochastic geometry can be used to characterize the signal-to-interference-plus-noise-ratio (SINR) at a typical user and hence the outage/coverage probability of a typical network deployment, see, e.g., [10] and [11]. Following this approach, [12] characterized the outage probability in a heterogeneous network without cooperation; [13] studied the impact of backhaul delay in a wireless network where CoMP takes place in the form of zero forcing beamforming at the cooperating BSs; [14] also investigated the role played by a non-ideal backhaul network and analyzed the performance of a specific two-base-station cooperative scheme based on rate-splitting, similar to the one proposed in [15] for the multiple access channel with conferencing encoders; finally, [16] analyzes a scheme where random clusters of BSs cooperate by nulling the intercell interference.

This paper presents a tractable stochastic geometry–based model for studying BS cooperation in downlink communication of heterogeneous networks. The model consists of K independent tiers of randomly located BSs, where each tier

is characterized by a different density of BSs and available power. Base stations within each network are assumed to be spatially distributed according to a Poisson point process (PPP). While this model can in principle be used to analyze arbitrary cooperation schemes, the paper focuses on the *joint transmission* scenario, where BSs belonging to different tiers jointly transmit data to the same user in a synchronous manner, as if they were forming a single distributed antenna system. Assuming that a user connects to a set of cooperating BSs and that the network operates in the interference-limited regime, i.e., the background thermal noise power is negligible compared to the aggregate interference power, we derive closed integral-form expressions for the coverage probability in the entire network in two different cases: (1) the cooperating BSs are those that result in the maximum average received power and (2) the cooperating BSs belong to different tiers and are the closest to the user. See Fig. 1 for an illustration of the two cases.

In both cases, the expressions derived for the coverage probability illustrate the impact of the underlying network parameters, such as the density of BSs, the available transmit powers, and the fading coefficients, on the overall system performance. An interesting observation is that in Case 1 the coverage probability does not depend on the number of network tiers, nor on the network density and available power (see Theorem 1). This means that the gains in coverage provided by cooperation can also be achieved by a single homogeneous network, provided that an ideal backhaul network allows cooperation among spatially distributed BSs and that users can connect to the BSs that result in the highest received power. On the contrary, in Case 2 the coverage probability does depend on the parameters describing each network tier (see Theorem 2) and tends to decrease as the density goes beyond a critical value because of the net increase in interference in the entire network (see, e.g., Fig. 4). The results obtained are used to quantify the benefits of cooperation. Numerical evaluation in the case where cooperation is limited to two BSs illustrates gains in coverage probability of up to about 30% for both cooperative cases compared to non-cooperative case (see Fig. 2).

This paper is organized as follows. Section II introduces the system model. Section III presents the main results of the paper, the coverage probabilities in the two cases described above. Section IV includes numerical evaluations of the derived expressions illustrating the gains of cooperation over the non-cooperative case. Section V concludes the paper. Throughout we denote by $\|\mathbf{u}\|_p$ the L^p-norm, $p > 1$, of a vector $\mathbf{u} = (u_1, u_2, \ldots, u_n) \in \mathbb{R}^n$, i.e., $\|\mathbf{u}\|_p = \left(\sum_{i=1}^{n} |u_i|^p\right)^{1/p}$, and we drop the subscript p in the special case $p = 2$ of Euclidean distance.

II. SYSTEM MODEL

We consider a heterogeneous wireless network composed by K independent network tiers of BSs with different deployment densities and transmit powers. It is assumed that the BSs belonging to the ith tier have transmit power P_i and are spatially

distributed according to a two-dimensional homogeneous PPP Φ_i of density λ_i, $i = 1, \ldots, K$. We focus on a typical user located at origin $(0,0) \in \mathbb{R}^2$ and assume that a subset of the total ensemble of BSs cooperate by jointly transmitting a message to this tagged receiver. In the following, we denote by $\mathcal{C} \subset \bigcup_{i=1}^{K} \Phi_i$ the set of locations of the cooperating BSs. In this setup, the received channel output at the typical receiver can be written as

$$\sum_{x \in \mathcal{C}} \frac{P_{\nu(x)}^{1/2}}{\|x\|^{\alpha/2}} h_x X + \sum_{x \in \mathcal{C}^c} \frac{P_{\nu(x)}^{1/2}}{\|x\|^{\alpha/2}} h_x X_x + Z, \qquad (1)$$

where $\nu(x)$ returns the index of network tier to which BS located at $x \in \mathbb{R}^2$ belongs, i.e. $\nu(x) = i$ iff $x \in \Phi_i$; h_x denotes the random fading coefficient between the BS located at x and the user located at the origin; $\alpha > 2$ denotes the path loss exponent; X denotes the channel input symbol that is non-coherently sent by the cooperating BSs; $\mathcal{C}^c := \bigcup_{i=1}^{K} \Phi_i \setminus \mathcal{C}$ denotes the locations of the BSs that are not in the set of cooperating BSs; X_x denotes the channel input symbol sent by the BS located at $x \in \mathcal{C}^c$; finally, $Z \sim \mathcal{CN}(0, \sigma^2)$ is a standard additive circular complex white Gaussian random variable modeling the background thermal noise at the typical receiver. Throughout the paper it is assumed that the fading coefficients $\{h_x\}$ are i.i.d. $\sim \mathcal{CN}(0,1)$ independent of everything else (i.e., Rayleigh fading assumption), a legitimate assumption in a rich scattering environment.

Assuming that the channel inputs $\{X_x\}$ and X in (1) are independent zero-mean random variables of unit variance, the SINR at the typical receiver for a given realization of the PPPs and the fading coefficients is given by

$$\text{SINR} = \frac{|\sum_{x \in \mathcal{C}} P_{\nu(x)}^{1/2} \|x\|^{-\alpha/2} h_x|^2}{\sigma^2 + \sum_{x \in \mathcal{C}^c} P_{\nu(x)} \|x\|^{-\alpha} |h_x|^2}$$

If we further assume that the network operates in the interference-limited regime, i.e., the background thermal noise power σ^2 is negligible compared to the total aggregate interference power, the resulting signal-to-interference-ratio (SIR), obtained ignoring the noise term in the SINR expression is given by

$$\text{SIR} = \frac{|\sum_{x \in \mathcal{C}} P_{\nu(x)}^{1/2} \|x\|^{-\alpha/2} h_x|^2}{\sum_{i=1}^{K} P_i I_i},$$

where we defined

$$I_i := \sum_{x \in \Phi_i \setminus \mathcal{C}} |h_x|^2 \|x\|^{-\alpha}$$

as the aggregate interference power due to the non-cooperative BSs in tier i.

The quantity of interest in this paper is the *coverage probability* P_c at the typical receiver, i.e., the probability that the SIR is greater than a given threshold θ

$$\mathsf{P}_\text{c} = \mathbb{P}(\text{SIR} > \theta). \qquad (2)$$

Assuming capacity-achieving Gaussian codebooks, the coverage probability can be directly related to the rate of communication from the cooperating BSs to the typical user. After taking the logarithm at both sides of the inequality in (2), P_c can be interpreted as the probability that an ergodic communication rate of $R(\theta) = \log_2(1 + \theta)$ is achievable.

III. Coverage Probabilities

Thus far we have made no assumption on the way the set of cooperating BSs is selected. In this section, we derive the coverage probability (2) at the typical receiver located at the origin in two cases of practical interest.

First, we consider the case where the typical receiver connects to the n BSs that result in the *strongest* average received power among the ensemble of BSs in the K network tiers. In this case $\mathcal{C}$ in (1) denotes the locations of the n BSs with strongest received power $P_{\nu(x)}\|x\|^{-\alpha}$, i.e.,

$$\mathcal{C} = \left\{ (x_1, \ldots, x_n) : \mathbf{x} = \arg \max_{\mathbf{x} \in (\bigcup_{j=1}^K \Phi_j)^n} \sum_{i=1}^n \frac{P_{\nu(x_i)}}{\|x_i\|^\alpha} \right\} \quad (3)$$

where $x_i \neq x_j$ for $i \neq j$. Notice that the cooperative BSs belong in general to different network tiers. We denote this as *Case 1*, see Fig. 1. Although not in the context of CoMP, this case has been previously considered in the literature, e.g., [17] considered a similar setup for non-cooperative homogeneous network ($K = 1$). As we will see, our result generalizes the one in [17] to the case $K > 1$. This case is applicable to wireless networks where users keep a list of the neighboring BSs with the strongest received power to initiate handoff requests.

Second, we consider a case where the typical receiver can only connect to the *nearest* BS from each network tier. In this case the set $\mathcal{C}$ is defined as

$$\mathcal{C} = \left\{ (x_1, \ldots, x_n) : x_i = \arg \max_{x \in \Phi_i} \|x\|^{-\alpha}, i \in \mathcal{I} \right\}, \quad (4)$$

where $\mathcal{I} \subseteq \{1, \ldots, K\}$ is an index set of cardinality $n \leq K$. Notice that if $j \notin \mathcal{I}$, then the typical terminal does not connect to any BS from the jth tier. This case is motivated by the fact that in practical deployments cooperation across network tiers (e.g., between a macro and pico cell) is instrumental for offloading users from the macro base stations (e.g. see [12]) and might be easier to implement than within BSs in the same network tier. We denote this as *Case 2*.

A. Case 1: Cooperation among the BSs with strongest received power

The main result for the coverage probability in this case is given in Theorem 1.

Theorem 1: Let the set $\mathcal{C}$ be defined as in (3). Then, the coverage probability P_c in (2) is

$$\int_{0 < u_1 < \ldots < u_n < \infty} \exp\left(-u_n \left(1 + 2 \frac{F(\|\tilde{\mathbf{u}}\|_{\alpha/2}^{1/2} \, \theta^{-1/\alpha})}{\|\tilde{\mathbf{u}}\|_{\alpha/2} \, \theta^{-2/\alpha}} \right) \right) d\mathbf{u}, \quad (5)$$

where $\tilde{\mathbf{u}} := \left(\frac{u_n}{u_1}, \frac{u_n}{u_2} \ldots, \frac{u_n}{u_n} \right)$ and $F(x) := \int_x^\infty \frac{r}{1+r^\alpha} dr$.

Proof: See Appendix A. ∎

Theorem 1 provides a general integral expression for the coverage probability at the typical receiver that only depends on the number of cooperating BSs n, the threshold θ, and the path-loss exponent α. Equation (5), in fact, is independent of the number of network tiers K and their respective power levels and deployment densities. A similar observation was made in [12, Eq. (3)] for non-cooperative interference-limited heterogeneous networks although in a slightly different setup. This means that in a practical deployment composed by multiple small cells, the coverage probability is independent of the intensity of BSs within each cell, contrary to the belief that a higher density of BSs leads to a greater amount of intercell interference and hence to a degraded network performance. Another implication of this result is that the same gains in coverage probability provided by cooperation in a heterogeneous network can also be achieved by a single homogeneous network, provided that an ideal backhaul network allows cooperation among spatially distributed BSs and that users can connect to the BSs that result in the highest received power. The intuition behind this result is that a variation in the number of network tiers or density of BSs leads to changes in the total received power as well as in the total aggregate interference power but the scaling of these two quantities is such that their ratio remains constant.

It should also be remarked that (5) depends on the semi-open integral defined in $F(x)$. Although in general $F(x)$ can not be solved explicitly, closed-form expressions exist for specific values of $\alpha > 1$. For example, it can be easily verified that if $\alpha = 3$, then

$$F(x) = \frac{1}{6} \log \left(1 + \frac{3x}{1 - x + x^2} \right) + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\sqrt{3}}{2x - 1} \right),$$

while

$$F(x) = \frac{1}{2} \tan^{-1}(x^{-2})$$

in the special case $\alpha = 4$.

Making use of Theorem 1, it is possible to derive an expression for the coverage probability in the case of no cooperation.

Corollary 1: In the special case $n = 1$, i.e., when the typical receiver connects to a single BS, the coverage probability P_c simplifies to

$$\frac{1}{1 + 2\theta^{2/\alpha} F(\theta^{-1/\alpha})}. \quad (6)$$

The expression for the coverage probability in Corollary 1 coincides with the one derived in [17, Theorem 2] in the special case of a homogeneous network. Therefore, in this case Theorem 1 provides a generalization of [17] to the general case $K > 1$ of a heterogeneous network.
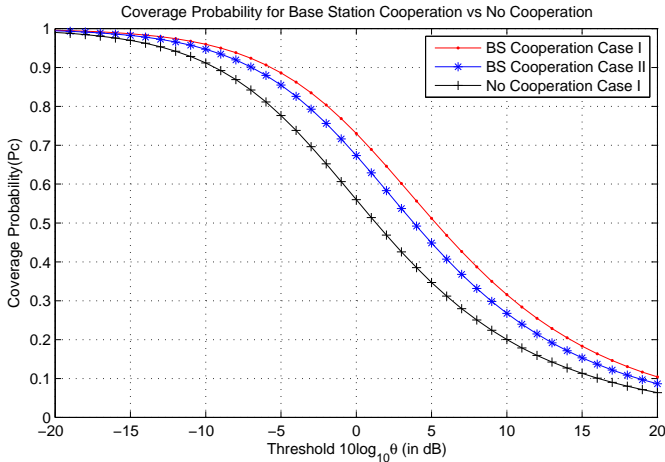
Fig. 2. Comparison of the coverage probabilities for BS cooperation Case 1 and 2 vs no cooperation against the threshold in dB, under the assumption that $\alpha = 4$, $K = 2$, $P_1 = 25$, $P_2 = 1$, $\lambda_1 = (500^2\pi)^{-1}$, and $\lambda_2 = 5(500^2\pi)^{-1}$.
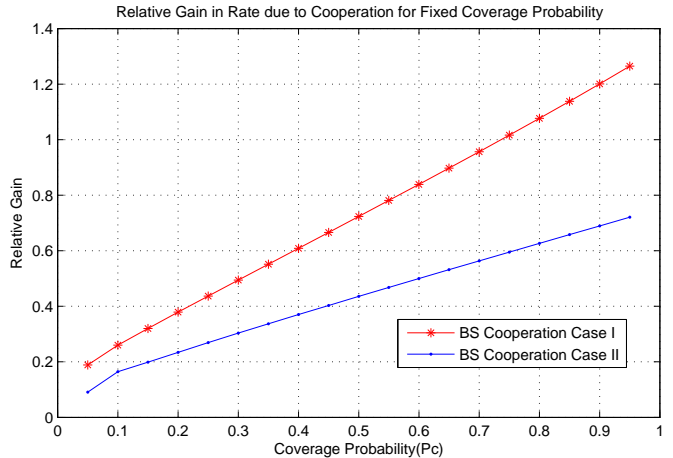

Fig. 3. Relative rate gain compared to no cooperation against fixed coverage probabilities for Case 1 and Case 2 with $K = 2$, $\alpha = 4$, $P_1 = 25$, $P_2 = 1$, $\lambda_1 = (500^2\pi)^{-1}$, and $\lambda_2 = 5(500^2\pi)^{-1}$

B. Case 2: Cooperation among the closest BS to the receiver in each network tier

The main result for the coverage probability in this case is given in Theorem 2.

Theorem 2: Let the set $\mathcal{C}$ be defined as in (4). Then, the coverage probability $\mathsf{P_c}$ in (2) is

$$\int_{\mathbb{R}^n_+} \prod_{i=1}^{K} \mathcal{L}_i \left(\frac{\theta P_i}{\sum_{j\in\mathcal{I}} P_j d_j^{-\alpha}} \right) \prod_{i\in\mathcal{I}} f_{D_i}(d_i)\, \mathrm{d}\mathbf{d} \qquad (7)$$

where $f_{D_i}(d_i) = 2\pi\lambda_i d_i e^{-\pi\lambda_i d_i^2}$ for $d_i \geq 0$ and

$$\mathcal{L}_i(s) = \begin{cases} e^{-2\pi\lambda_i s^{2/\alpha} F(d_i s^{-1/\alpha})}, & i \leq n, \\ e^{-\pi\lambda_i s^{2/\alpha} \mathrm{sinc}^{-1}(2/\alpha)}, & i > n. \end{cases}$$

Proof: See Appendix B. ∎

Note that unlike (5) the expression for the coverage probability given in Theorem 2 does depend on the parameters describing each network tier, namely the spatial densities and the available transmit powers at each BS. As an application of Theorem 2, it is possible to derive an expression for the coverage probability in the case of no cooperation, i.e., $n = 1$. Without loss of generality, suppose that $\mathcal{I} = \{1\}$, so the typical receiver only connects to the nearest BS from network tier 1. Then, we have the following result.

Corollary 2: In the special case where $\mathcal{I} = \{1\}$, (7) simplifies to

$$\frac{1}{1 + \theta^{2/\alpha} 2F(\theta^{-1/\alpha}) + \frac{\theta^{2/\alpha}}{\mathrm{sinc}(2/\alpha)} \sum_{i=2}^{K} \frac{\lambda_i}{\lambda_1} \frac{P_i^{2/\alpha}}{P_1^{2/\alpha}}}. \qquad (8)$$

By comparing (6) with (8), notice that the former does not contain the sum of terms dependent on powers and densities

that appears in the latter. Since each term in the sum is nonnegative, it follows that the coverage probability in (6) is in general larger than the one in (8). This is consistent with the fact that (6) was derived under the assumption that the user connects to the strongest BS in the entire network and not only within network tier 1. Clearly, (8) is equal to (6) when $K = 1$.

IV. NUMERICAL RESULTS

In this section, we present numerical evaluations of the integral expressions for the coverage probability derived in Section III. We focus on the special case of two network tiers consisting of a macro–tier overlaid with a pico–tier. Specifically, we assume that $\alpha = 4$ and that the first tier has spatial intensity $\lambda_1 = (500^2\pi)^{-1}$ and available power $P_1 = 25$, while the second tier has spatial intensity $\lambda_2 = 5\lambda_1$ and available power $P_2 = P_1/25$.

Fig. 2 illustrates the effect of the SIR threshold θ on the coverage probability (2). By comparing the performance of the cooperative schemes in Case 1 and Case 2 to the baseline case of no cooperation in (6), we observe that around 0 dB cooperation yields relative gains in coverage probability of up to about 30% compared to non-cooperative case in Case 1, while the relative gain in Case 2 is of about 20%.

As pointed out at the end of Section II, the coverage probability can be directly related to the ergodic rate of communication from the cooperating BSs to the typical receiver. By replacing θ by $2^R - 1$ in (5), (6), and (7), setting the resulting expression equal to $\mathsf{P_c}$ and solving for R, the expressions derived in Section III yield the maximum ergodic rates $R^{(1)}(\mathsf{P_c})$, $R^{(n)}(\mathsf{P_c})$, and $R^{(2)}(\mathsf{P_c})$, that can be achieved with probability $\mathsf{P_c}$ in Case 1, no-cooperation, and Case 2, respectively. Fig. 3 illustrates the relative rate *gain* for Case j, $j = 1, 2$, over the no-cooperation case, which is computed as

$$\frac{R^{(j)}(\mathsf{P_c}) - R^{(n)}(\mathsf{P_c})}{R^{(n)}(\mathsf{P_c})}, \quad j = 1, 2.$$
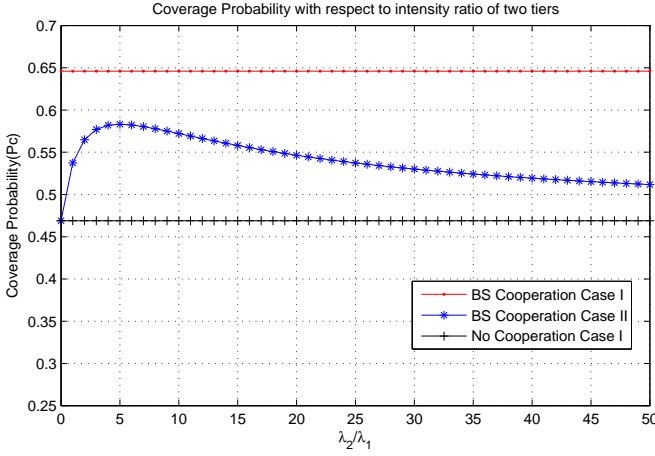
Fig. 4. Comparison of the coverage probabilities for BS cooperation Case 1 and 2 vs no cooperation against the intensity ratio of two tiers with $K = 2$, under the assumption that $\alpha = 4$, $P_1 = 25$, $P_2 = 1$, $\lambda_1 = (500^2\pi)^{-1}$, and $\theta = 2$ dB

Notice that the rate gains of cooperation increase with P_c and when $P_c \approx 1$ the relative gain is more than 100% in Case 1 and more than 60% in Case 2.

Finally, Fig. 4 illustrates the coverage probability as a function of the ratio λ_2/λ_1, under the assumption that λ_1 is kept fixed at $(500^2\pi)^{-1}$ and $\theta = 2$ dB. As expected, the coverage probability in Case 1 does not depend on the intensity of the tiers and yields a gain in coverage probability of about 35% compared to no cooperation case. Notice that the coverage probability in Case 2 reaches a maximum at a critical value for λ_2, after which it decreases because of the net increase in interference in the entire network. Also notice that the curve for Case 2 always lies between the ones for Case 1 and for the no-cooperation case and it can be proved that the lower bound is attainable in the special case where $\lambda_2 = 0$ and $\lambda_2 \to \infty$.

V. Conclusion

In this paper, we considered the problem of joint transmission in heterogeneous cellular networks. Using tools from stochastic geometry, we derived an integral expression for the coverage probability in two cases: 1) a typical receiver located at the origin can combine the received signal from the BSs with maximum average received power strength, and 2) the set of cooperating BSs includes the BS in each network tier that is closest to the origin. The analysis presented in this paper assumes no channel state information at the transmitters and that all BSs and the receiving user are equipped with a single antenna. Future work includes the generalization to the MIMO case as well as to the case where cooperating BSs have partial or perfect channel state information. Finally, the presented results do not take into account the possibility of linear precoding at the cooperating BSs and the cost of establishing cooperation among BSs belonging to different tiers. This topic is currently being investigated.

For every $i = 1, \ldots, K$, let $\Xi_i = \{\|x\|^\alpha/P_i, x \in \Phi_i\}$ denote the normalized path loss between each BS in Φ_i and the typical receiver located at the origin. By the mapping theorem [18, Theorem 2.34], Ξ_i is a non-homogeneous PPP with intensity $\lambda_i(x) = \lambda_i \frac{2\pi}{\alpha} P_i^{2/\alpha} x^{2/\alpha-1}$, $x \in \mathbb{R}$. From the independence of the PPPs $\Phi_1, \cdots, \Phi_K$, it follows that $\Xi_1, \cdots, \Xi_K$ are also independent and thus the process $\Xi = \bigcup_{i=1}^K \Xi_i$ is a non-homogeneous PPP with density $\lambda(x) = \sum_{i=1}^K \lambda_i(x)$. Without loss of generality, suppose that the elements of Ξ are re-indexed in increasing order of magnitude, such that $\|x_1\|^\alpha/P_{\nu(x_1)} \leq \|x_2\|^\alpha/P_{\nu(x_2)} \leq \|x_3\|^\alpha/P_{\nu(x_3)} \leq \cdots$, and define $\gamma_k : \|x_k\|^\alpha/P_{\nu(x_k)}$ as the normalized path loss between the typical receiver and the k-th BS in the ordered list. Since the typical receiver connects to the n BSs with strongest average received power, it follows that the normalized path loss of the cooperating BSs in $\mathcal{C}$ is given by $\gamma = \{\gamma_1, \ldots, \gamma_n\}$. Then, by defining $g_k := |h_{x_k}|^2$ and $I = \sum_{k>n} g_k \gamma_k^{-1}$, the coverage probability in (2) can be re-written as:

$$
\begin{aligned}
P_c &= \mathbb{P}\left(\left|\sum_{k\leq n} \gamma_k^{-1/2} h_k\right|^2 > \theta \sum_{k>n} g_k \gamma_k^{-1}\right) \\
&\stackrel{(a)}{=} \mathsf{E}_{\gamma,I}\left(\exp\left(-\frac{\theta I}{\sum_{k=1}^n \gamma_k^{-1}}\right)\right) \\
&\stackrel{(b)}{=} \mathsf{E}_{\gamma}\left(\mathcal{L}\left(\frac{\theta}{\sum_{k=1}^n \gamma_k^{-1}}\right)\right) \\
&= \int_{\gamma_1 < \ldots < \gamma_n} \mathcal{L}\left(\frac{\theta}{\sum_{k=1}^n \gamma_k^{-1}}\right) f_\Gamma(\gamma)\, d\gamma, \quad (9)
\end{aligned}
$$

where (a) follows from the fact that $|\sum_{k\leq n} \gamma_k^{-1/2} h_k|^2 \sim \exp\left(\left(\sum_{k=1}^n \gamma_k^{-1}\right)^{-1}\right)$, because of the Rayleigh fading assumption, and the fact that $(h_1, \ldots, h_n)$ are mutually independent, while (b) makes use of the Laplace transform of I $\mathcal{L}(s) = \mathsf{E}(e^{-sI})$.

The joint distribution of γ can be obtained by following similar steps as in the derivation of the joint distribution of the nearest points in a homogeneous PPP [19]. It can be easily verified that for any $0 < \gamma_1 < \ldots < \gamma_n < \infty$,

$$
f_\Gamma(\gamma) = \left(\pi\delta\sum_{i=1}^K \lambda_i P_i^\delta\right)^n e^{-\pi\sum_{i=1}^K \lambda_i P_i^\delta \gamma_n^\delta} \prod_{i=1}^n \gamma_i^{\delta-1} \quad (10)
$$

where $\delta = 2/\alpha$.

Next, notice that the Laplace transform of I can be re-written as follows

$$
\begin{aligned}
\mathcal{L}(s) &= \mathsf{E}\left(e^{-sI}\right) \\
&\stackrel{(a)}{=} \mathsf{E}_\Xi\left(e^{-s\sum_{k>n} g_k \gamma_k^{-1}}\right) \\
&= \mathsf{E}_\Xi\left(\prod_{k>n} \mathsf{E}_{g_k}\left(e^{-sg_k\gamma_k^{-1}}\right)\right) \\
&\stackrel{(b)}{=} \mathsf{E}_\Xi\left(\prod_{k>n} \frac{1}{1 + s\gamma_k^{-1}}\right)
\end{aligned}
$$

$$\overset{(c)}{=} \exp\left(-\int_{\gamma_n}^{\infty}\left[1-\frac{1}{1+sx^{-1}}\right]\lambda(x)\,\mathrm{d}x\right)$$

$$\overset{(d)}{=} \exp\left(-2\pi s^{2/\alpha}F((\gamma_n s^{-1})^{1/\alpha})\sum_{i=1}^{K}\lambda_i P_i^{2/\alpha}\right), \quad (11)$$

where (a) uses the definition of I; (b) uses the fact that the moment generating function of an exponential random variable with parameter a is equal to $(1-t/a)^{-1}$; (c) is due to the probability generating functional for a PPP [18, Theorem 4.9]; (d) follows from the transformation $x = st^{\alpha}$ and the definition of $F(x)$.

Finally, after substituting (10) and (11) into (9) and performing the change of variable $u_i = \sum_{j=1}^{K}\lambda_j P_j^{\delta}\pi x_i^{\delta}$, $i = 1,\dots,n$, it can be easily verified that (9) reduces to (5).

APPENDIX B
PROOF OF THEOREM 2

Without loss of generality, permute the indices of the K network tiers such that $\mathcal{I} = \{1,\cdots,n\}$. For every $i \in \mathcal{I}$, let d_i denote the distance between the cooperating BS from the ith network tier and the typical receiver. Then, (2) can be re-written as

$$\mathsf{P_c} = \mathsf{E_{I,d}}\left(\exp\left(-\theta\frac{\sum_{i=1}^{K}P_i I_i}{\sum_{i\in\mathcal{I}}P_i d_i^{-\alpha}}\right)\right)$$

$$= \mathsf{E_d}\left(\prod_{i=1}^{K}\mathcal{L}_i\left(\frac{\theta P_i}{\sum_{j\in\mathcal{I}}P_j d_j^{-\alpha}}\right)\right), \quad (12)$$

where the second equality is written in terms of the Laplace transform of I_i and follows from the mutual independence of the PPPs $\{\Phi_i\}$, which implies the independence of $I_1,\dots,I_K$. By following similar steps as in (11), we write

$$\mathcal{L}_i(s) = \mathsf{E}\left(e^{-sI_i}\right)$$

$$= \mathsf{E}_{\Phi_i}\left(e^{-s\sum_{x\in\Phi_i\setminus x_i}g_x\|x\|^{-\alpha}}\right)$$

$$= \mathsf{E}_{\Phi_i}\left(\prod_{x\in\Phi_i\setminus x_i}\mathsf{E}_{g_x}\left(e^{-sg_x\|x\|^{-\alpha}}\right)\right)$$

$$= \mathsf{E}_{\Phi_i}\left(\prod_{x\in\Phi_i\setminus x_i}\frac{1}{1+s\|x\|^{-\alpha}}\right)$$

$$= \exp\left(-\int_{\|x\|>d_i}\left[1-\frac{1}{1+s\|x\|^{-\alpha}}\right]\lambda_i\,\mathrm{d}x\right). \quad (13)$$

Next, notice that for every $i > n$, x_i is the null set and hence $d_i = 0$. Therefore, by switching to polar coordinates

$$\mathcal{L}_i(s) = \exp\left(-2\pi\lambda_i\int_0^{\infty}\frac{sr^{-\alpha}}{1+sr^{-\alpha}}r\,\mathrm{d}r\right)$$

$$= \exp\left(-\frac{\pi\lambda_i s^{2/\alpha}}{\mathrm{sinc}(2/\alpha)}\right). \quad (14)$$

On the other hand, notice that $|x| \geq d_i$ for every $x \in \Phi_i \setminus x_i$ and $i \leq n$, so in this case

$$\mathcal{L}_i(s) = \exp\left(-2\pi\lambda_i\int_{d_i}^{\infty}\frac{sr^{-\alpha}}{1+sr^{-\alpha}}r\,\mathrm{d}r\right)$$

$$= \exp\left(-2\pi\lambda_i s^{2/\alpha}F(d_i s^{-1/\alpha})\right). \quad (15)$$

Finally, substituting the values of $\mathcal{L}_i(s)$ into (12) and using the fact that the nearest distances d_i's in a PPP are Rayleigh distributed [18, Eq. (2.12)] gives the desired result as stated in (7).

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